A Frequency Domain Correlation Technique for Model Correlation and Updating

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ABSTRACT

Up to now, existent FRF based model updating methods use the differences between measured and analytical FRFs at a fixed frequency, as residual to minimize. This approach does not take into account that FRFs between a reference model (experimental) and a perturbed one (a finite elements model not yet updated), displace in two axes : amplitude and frequency.
A more physical correlation, then, uses also the frequency shift . The problem is how to find it.
Taking as base the well known Modal Assurance Criterion (MAC), the Modal Scale Factor (MSF), and the concept of frequency shift, two correlation techniques, the Frequency Domain Assurance Criterion (FDAC) and the Frequency Response Scale Factor (FRSF), are presented. They help to quantify the level of correlation between responses coming from the two models, determine the frequency shift at all measured frequencies, and establish a suitable set of frequencies to use during the updating procedure.

NOMENCLATURE

$[Z]=[K]-\omega^2[M]$: Dynamic stiffness matrix
$[H(\omega)]=[Z\ (\omega)]^{-1}$: Frequency response matrix
FRF : Frequency Response Function (any column of *[H]*)
$\{A\}_j$: j-th column of *[A]*
a : analytic
x : experimental
dof : degree of freedom
FE : Finite Element Method

1. INTRODUCTION

The importance of the structural dynamic behavior in the automobile and aerospace industries have made necessary the construction of predictive models that may come from experimental tests (modal analysis) or from numerical models (finite elements). Both techniques suffer limitations : modal analysis results are valid only for the test conditions and the amount of information is limited due to the number and nature of measured points (only translational degrees of freedom). The Finite Element method presents problems if some parameters are poorly estimated. This causes a distance between analytical and measured responses.
In order to solve these problems, model updating techniques use experimental data (mainly from dynamic tests) as reference to correct the analytical model. The result is a model that better represents reality and that can be modified to fulfill the design requirements with a higher level of confidence.

The use of FRFs as input for the updating procedure seems attractive since direct measures have a good level of confidence, there is a lot of available information (even if redundant), and the modal parameter identification cost is avoided.

Lin & Ewins [1] proposed to update the global matrices *[M], [C]* and *[K]* by adding perturbation matrices formed by linear combination of certain chosen elementary matrices. In order to find the corresponding weighting factors, a matrix inversion property is exploited. Convergence is obtained by an iterative process.

Larsson [2] uses a linearization of *[Z]* in terms of the design physical parameters at an elementary level. The use of *[Z]*, instead of *[H]* assures a better numerical stability for the sensitivity calculation.

Both methods result in an over-determined system of equations using the residues between the experimental and analytical FRFs at the same frequency(ies). In what follows a 'physical' objection to these approaches will be made.

2. CHOICE OF COMPARISON FREQUENCIES

Interesting conclusions can be stated for a conservative system for which the stiffness matrix $\left[K\right]$ is *perturbed* by a coefficient α :

$$\left[K_{per}\right]=\alpha\left[K_{ref}\right] \tag{1}$$

Then, it can be easily proven [6] that:

$$\left[H_{per}\left(\sqrt{\alpha}\omega\right)\right]=\frac{1}{\alpha}\left[H_{ref}\left(\omega\right)\right] \tag{2}$$

Equation (2) shows that a shift *in frequency and a new scale factor* appears on all FRFs and, what is the most important, *a direct correlation exists for two FRFs if **the frequency shift** is taken into account*. As shown in **Fig. 1,** if updating or correlation are made at the same frequency ω_{upd} Hz, it will compare two vectors with a *phase lag* of 180° and with completely different modal participations. Otherwise, if $\omega_{upd}{}'$ Hz is used in the model and ω_{upd} in the experimental FRFs as comparison frequencies, both FRFs will be in phase and will have a high degree of correlation.

In a more general situation, and referring to **Fig. 2**, if ω_P, ω_Q, and ω_R are used as updating frequencies, points *P'', Q''*, and *R''* will be updated towards the references *P, Q,* and *R* respectively instead of using *P', Q'*, and *R'* which are the true corresponding points. In this way, no exploitation is done of the natural correlation on the frequency axis.

Model updating methods based on modal information implicitly take the frequency shift into account, since they pair modes at different frequencies: in **Fig. 2** for example,

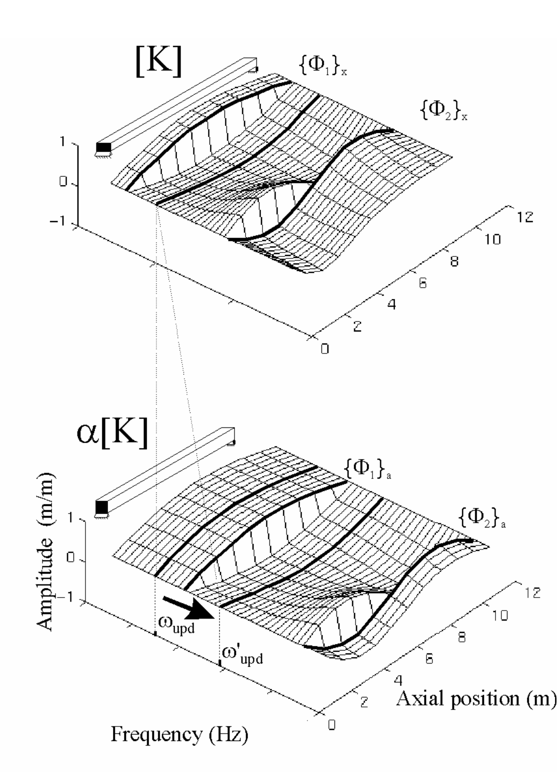

Fig 1. FRF frequency shift on a beam

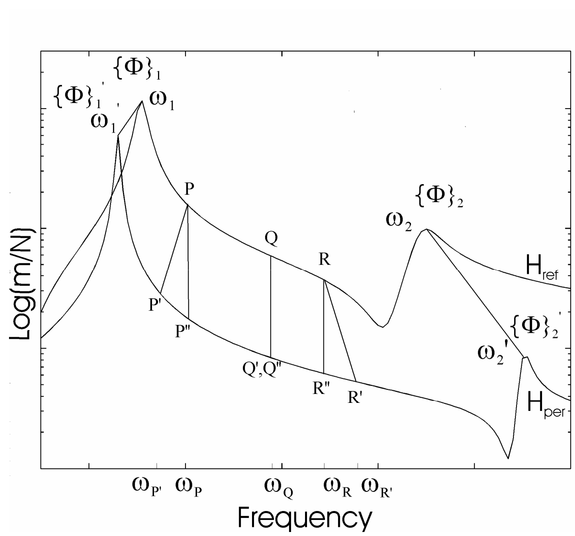

Fig. 2. Reference and perturbed system FRFs

$\left\{\omega_1', \omega_2', \{\Phi'\}_1, \{\Phi'\}_2\right\}$ will be updated to $\left\{\omega_1, \omega_2, \{\Phi\}_1, \{\Phi\}_2\right\}$.

It should be pointed out that usually, as several parameters at elementary level are perturbed, each of these parameters will shift the frequency in different directions so that an average frequency shifting exists for each point. In **Fig. 2**, P',Q' and R' are unknown. A new frequency domain correlation technique that allows the estimation of such points in a global sense will be presented hereafter.

3. FREQUENCY DOMAIN ASSURANCE CRITERION (FDAC) [3]

Up to now, the most common technique used to assess the results of a FE model is the MAC [4]. It gives quantitatively a good idea of the global closeness between experimental and FE mode shapes. Nefske [5] proposed to measure the correlation between two FRFs in a similar way but he performs the comparison at the same frequencies, obtaining a vector of correlation.

Based on these techniques and on the concept of frequency shifting mentioned above, it was proposed in [3] to measure the closeness between measured and synthesized FRFs by using the following correlation criterion :

$$FDAC(\omega_a, \omega_x, j) = \frac{\left(\{H_a(\omega_a)\}_j^T \{H_x(\omega_x)\}_j\right)^2}{\left(\{H_a(\omega_a)\}_j^T \{H_a(\omega_a)\}_j\right)\left(\{H_x(\omega_x)\}_j^T \{H_x(\omega_x)\}_j\right)} \tag{3}$$

where

j corresponds to the measured column of *[H]* ;
ω_a corresponds to the frequency at which $\{H_a\}_j$ is calculated ;
ω_x corresponds to one frequency at which the FRF was measured experimentally.

The so-called Frequency Domain Assurance Criterion (FDAC) can be regarded as equivalent to MAC in the FRF domain, as the Frequency Response Assurance Criterion (FRAC) [6] is related to the COordinate Modal Assurance Criterion (COMAC) [7] **(Table I).**

	Level of Correlation	
Domain	Local (d.o.f.)	Global
Modal	**COMAC**	**MAC**
Frequency	**FRAC**	**FDAC**

Table I.
Correlation techniques

It comes from **equation (3)** that FDAC values are limited to the interval $[0,1]$. As for MAC, a value of 1 means perfect correlation, and 0 no correlation at all.

In terms of FDAC, the frequency shift at a fixed frequency ω_a in the FE model can be defined as the difference :

$$\Delta\omega_{SHIFT}(\omega_a) = \omega_x^* - \omega_a \tag{4}$$

where ω_x^* is the frequency at which FDAC reaches its maximum for all measured frequencies.

3.1. AN IMPROVED FDAC

A drawback of FDAC calculated from **equation (3)** relies in its insensibility to the phase lag between the FRFs. It allows pairing between FRFs that have a relative phase of 180°, a result that has no physical meaning : two corresponding FRFs should be at least in the same semi-plane with respect to the excitation force. In order to avoid this situation it is better to redefine FDAC as the *cosine* between the FRFs , so:

$$FDAC(\omega_a,\omega_x,j)=\frac{\left(\left\{H_a(\omega_a)\right\}_j^T\left\{H_x(\omega_x)\right\}_j\right)}{\left|\left\{H_a(\omega_a)\right\}_j\right|\,\left|\left\{H_x(\omega_x)\right\}_j\right|} \tag{5}$$

In this way, FDAC may take values in the range
[-1,1]. A value FDAC>0 means that both FRFs are in the same semi-plane (in phase). A value FDAC near 1 means a high shape similarity.

In the context of a FRF based updating procedure, FDAC helps in the choice of the frequencies that should be used. It allows to select the intervals where the model is close enough to the experiments (a high value of FDAC, and *no shifting*).

If **equation (5)** is evaluated for a given set of analytical frequencies and for all measured frequencies , the FDAC matrix is obtained. A perfectly updated model will have only positive unitary values on the axis $\omega_x = \omega_a$. As a consequence, the criteria of shape and phase similarity, and equal frequency pairing will be satisfied by the FE model.

4. FREQUENCY RESPONSE SCALE FACTOR (FRSF)

Regarding to the definition, it comes that FDAC is insensible to the existent scale factor between both analytical and experimental FRFs.

For this reason, a Frequency Response Scale Factor (FRSF) can be defined (by analogy with the Modal Scale Factor MSF [8]) :

$$FRSF(\omega_a,j)=\frac{\left(\left\{H_a(\omega_a)\right\}_j^T[W]\left\{H_a(\omega_a)\right\}_j\right)}{\left(\left\{H_x(\omega_x^*)\right\}_j^T[W]\left\{H_x(\omega_x^*)\right\}_j\right)} \tag{6}$$

where

ω_x^* is the experimental frequency corresponding to ω_a (i.e. maximum FDAC);
j refers to the measured column of [H] ;
[*W*] is a weighting matrix.

For a perfect model, all the components of the FRSF vector should be equal to 1.

However since damping is not taken into account in the model, it is expected to find FRSF values greater than 1 near the resonances. Zones where the energy ratio is too high should be avoided as candidates for updating purposes.

5. EXPERIMENTAL EXAMPLE

The proposed correlation technique was tested on the experimental test case shown in **Fig. 3**. It consists of a 3D space frame 2.82 m long. The reference structure has lost one of its beams, while the analytical (initial) model is complete. The model has 1044 dofs and 192 elements. Damping was not modelled.

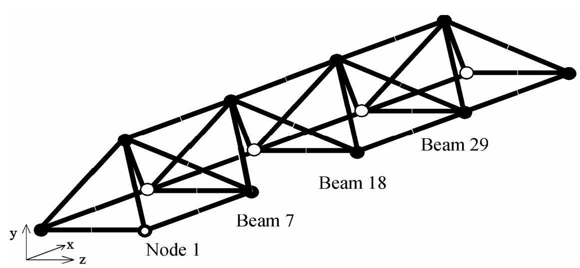

Fig. 3. Experimental test case

The translational degrees of freedom on 45 nodes were measured, at the extreme and in the middle of each beam. So, a total of 135 measured dofs are avalaible. The excitation force is applied on node 1 in the -y- direction. The frequency range covers from 0 to 200 Hz.

In **Fig. 4** the initial MAC matrix is shown. The first experimental mode does not appear on the model. After updating (**Fig. 5**) the modal properties are quite closer.

The initial and updated FDAC matrices are shown in **Fig. 6** and **Fig. 7** respectively. The clearer zones show high correlation and minimal phase lag.

The points of maximal correlation at each FE frequency $\left(\omega_x^*\right)$ are plotted with black points. As it is expected, at each resonance the frequency shift corresponds to the natural frequency shift. In the frequency range from 50 to 150 Hz the shift is quite important, and above 150 Hz the correlation is very poor (the measured FRFs show strong local mode shape influence, see **Fig. 8**). For these reasons the interval [50,200] Hz was excluded from the updating procedure.

The analysis of the frequency shift in the range [0,50] Hz gives the following possible set of frequencies for use in a FRF based model updating procedure ([1] or [2]): 6, 13 , 27 and 39 Hz (the arrows in **Fig. 9**). The energy check on these frequencies given by the FRSF (**Fig. 10**) also show good values (≈ 1). After 'deleting' beam 18 from the model, FDAC takes higher values, and the frequency shift is quite reduced as shown in **Fig.7**.

6. CONCLUDING REMARKS

FDAC represents a powerful tool that can be used advantageously for several purposes in a FRF based model updating :

As a global correlation tool, FDAC evaluates quantitatively the closeness between measured and simulated FRFs. It is very helpful for the engineer, who is frequently asked to reduce vibrations in terms of the dynamic responses. Frequency zones where the model show poor results are easily detected. No identification is needed, since the measures are used directly.

For error localization or damage detection using the approach presented in [9], FDAC allows better results due to the correct FRF frequency pairing. For the same reason, the use of the FRF frequency shift concept is very promising for the development of model updating methods.

In its improved form, FDAC is sensible to three characteristic parameters of the compared FRFs : shape, phase and frequency. Another property is the scale factor, which is affected mainly by damping and modelling errors. Such errors can be the cause of no convergence to a solution in the updating process. Frequencies where the perturbations are strong (which are detected by FRSF) must be avoided.

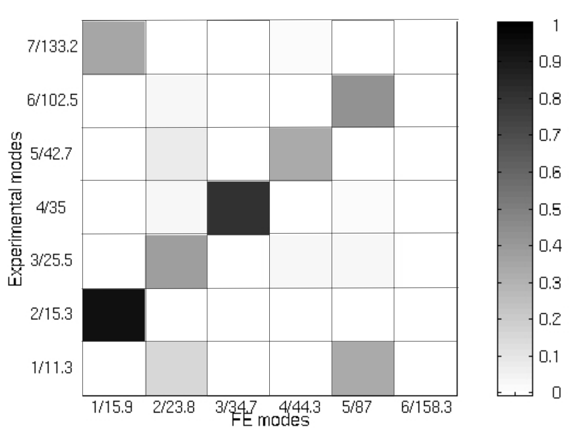

Fig. 4. Initial MAC

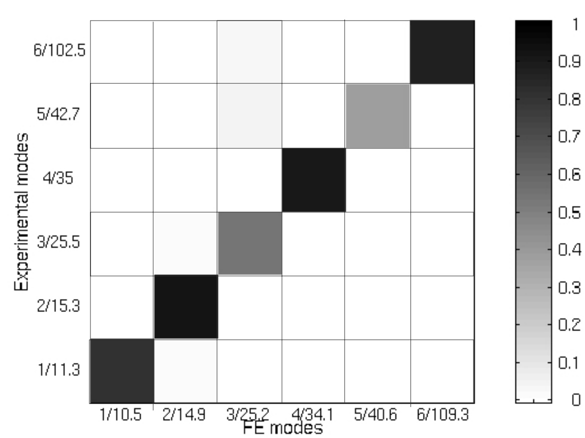

Fig. 5. MAC after updating

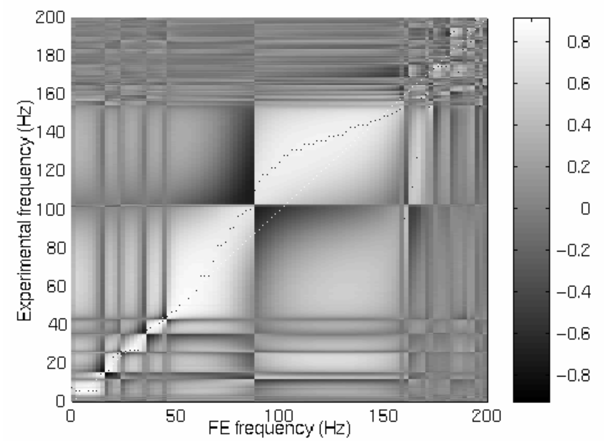

Fig. 6. Initial FDAC (all the frequency range)

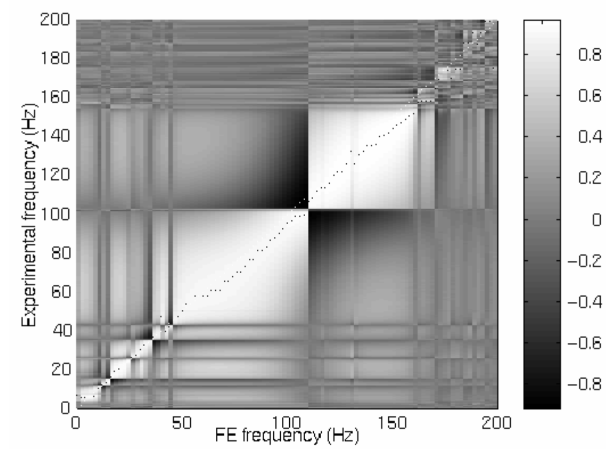

Fig. 7. FDAC after 'removing' beam 18

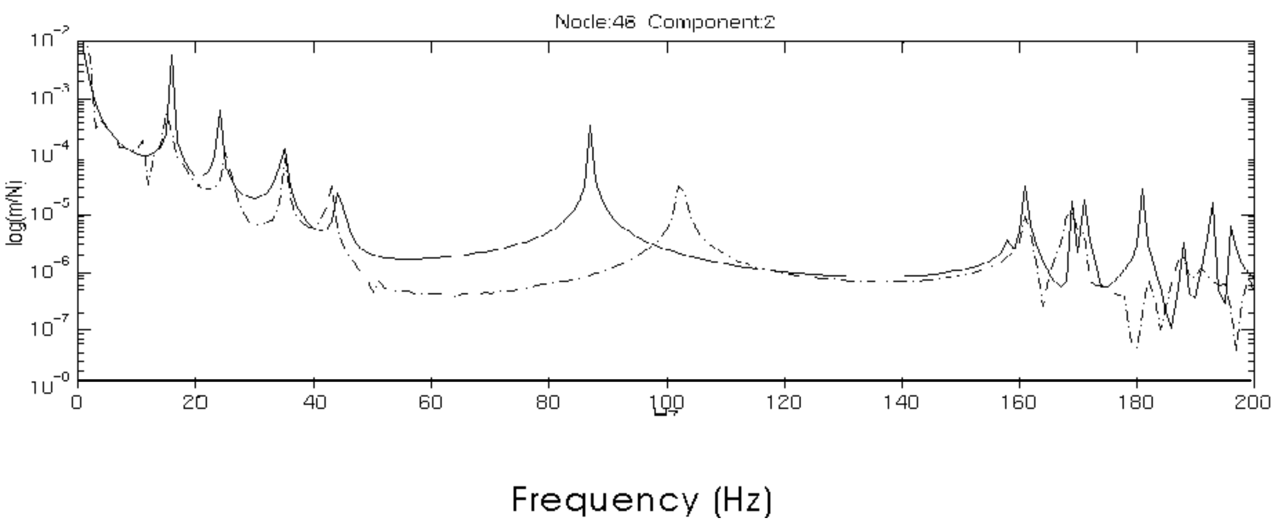

Fig. 8. Typical FRFs (-. Experimental, - initial FE)

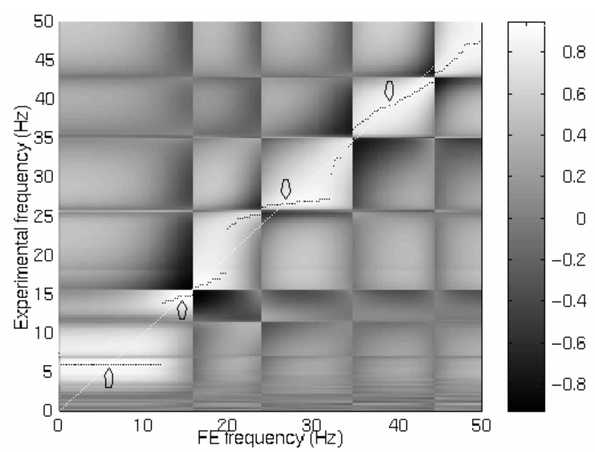

Fig. 9 Initial FDAC (lower frequency range)

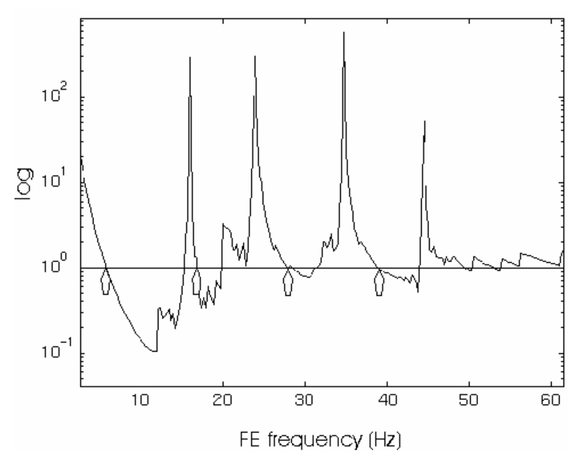

Fig. 10. Initial FRSF

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