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Compliance Calibration of Specimens Used in the R-Curve Practice

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ABSTRACT: The compliance calibrations for the compact (CS) and crack-line-wedge-loaded (CLWL) specimens have been determined by experimental measurements and by boundary-collocation analysis. The CS and CLWL specimen configurations were modeled more accurately than those used in previous analytical investigations. Polynomial expressions for the compliance at various stations along the crack line for the CS and CLWL specimens are presented. The compliance calibrations for the center-crack tension (CCT) specimen have been determined theoretically by boundary-collocation and finite-element analysis. The calculated compliance values for the CCT specimen are compared with values obtained from the Irwin-Westergaard expression and from a modification to the Irwin-Westergaard expression proposed by Eftis and Liebowitz. The Eftis-Liebowitz expression was found to be in good agreement (±2 percent) with both analyses for crack aspect ratios up to 0.8 and for gage half-span to specimen width ratios up to 0.5.

KEY WORDS: fracture properties, mechanical properties, calibration, stresses, strains, crack propagation

The compliance relationships presented in the 1974-1975 "Proposed Method for R-Curve Determination" [*1*][3] for the three recommended specimens were taken from information considered to be the best available at the time. Solutions for the center-cracked tension (CCT), compact (CS), and crack-line-wedge-loaded (CLWL) specimens were obtained from complex analytical solutions or boundary collocation analyses. Assumed loading and boundary conditions were slightly variant from those

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used in practice. The CCT specimen compliance was obtained from the analytical solution for an infinite plate with a periodic colinear array of cracks [*2*]. Although the accuracy was known to be suspect for large crack aspect ratios, $2a/W$, it was chosen for its versatility in handling variable gage spans. The CS and CLWL compliance relationship was obtained from a boundary collocation analysis of a configuration without pin-loading holes and subjected to only externally applied loads [*3,4*]. Since both specimens have the standard compact specimen shape and share a common load line, the compliance was considered to be common to both specimens. Experimental verification for the CS specimen only was provided by W. F. Brown, Jr. [*5*].

Recently, a finite element computation made on the CLWL specimen by Ratwani [*6*] indicated a potential problem in using a common (CS and CLWL) compliance record. This provided the stimulus to reexamine the complete compliance package given in Ref *1*. An ad hoc committee was formed within ASTM E24.01.04 to make this study, and the present report represents the findings of this group.

Analysis of the CS and CLWL Specimens

In order to represent the configuration and loading conditions for CS and CLWL specimens more accurately, an improved method of boundary collocation [*7,9*] was herein applied in Ref *10* to the two dimensional stress analysis of these specimens. The improved method included the effects of the pin-loaded holes where previous collocation analyses did not include these holes. The improved solutions were based on the complex variable method of Muskhelishvili [*11*]. The complex-series stress functions for the specimens were constructed so that the boundary conditions on the crack surfaces were satisfied exactly, while the conditions on the external boundary and the circular-hole boundaries were satisfied approximately. The present paper presents only the solutions for plane-stress displacements ($\mu = 0.3$). The plane-strain displacements can be obtained from the plane-stress displacements for a given position, x, along the crack plane by multiplying the plane-stress displacements by $[(1 + \kappa)(7 - \kappa)/16]$ [*9*].

For the CS and CLWL specimen configurations, the model consists of a semi-infinite crack located along the x-axis in an infinite plate subjected to a uniformly distributed line load, P, as shown in Fig. 1. The dashed lines L_1 (rectangular) and L_2 (circular holes) define the boundaries of the specimen. The boundaries L_1 and L_2 may have any simple shape and may be subjected to any boundary conditions which are symmetric about the x-axis. Further details on the analysis can be obtained in Ref *10*. Figures 2 and 3 show the locations along the crack-line (V_0, V_1, V_{LL}, and V_2) for

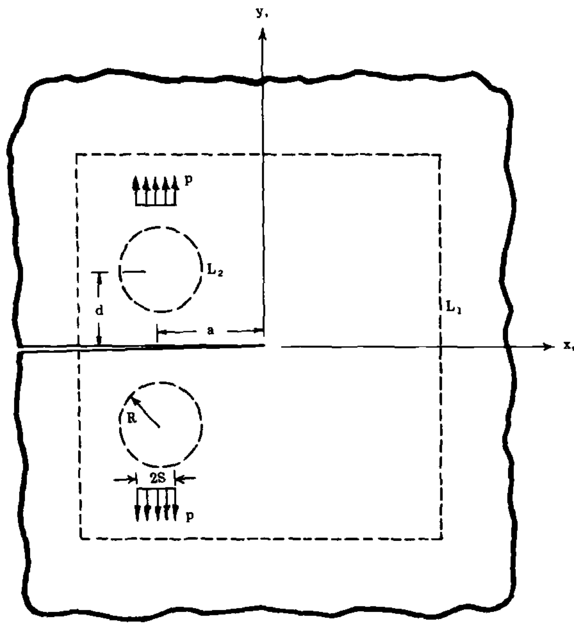

FIG. 1—*Semi-infinite crack in an infinite plate subjected to a uniformly distributed internal line load.*

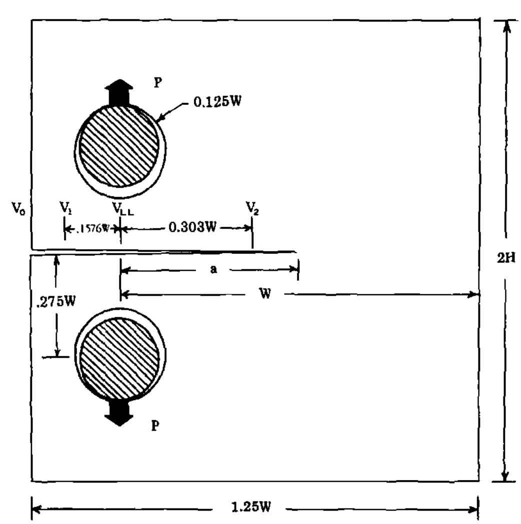

FIG. 2—*CS specimen subjected to pin loading.*

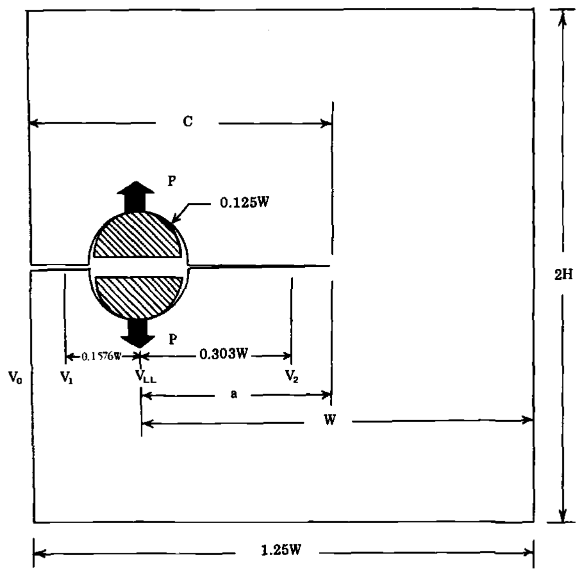

FIG. 3—*CLWL specimen subjected to pin loading.*

which the compliance has been calculated. These values are listed in Tables 1 and 2.

Experimental calibrations were made on CS and CLWL specimens having W dimension of 8.25 in. (209 mm). The materials used were ¼-in. (6-mm) thick 2024-T3 aluminum, ¼-in. (6-mm) thick 7075-T6 aluminum (CLWL only), and 0.084-in. (2-mm) thick PH15-7Mo steel (CLWL only). Displacement was measured at locations V_1 and V_2 using the precision clip gages, shown in Fig. 4, with an initial span of 0.8 in. (20 mm). They are enlarged versions of the NASA type clip gage, specifically designed for

TABLE 1—*Crack-line compliance for the compact specimen as a function of* a/W *for plane-stress conditions* (μ = 0.3).

$\frac{a}{W}$	$\frac{EB2V_0}{P}$	$\frac{EB2V_1}{P}$	$\frac{EB2V_{LL}}{P}$	$\frac{EB2V_2}{P}$
0.30	24.90	21.24	14.28	...
0.35	29.89	25.78	18.09	4.64
0.40	36.18	31.51	22.86	8.05
0.45	44.23	38.83	28.96	12.07
0.50	54.76	48.44	36.99	17.24
0.55	69.00	61.44	47.90	24.26
0.60	89.04	79.78	63.35	34.26
0.65	118.7	107.0	86.36	49.27
0.70	165.5	150.0	122.8	73.29

TABLE 2—*Crack-line compliance for the CLWL specimen as a function of a/W for plane-stress conditions* (μ = 0.3).

$\frac{a}{W}$	$\frac{EB2V_0}{P}$	$\frac{EB2V_1}{P}$	$\frac{EB2V_{LL}}{P}$	$\frac{EB2V_2}{P}$
0.30	19.90	18.14	15.51	. . .
0.35	25.12	22.80	19.37	4.80
0.40	31.58	28.64	24.19	8.25
0.45	39.73	36.05	30.34	12.29
0.50	50.33	45.70	38.42	17.47
0.55	64.59	58.73	49.36	24.49
0.60	84.63	77.07	64.85	34.48
0.65	114.3	104.2	87.91	49.47
0.70	161.0	147.2	124.0	73.45

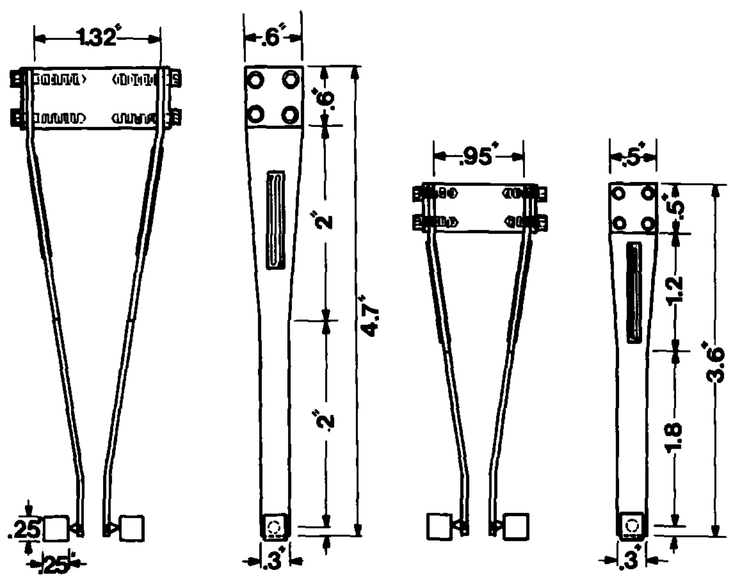

FIG. 4—*Front and side views of* V_1 *and* V_2 *clip gages.*

measuring the large displacements encountered in R-curve work. All specimens were aligned carefully and restrained from buckling using antibuckling cover plates. Friction from the antibuckling guides was prevented through the use of ball bearing pads sandwiched between the specimen and guides. Each experimental value given in the present report was averaged from at least three replicate determinations.

Figures 5 and 6 show the compliance, $EB2v1/P$, for the CS and CLWL specimens as a function of crack-length to width ratio. The curves represent calculated values, and the symbols are experimental measurements.

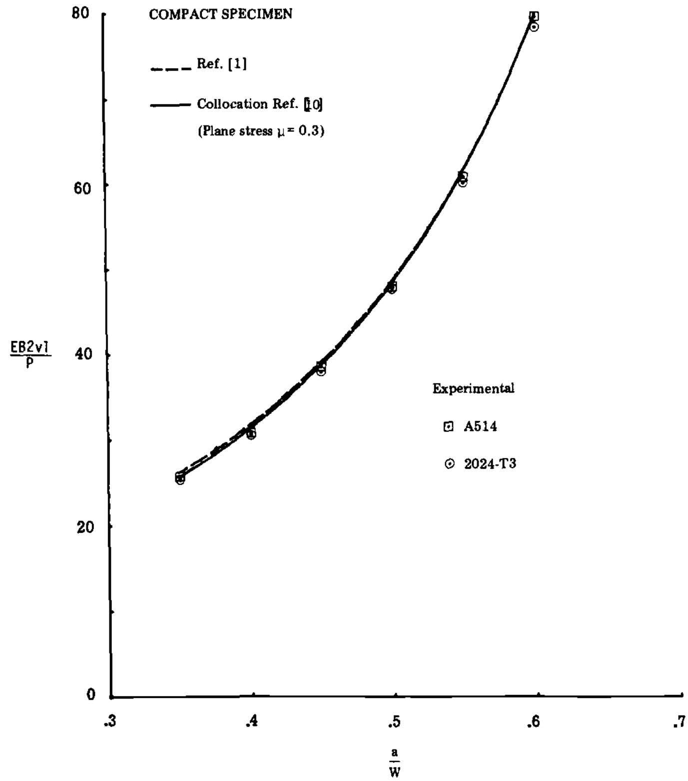

FIG. 5—*Comparison of experimental and theoretical compliance at the* V1 *location for the CS specimen.*

The solid curves represent the collocation results [*10*], and the dashed curves show the values used in the proposed R-curve practice. The collocation results in Fig. 5 are only slightly lower (2 percent) than the dashed curve at small a/W, indicating that the early results of Ref *4* for all intents and purposes were satisfactory for use with the CS specimen. For the CLWL specimen the collocation results are considerably lower (13 to 14 percent) than the dashed curve representing the formerly recommended values. The calculated and experimental values are in essential agreement and demonstrate the fundamental difference in compliance behavior between the two loading configurations.

Figure 7 shows the ratio between v_1 and v_2 displacements for the CS and CLWL specimens. This relationship is used in the double compliance method of crack length determination and again a fundamental difference which also had not been envisioned is shown between the two specimens. The solid and open symbols represent the experimental data. The solid curves show the collocation results [*10*], and the dashed curve shows the

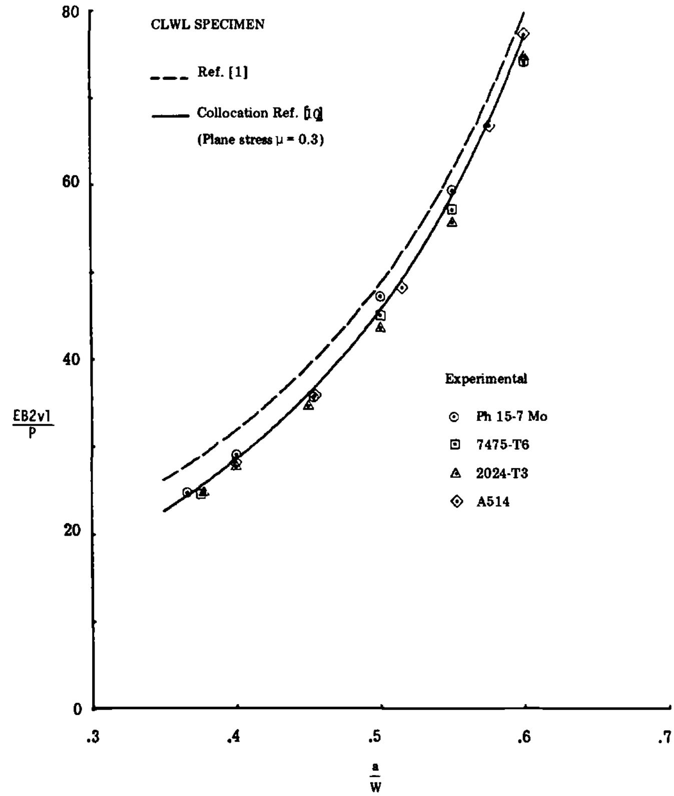

FIG. 6—*Comparison of experimental theoretical compliance at the* V1 *location for the CLWL specimen.*

v_1/v_2 displacement ratios used in the R-curve method. In this case Ref *1* values compare favorably with the new values because they were obtained initially by experimental calibration of CLWL specimens.

Polynomial expressions of the following form were fit to the boundary collocation results

$$EB2v/P = A_0 + A_1(a/W) + A_2(a/W)^2 + A_3(a/W)^3 + A_4(a/W)^4$$

The coefficients Ai for both specimen types for the four locations considered are given in Table 3. These expressions are within ±0.4 percent of the collocation results for $0.35 \leqslant a/W \leqslant 0.6$. Experimental displacement measurements had been made over a span of 0.8 in. instead of on the crack line as per the calculations. This was determined in Ref *10* to produce less than ½ percent error in the displacements and for all practical purposes relevant to experimental accuracy can be ignored.

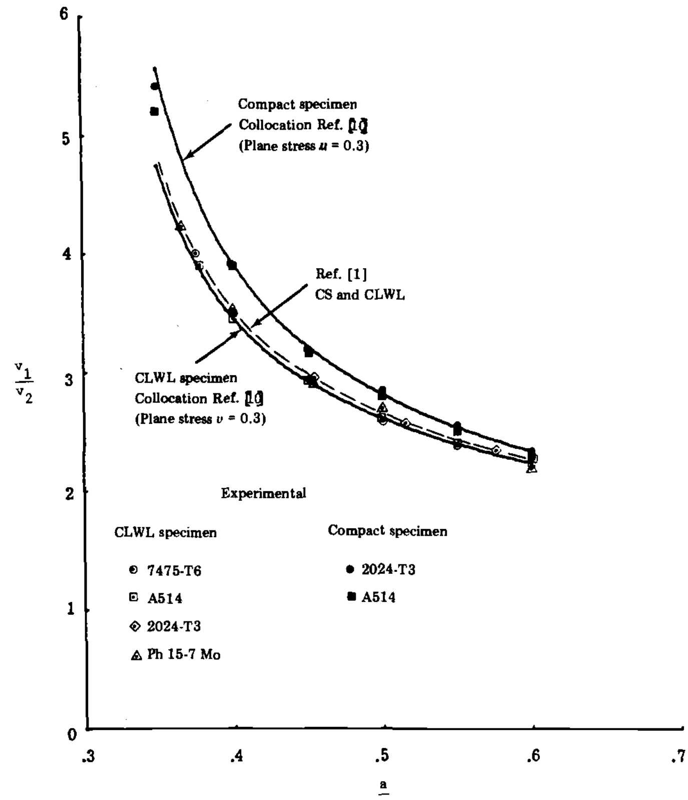

FIG. 7—*Comparison of experimental and theoretical displacement ratio* (V1/V2) *for the CS and CLWL specimens.*

TABLE 3—*Polynomial expression coefficients for the crack-line compliance of CS and CLWL specimens (plane-stress conditions with μ = 0.3).*

Specimen Type	Location	A_0	A_1	A_2	A_3	A_4
Compact	V_0	120.7	−1065.3	4098.0	−6688.0	4450.5
	V_1	103.8	−930.4	3610.0	−5930.5	3979.0
	V_{LL}	84.9	−794.0	3082.0	−5074.5	3406.0
	V_2	5.75	−190.3	1081.5	−2150.5	1680.5
CLWL	V_0	109.5	−1021.6	3986.5	−6553.0	4386.0
	V_1	101.9	−948.9	3691.5	−6064.0	4054.0
	V_{LL}	92.8	−843.2	3210.0	−5210.0	3455.0
	V_2	6.48	−198.7	1117.0	−2207.5	1712.5

$EB2v/P = A_0 + A_1(a/W) + A_2(a/W)^2 + A_3(a/W)^3 + A_4(a/W)^4$
Accuracy: ±0.4% $0.35 \leq a/W \leq 0.6$

Analysis of the CCT Specimen

Boundary Collocation

For a CCT specimen configuration, the boundary collocation analysis [10] considered a crack located along the x-axis in an infinite plate as shown in Fig. 8. The dashed lines, denoted L, define the boundary of the specimen. The boundary L, may have any simple shape (symmetric about the x- and y-axes) and be subjected to any boundary conditions which are also symmetric about the x- and y-axes. The stress functions for this configuration automatically satisfy stress-free boundary conditions on the crack surfaces, and the boundary conditions on L were satisfied approximately. In the following section, the plane-stress displacements ($\mu = 0.3$) are presented for various gage half-span to width ratios, (Y/W), along the centerline of the specimen. These displacements are given in Table 4.

Finite Element Analysis

An ANSYS general purpose finite element program was applied to the two dimensional stress analysis of the CCT specimen with 8-noded isoparametric elements. The intended objective of this finite element analyses was to obtain the elastic compliance as a function of crack length to width ratio under the plane-stress state at different centerline locations of a

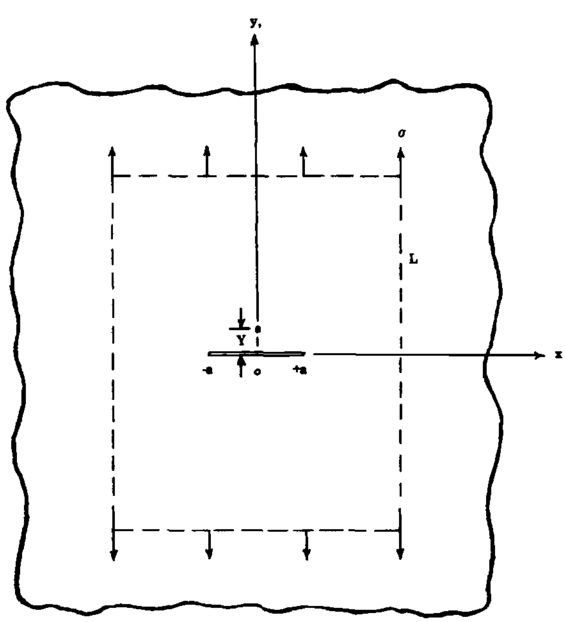

FIG. 8—*A crack in an infinite plate.*

TABLE 4—*Centerline compliance,* E2v/σW, *for the CCT specimen as a function of* Y/W *and* 2a/W *for plane-stress conditions* (μ = *0.3*).

Boundary Collocation								
Y/W \ 2a/W	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0	0.201	0.410	0.635	0.886	1.182	1.548	2.037	2.761
0.25	0.536	0.638	0.801	1.019	1.298	1.658	2.145	2.870
0.50	1.021	1.086	1.197	1.359	1.583	1.890	2.328	3.003
0.75	1.517	1.571	1.665	1.804	2.002	2.282	2.690	3.335
1.00	2.016	2.067	2.156	2.289	2.479	2.751	3.150	3.787

CCT specimen. In order to account for the singularity behavior, a high degree of grid refinement with 391 nodes and 732 degrees of freedom was used in the region of the crack tip. This accommodates high-stress gradient and can provide adequate elastic displacement or compliances for a given crack length to width ratio at different centerline spans. The fine mesh refinement in the crack-tip region may not be academically desirable but can be technically acceptable for simple geometries (only one quarter of CCT specimen is idealized for symmetry reasons) and loading of the CCT specimen. The mesh idealization was made in accordance with a calibration specimen of 16 in. (406 mm) width with uniform applied load at $1.1W$ away from the crack plane. The nodal point displacement along the centerline for a given load was used for evaluating the elastic compliance. The crack length to width ratio range of $0 \leqslant 2a/W \leqslant 0.56$ was studied under the plane-stress state with $\mu = 0.33$, and the results are reported in Table 5.

Figure 9 shows the compliance, $E2v/\sigma W$, for three values of Y/W. The collocation and finite element results are shown as data points. The curves show the calculated relationship as set forth in the early version of the R-curve method, taken from Irwin [*2*].

TABLE 5—*Centerline compliance,* E2v/σW, *for the CCT specimen as a function of* Y/W *and* 2a/W *for plane-stress conditions.*

Finite Element						
Y/W \ 2a/W	0.1	0.2	0.3	0.4	0.5	0.5625
0.00006	0.200	0.415	0.640	0.886	1.182	1.400
0.250	0.540	0.634	0.806	1.016	1.298	1.520
0.50	0.980	1.042	1.162	1.320	1.548	1.732
0.75	1.494	1.548	1.644	1.774	1.984	2.142
1.0	2.012	2.060	2.150	2.280	2.470	2.630

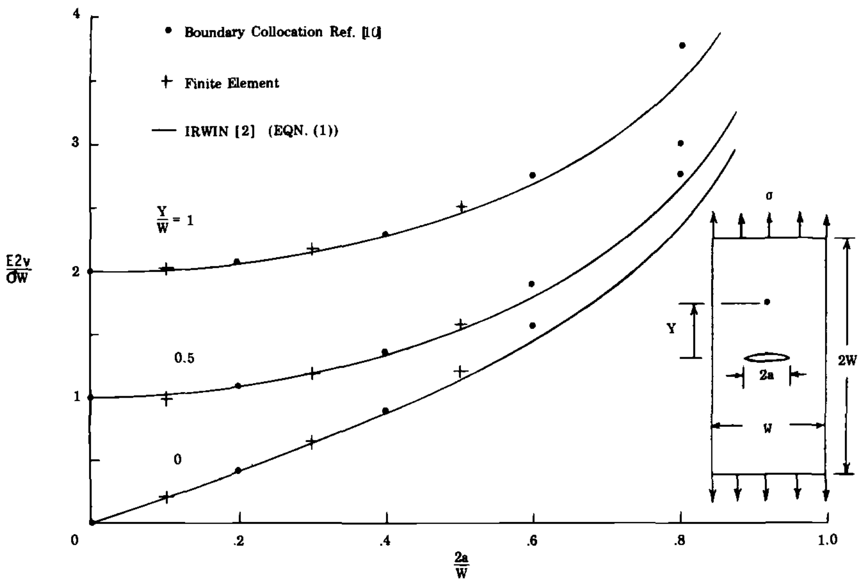

FIG. 9—*Comparison of compliance from the collocation and finite element analysis and Irwin's equation for the CCT specimen.*

$$\frac{E2v}{\sigma W} = 2\left[\frac{2W}{\pi Y}\cosh^{-1}\left(\frac{\cosh \pi Y/W}{\cos \pi a/W}\right) - \frac{1+\mu}{\left[1+\left(\frac{\sin \pi a/W}{\sinh \pi Y/W}\right)^2\right]^{1/2}} + \mu\right]\frac{Y}{W} \quad (1)$$

Equation 1 agrees within 3 percent of the analytical results for $2a/W$ less than 0.4 for all Y/W considered. However, the agreement becomes unacceptable at larger $2a/W$. Eftis and Liebowitz [*12*] proposed a modification to Eq 1 to account for the effects of finite specimen width which is given by

$$\frac{E2v}{\sigma W} = [\pi a/W \,/\, \sin(\pi a/W)]^{1/2} \times \text{Eq 1} \quad (2)$$

Figure 10 shows a comparison between the results of Eq 2 and the analytical results. There is good agreement in the range $0 \leqslant Y/W \leqslant 0.5$ and for $0 \leqslant 2a/W \leqslant 0.8$.

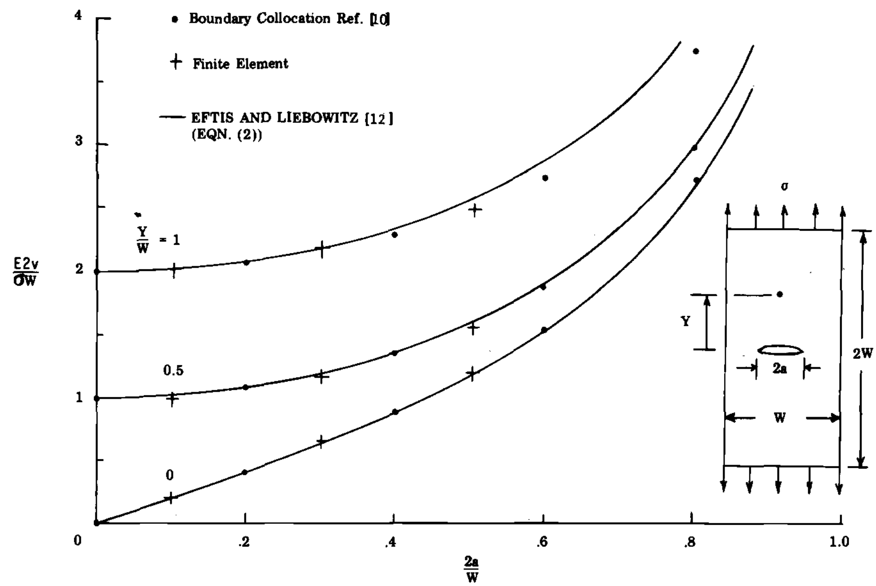

FIG. 10—*Comparison of compliance from the collocation analysis and the Eftis-Liebowitz equation for the CCT specimen.*

Experimental Measurements

Experimental calibrations were made relating the elastic compliance to crack aspect ratio ($2a/W$) for a 16-in (406-mm) wide, 0.249-in. (6.4-mm) thick, 7075-T6 aluminum CCT specimen. A modified SR-4 gage, accurate within 1 percent of the displacement range, was used for displacement measurements at a $Y/W = 0.3478$ location centered on the $2a/W$ dimension. The experimentally measured compliance data were determined by averaging at least three repeated measurements of deflection versus applied load slopes in the elastic range.

Figure 11 and Table 6 show the comparison between experimental elastic compliance data, the analytical results of Eqs 1 and 2, and finite element data for $Y/W = 0.3478$. The compliance as predicted from Eq 2 and the experimental compliance values as obtained from the calibration results are in essential agreement over all crack aspect ratios. Therefore, the compliance data of both finite element and boundary collocation calculations and experimental calibrations at $Y/W = 0.3478$ support the use of Eq 2 for all Y/W less than 0.5.

Conclusions and Comments

The analytical approaches of boundary collocation and finite element

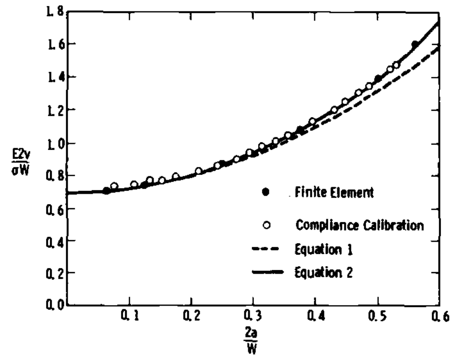

FIG. 11—*Comparison of compliance calibrations and analytical compliance data of a CCT specimen at* Y/W = *0.3478 location.*

TABLE 6—*Comparison between finite element, Eq 2, and experimental compliance for 16-in.-wide (406-mm) CCT specimen with measurements taken at* Y/W = *0.3478.*

$E2v/\sigma W$ \ $2a/W$	0.0625	0.125	0.250	0.375	0.500	0.5625
Finite element	0.7075	0.7397	0.8660	1.0788	1.3888	1.5897
Eftis and Liebowitz	0.7064	0.7387	0.8670	1.0810	1.3909	1.5909
Test	0.723	0.760	0.878	1.089	1.396	1.586

analysis were applied to the compliance problems of the CCT, CS, and CLWL specimens. It was determined that despite the many common features of the CS and CLWL specimens, they have distinctly different compliance character. This was confirmed experimentally.

Review of the CCT specimen situation by the analytical approached demonstrated a need for a modification to the well known Irwin-Westergaard expression. The development of Eftis and Liebowitz was found to be more suitable for use under most practical experimental conditions.

The Task Group on R-curves under the direction of ASTM Committee E-24.01 has produced a proposed recommended practice for R-curve determination which represents the consensus of practice from those who have held an interest in R-curve work. As this report demonstrates, the status of recommendations can change as new information is developed and the ASTM procedure for method development is geared to provide for such contingencies. This document represents, therefore, a presentation of the background information that was developed for recommending a revision that has since been adopted.

Acknowledgments

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References

[1] ASTM Proposed Recommended Practice for R-Curve Determination, *Annual Book of ASTM Standards*, Part 10, 1975.

[2] Irwin, G. R., "Fracture Testing of High-Strength Sheet Materials Under Conditions Appropriate for Stress Analysis," NRL Report 5486, National Research Laboratory, July 1960.

[3] Wilson, W. K., *Engineering Fracture Mechanics Journal*, Vol. 2, No. 2, Nov. 1970.

[4] Gross, B., Roberts, E., Jr., and Srawley, J. E., *International Journal of Fracture Mechanics*, Vol. 4, No. 3, Sept. 1968.

[5] Brown, W. F., Jr., Note for ASTM E-24 Sub I "Effects of Some Dimensional Variables on Compact Tension Specimens," Sept. 1969.

[6] Ratwani, M. M. and Wilhem, D. P., "Development and Evaluation of Methods of Plane Stress Fracture Analysis," Part II, Vol. I, AFFDL TR-73-42, Air Force Flight Defense Laboratory, Dayton, Ohio, April 1975.

[7] Newman, J. C., Jr., "Stress Analysis of Simply and Multiply Connected Regions Containing Cracks by the Method of Boundary Collocation," M.S. thesis, Virginia Polytechnic Institute, Blacksburg, Va., May 1969.

[8] Newman, J. C., Jr., "An Improved Method of Collocation for the Stress Analysis of Cracked Plates with Various Shaped Boundaries," NASA TN D-6376, National Aeronautics and Space Administration, Washington, D.C., Aug. 1971.

[9] Newman, J. C., Jr., in *Fracture Analysis, ASTM STP 560,* American Society for Testing and Materials, 1974, pp. 105–121.

[10] Newman, J. C., Jr., "Crack-Opening Displacements in the Center-Crack, Compact, and Crack-Line-Wedge-Loaded Specimens," NASA TN, National Aeronautics and Space Administration, Washington, D.C., 1976.

[11] Muskhelishvili, N. I., (J. R. M. Rodak, translator), *Some Basic Problems of the Mathematical Theory of Elasticity,* 3rd edition, P. Noordhoff, Ltd., Groningen, 1953.

[12] Eftis, J. and Liebowitz, H., *International Journal of Fracture Mechanics,* Vol. 8, No. 4, Dec. 1972.