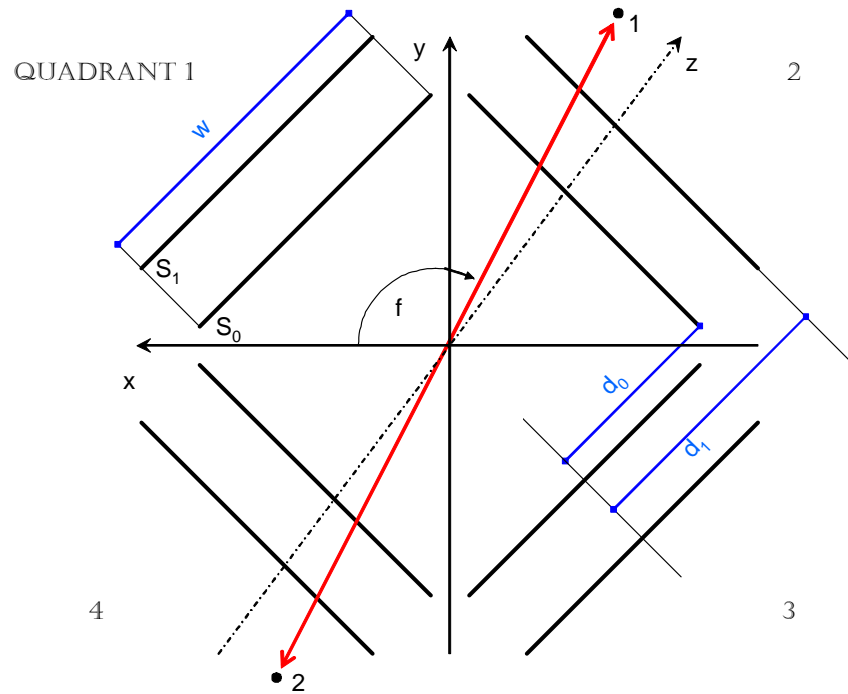


## Event Simulation for pp or pbarp elastic scattering

A The input is given by the following detector setup, which )



1. For pp scattering a convenient choice to identify the two protons is to associate the smaller scattering angle  $\theta$  with particle 1. No information is lost, the two protons are not distinguishable and the angular distributions of all observables extend to 90 degree in the cm.
2. In pbarp scattering the two particles are distinguishable. The scattering angle  $\theta$  associated to the imping particle.

### Silicon Detector Dimensions and Positions

$$w := 100 \text{ mm} \quad d := \begin{pmatrix} 60 \\ 80 \end{pmatrix} \text{ mm}$$

Detectors are infinite along z coordinate with a strip pitch of

$$ds := 0.8 \text{ mm}$$

and a thickness of  $t := 0.03 \text{ cm}$

which corresponds to  $t \cdot 10^4 = 300 \text{ } \mu\text{m}$

### Phi Acceptance for the different detectors in the four quadrants

$$\varphi_{\text{acc}}(1, q) := \begin{bmatrix} \frac{\pi}{4} \cdot (2 \cdot q - 1) - \arccos \left[ \frac{d_1}{\sqrt{(d_1)^2 + \left(\frac{w}{2}\right)^2}} \right] \\ \frac{\pi}{4} \cdot (2 \cdot q - 1) + \arccos \left[ \frac{d_1}{\sqrt{(d_1)^2 + \left(\frac{w}{2}\right)^2}} \right] \end{bmatrix}$$

Phi Acceptance for hits in all four layers

### Inner Layer

### Outer Layer

Quadrant	Inner Layer $\varphi_{\text{acc}}(0, q)$	Outer Layer $\varphi_{\text{acc}}(1, q)$
Quadrant 1	$\begin{pmatrix} 5.194 \\ 84.806 \end{pmatrix} \cdot \text{deg}$	$\begin{pmatrix} 12.995 \\ 77.005 \end{pmatrix} \cdot \text{deg}$
Quadrant 2	$\begin{pmatrix} 95.194 \\ 174.806 \end{pmatrix} \cdot \text{deg}$	$\begin{pmatrix} 102.995 \\ 167.005 \end{pmatrix} \cdot \text{deg}$
Quadrant 3	$\begin{pmatrix} 185.194 \\ 264.806 \end{pmatrix} \cdot \text{deg}$	$\begin{pmatrix} 192.995 \\ 257.005 \end{pmatrix} \cdot \text{deg}$
Quadrant 4	$\begin{pmatrix} 275.194 \\ 354.806 \end{pmatrix} \cdot \text{deg}$	$\begin{pmatrix} 282.995 \\ 347.005 \end{pmatrix} \cdot \text{deg}$

$$\frac{(\varphi_{\text{acc}}(1, 1)_1 - \varphi_{\text{acc}}(1, 1)_0) \cdot 4}{2 \cdot \pi} = 0.711$$

$$\varphi_{\text{quad}}(q) := \left[ \frac{\pi}{4} \cdot (2 \cdot q - 1) \right]$$

- $\varphi_{\text{quad}}(1) = 45 \cdot \text{deg}$
- $\varphi_{\text{quad}}(2) = 135 \cdot \text{deg}$
- $\varphi_{\text{quad}}(3) = 225 \cdot \text{deg}$
- $\varphi_{\text{quad}}(4) = 315 \cdot \text{deg}$

In total, `nevent` are generated

in `nbin` bins

`nevent := 25000`    `i := 0..nevent - 1`

`nbin := 40`

`x0` and `y0` offsets are ranging from

`x0_min := -5 mm`    `x0_max := 5 mm`

`x0_i := rnd(x0_max - x0_min) - x0_max`

`Hx0 := histogram(nbin, x0)`

`y0_min := -5 mm`    `y0_max := 5 mm`

`y0_i := rnd(y0_max - y0_min) - y0_max`

`Hy0 := histogram(nbin, y0)`

The vertex is located between

`z0_min := -200 mm`    `z0_max := 200 mm.`

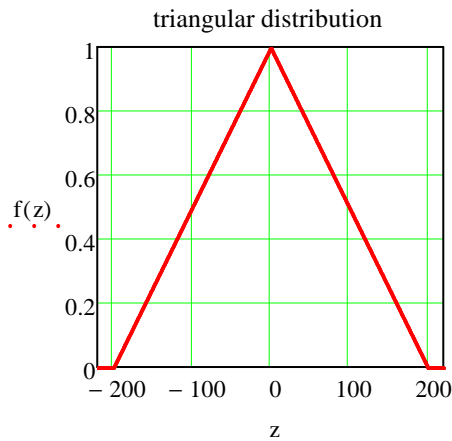
normalized probability distribution function for a triangular shape

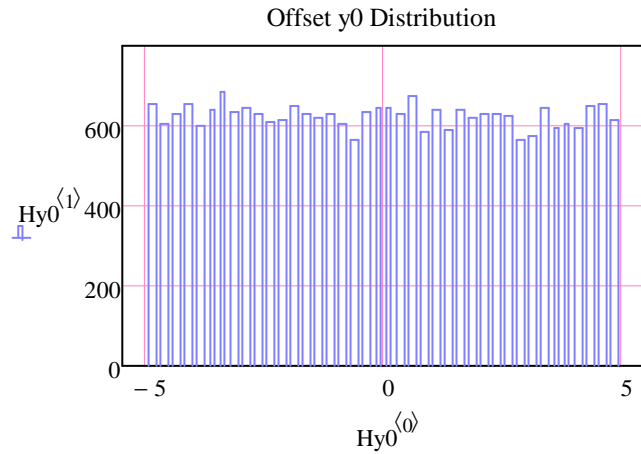
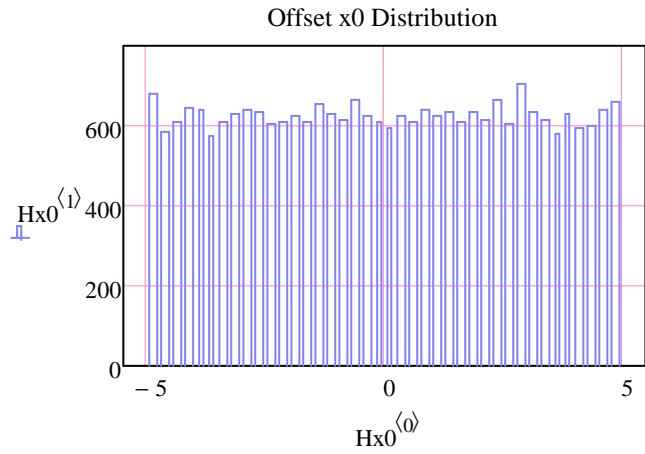
$$f(z) := \text{if} \left( z0_{\min} \leq z < z0_{\max}, 1 - \frac{\sqrt{z^2}}{z0_{\max}}, 0 \right)$$

`Hz0 := histogram(nbin, z0)`

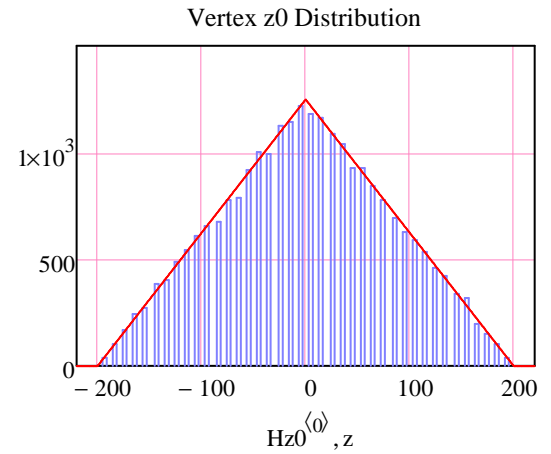
```

z0 :=
| i ← 0
| while i < nevent
|   random ← rnd(z0_max - z0_min) - z0_max
|   if f(random) ≥ rnd(1)
|     z0_i ← random
|     i ← i + 1
| z0
    
```





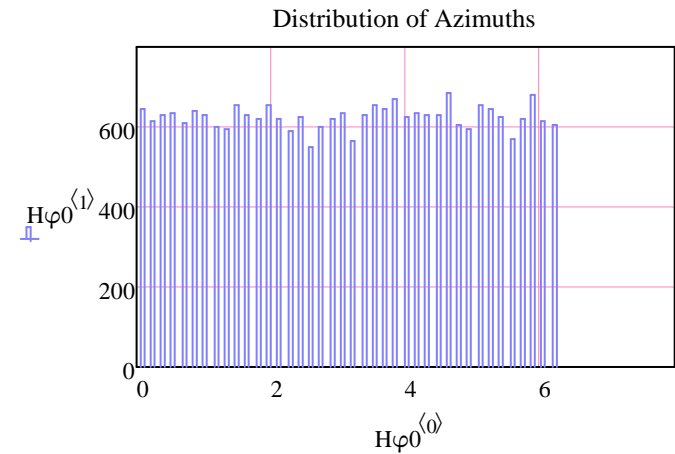
$H_{z0}^{(1)}$   
 $f(z) \cdot \frac{\text{nevent}}{\text{nbin}}$



also uniformly distributed are the generated events in  $\varphi$

$$\varphi_1 := \text{rnd}(2\pi)$$

$$H_{\varphi 0} := \text{histogram}(\text{nbin}, \varphi_0)$$



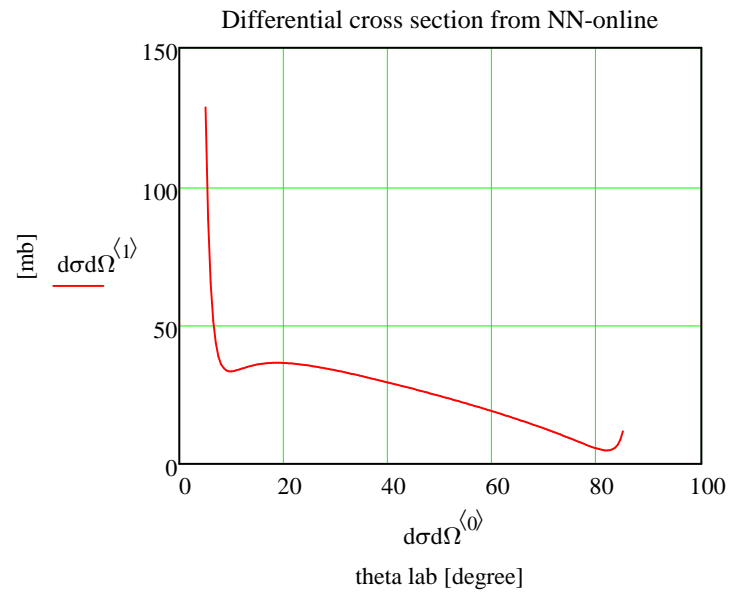
pick before implementation of a real cross section a simple distribution function

$$d\sigma d\Omega_{45} := \text{pp-45MeV\_pwa93.txt}$$

$$d\sigma d\Omega_{200} := \text{pp-200MeV\_pwa93.txt}$$

specify input data in use

$$d\sigma d\Omega := d\sigma d\Omega_{45}$$



$$\theta_{\min} := 5 \text{ degree}$$

$$\theta_{\max} := 85 \text{ degree}$$

$$\text{degrad} := \frac{\pi}{180}$$

conversion from degree to radian

$$\text{radeg} := \frac{1}{\text{degrad}}$$

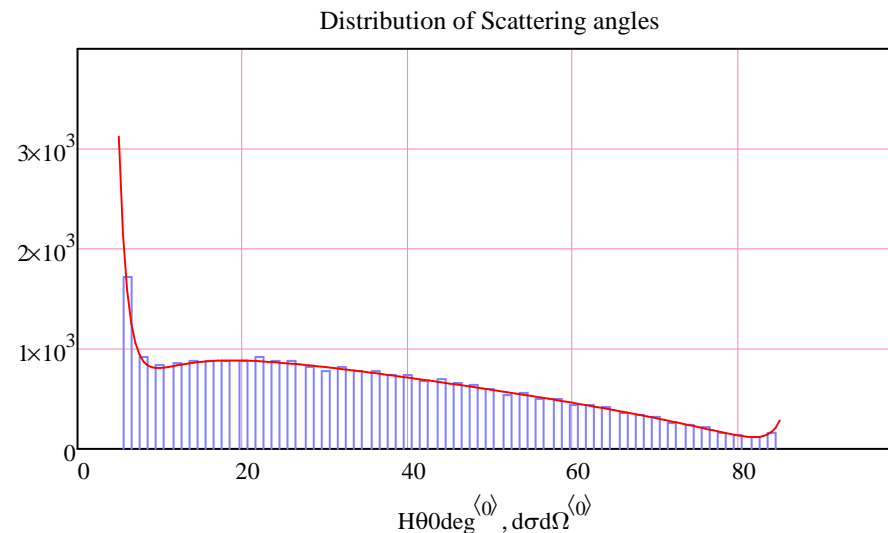
```

θ0 := i ← 0
while i < nevent
  θout ← rnd(θ0_max·degrad - θ0_min·degrad) + θ0_min·degrad
  j ←  $\frac{\text{round}\left[2 \cdot \left(\frac{\theta_{\text{out}}}{\text{degrad}}\right)\right]}{2} \cdot \frac{1}{0.5} - 10$ 
  if (dσdΩ(1))j ≥ rnd(max(dσdΩ(1)))
    θ0i ← θout
    i ← i + 1
θ0
    
```

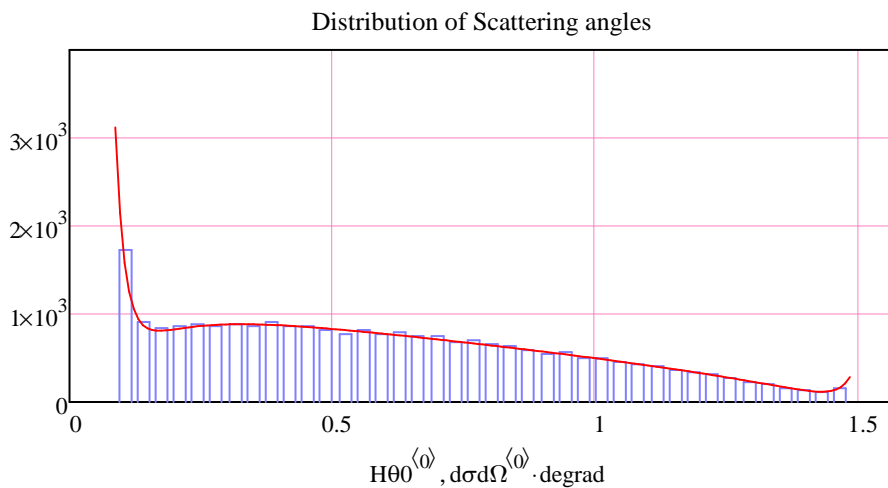
```
Hθ0deg := histogram(nbin, θ0·radeg)
```

```
Hθ0 := histogram(nbin, θ0)
```

$$H\theta_0^{(1)} = \frac{d\sigma d\Omega^{(1)}}{\max(d\sigma d\Omega^{(1)}) \cdot nbin} \cdot \frac{\text{nevent}}{5}$$



$$H\theta_0^{(1)} = \frac{d\sigma d\Omega^{(1)}}{\max(d\sigma d\Omega^{(1)}) \cdot nbin} \cdot \frac{\text{nevent} - 5}{1}$$



Now we can compose the input vector from the five parameters  $\theta_0, \varphi_0, x_0, y_0, z_0$   $X_1 := (\theta_0, \varphi_0, x_0, y_0, z_0)$

$$X := \text{augment}(\theta_0, \varphi_0, x_0, y_0, z_0)$$

Kinematics for Forward and Recoil

kinetic energy  
proton rest mass

$$T_p := 45 \quad \text{MeV}$$

$$m_p := 938.272 \quad \text{MeV}$$

$$\beta_{\text{lab}} := \frac{\sqrt{T_p^2 + 2 \cdot m_p \cdot T_p}}{m_p + T_p}$$

$$\gamma_{\text{lab}} := \frac{1}{\sqrt{1 - \beta_{\text{lab}}^2}}$$

$$fk := \frac{2}{1 + \gamma_{\text{lab}}} \quad fk = 0.97658$$

Kinematics of pp elastic scattering

kinematics calculations

Lab projectile energy and momentum

$$\beta_{\text{cm}} := \frac{\beta_{\text{lab}} \cdot \gamma_{\text{lab}}}{1 + \gamma_{\text{lab}}}$$

$$\gamma_{\text{cm}} := \frac{1}{\sqrt{1 - \beta_{\text{cm}}^2}}$$

$$\beta_{\text{cm}} = 0.153 \quad \gamma_{\text{cm}} = 1.012$$

$$p_{\text{lab}} := \gamma_{\text{lab}} \cdot \beta_{\text{lab}} \cdot m_p$$

$$E_{\text{lab}} := \gamma_{\text{lab}} \cdot m_p$$

CM energy  $\sqrt{s}$

$$p_{\text{lab}} = 294.057 \quad \text{MeV/c}$$

$$\sqrt{s} := m_p^2 + 2 \cdot E_{\text{lab}} \cdot m_p + m_p^2$$

$$\sqrt{s} = 1898.911 \quad \text{MeV}^2$$

$$p_{\text{cm}} := \frac{p_{\text{lab}} \cdot m_p}{\sqrt{s}}$$

$$p_{\text{cm}} = 145.297 \quad \text{MeV/c}$$

$$\beta(T) := \frac{\sqrt{T^2 + 2 \cdot m_p \cdot T}}{m_p + T}$$

for speed calculation

For pp elastic scattering  $\text{tg}(\theta_{\text{rec}}) = fk / \text{tg}(\theta)$

B) Event simulation

An event is defined by a straight line through a vertex point  $(x_0, y_0, z_0)$  that has the general form  $(x-x_0)/A = (y-y_0)/B = (z-z_0)/C$ . The recoil prong also originates from the vertex and

has a direction given by kinematics. Here, we assume values for the five independent parameters that describe an event  $(\theta, \phi, x_0, y_0, z_0)$  and calculate the appropriate **four** silicon detector coordinates.

$\theta_0 = 16.895 \cdot \text{deg}$	$\varphi_0 = 351.094 \cdot \text{deg}$	<b>x</b>	$A1_i := \cos(\varphi_{0_i}) \cdot \sin(\theta_{0_i})$	$A1_0 = 0.287$	$x0_0 = -1.668 \text{ mm}$
	$\varphi_{r_i} := \varphi_0 + \pi$	<b>y</b>	$B1_i := \sin(\varphi_{0_i}) \cdot \sin(\theta_{0_i})$	$B1_0 = -0.045$	$y0_0 = 1.775 \text{ mm}$
	$\varphi_{r_0} = 531.094 \cdot \text{deg}$	<b>z</b>	$C1_i := \cos(\theta_{0_i})$	$C1_0 = 0.957$	$z0_0 = -153.218 \text{ mm}$

Note:

- $\theta$  and  $\phi$  can be converted into A and B, and C (C is not an independent quantity)

check  $\text{angle}(A1_0, B1_0) = 351.094 \cdot \text{deg}$

$$\text{atan} \left[ \frac{\sqrt{(A1_0)^2 + (B1_0)^2}}{\sqrt{1 - \left(\frac{A1_0}{\cos(\varphi_0)}\right)^2}} \right] = 16.895 \cdot \text{deg}$$

straight line of first track

straight line of second track

$$\theta_{r_i} := \text{atan} \left( \frac{fk}{\tan(\theta_{0_i})} \right) \quad \theta_{r_0} = 72.723 \cdot \text{deg}$$

$$\text{first}(\gamma, i) := \begin{pmatrix} x0_i \\ y0_i \\ z0_i \end{pmatrix} + \gamma \cdot \begin{pmatrix} A1_i \\ B1_i \\ C1_i \end{pmatrix}$$

<b>x</b>	$A2_i := \cos(\varphi_{0_{r_i}}) \cdot \sin(\theta_{0_{r_i}})$	$A2_0 = -0.943$
<b>y</b>	$B2_i := \sin(\varphi_{0_{r_i}}) \cdot \sin(\theta_{0_{r_i}})$	$B2_0 = 0.148$
<b>z</b>	$C2_i := \cos(\theta_{0_{r_i}})$	$C2_0 = 0.297$

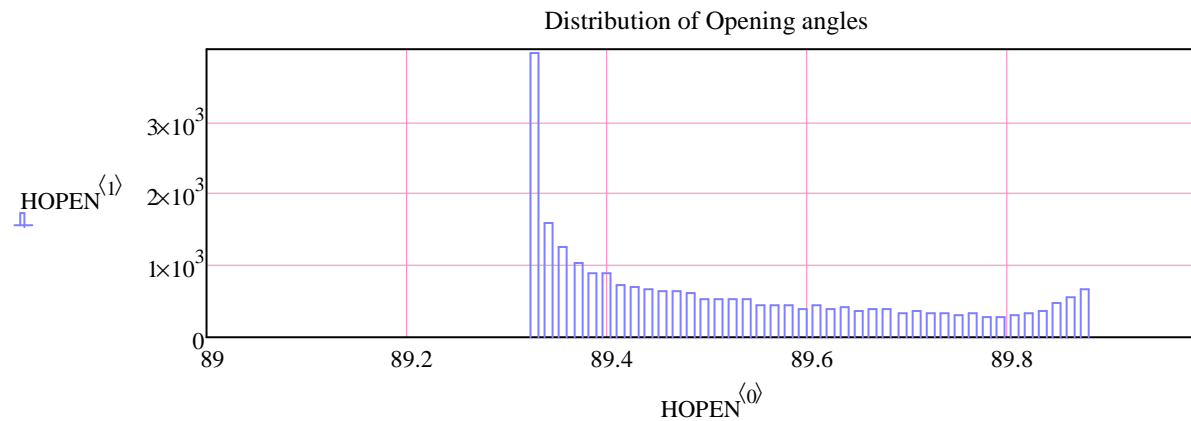
Opening angle

$$\text{second}(\delta, i) := \begin{pmatrix} x0_i \\ y0_i \\ z0_i \end{pmatrix} + \delta \cdot \begin{pmatrix} A2_i \\ B2_i \\ C2_i \end{pmatrix}$$

$$\text{OPEN}_i := \text{atan} \left( \frac{fk}{\tan(\theta_{0_i})} \right) + \text{atan} \left[ \frac{\sqrt{(A1_i)^2 + (B1_i)^2}}{\sqrt{1 - \left(\frac{A1_i}{\cos(\varphi_{0_i})}\right)^2}} \right]$$

$$\text{OPEN}_0 = 89.619 \cdot \text{deg}$$

HOPEN := histogram(nbin, OPEN·radeg)



Express the angle  $\theta_{rec}$  in terms of  $\theta$  (lab system), the angle of the forward scattered proton (based on *Particle Kinematics* by E. Byckling and K. Kajantie, pp.74.

for Definition:  $\theta := \theta_0$        $\theta_r := \theta_{r_0}$

## Lab momentum of the particles as function of the emission angle

$$P_{\text{lab}}(\theta) := \left[ p_{\text{lab}} \cdot \left[ m_p \cdot E_{\text{lab}} + 0.5 \cdot (m_p^2 + m_p^2 + m_p^2 - m_p^2) \right] \cdot \cos(\theta) + (E_{\text{lab}} + m_p) \cdot \left[ (m_p \cdot E_{\text{lab}})^2 - m_p^2 \cdot m_p^2 - m_p^2 \cdot p_{\text{lab}}^2 \cdot \sin^2(\theta) \right]^{0.5} \right] \cdot \left[ (E_{\text{lab}} + m_p)^2 - p_{\text{lab}}^2 \cdot \cos^2(\theta) \right]^{-1}$$

$$P_{\text{lab}}(\theta) = 280.796 \text{ MeV/c} \quad P_{\text{lab}}(\theta_r) = 85.462 \text{ MeV/c}$$

## Lab kinetic energy of the particles as function of the proton scattering angle

$$TT_p(\theta) := \left[ (E_{\text{lab}} + m_p) \cdot \left[ m_p \cdot E_{\text{lab}} + 0.5 \cdot (m_p^2 + m_p^2 + m_p^2 - m_p^2) \right] + p_{\text{lab}} \cdot \cos(\theta) \cdot \left[ (m_p \cdot E_{\text{lab}})^2 - m_p^2 \cdot m_p^2 - m_p^2 \cdot p_{\text{lab}}^2 \cdot \sin^2(\theta) \right]^{0.5} \right] \cdot \left[ (E_{\text{lab}} + m_p)^2 - p_{\text{lab}}^2 \cdot \cos^2(\theta) \right]^{-1} - m_p$$

$$TT_p(\theta) = 41.116 \text{ MeV} \quad TT_p(\theta_r) = 3.884 \text{ MeV} \quad TT_p(\theta) + TT_p(\theta_r) = 45 \text{ MeV} \quad \text{aha, correct!}$$

## With the given geometry, hits appear in the following detector layers

$$\text{hitq}(l, \varphi) := \begin{cases} 1 & \text{if } \left( \varphi_{\text{acc}(1,1)}{}_0 < \varphi < \varphi_{\text{acc}(1,1)}{}_1 \right) \\ 2 & \text{if } \left( \varphi_{\text{acc}(1,2)}{}_0 < \varphi < \varphi_{\text{acc}(1,2)}{}_1 \right) \\ 3 & \text{if } \left( \varphi_{\text{acc}(1,3)}{}_0 < \varphi < \varphi_{\text{acc}(1,3)}{}_1 \right) \\ 4 & \text{if } \left( \varphi_{\text{acc}(1,4)}{}_0 < \varphi < \varphi_{\text{acc}(1,4)}{}_1 \right) \\ 1 & \text{if } \varphi_{\text{acc}(1,1)}{}_0 + 2 \cdot \pi < \varphi < \varphi_{\text{acc}(1,1)}{}_1 + 2 \cdot \pi \\ 2 & \text{if } \varphi_{\text{acc}(1,2)}{}_0 + 2 \cdot \pi < \varphi < \varphi_{\text{acc}(1,2)}{}_1 + 2 \cdot \pi \\ 3 & \text{if } \varphi_{\text{acc}(1,3)}{}_0 + 2 \cdot \pi < \varphi < \varphi_{\text{acc}(1,3)}{}_1 + 2 \cdot \pi \\ 4 & \text{if } \varphi_{\text{acc}(1,4)}{}_0 + 2 \cdot \pi < \varphi < \varphi_{\text{acc}(1,4)}{}_1 + 2 \cdot \pi \\ 0 & \text{otherwise} \end{cases}$$



Define equations for the eight detector planes

$$\text{Si}(\alpha, \beta, l, q) := \begin{pmatrix} \cos(\varphi_{\text{quad}}(q)) \cdot d_l \\ \sin(\varphi_{\text{quad}}(q)) \cdot d_l \\ 0 \end{pmatrix} + \alpha \cdot \begin{pmatrix} -\cos(\varphi_{\text{quad}}(q)) \\ \sin(\varphi_{\text{quad}}(q)) \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Determine intercept of first prong and detectors

$$\text{M1}(1, i) := \begin{pmatrix} -\cos(\varphi_{\text{quad}}(\text{hitq}(1, \varphi_{0_i}))) & 0 & -A1_i \\ \sin(\varphi_{\text{quad}}(\text{hitq}(1, \varphi_{0_i}))) & 0 & -B1_i \\ 0 & 1 & -C1_i \end{pmatrix} \quad \text{v1}(1, i) := \begin{pmatrix} x0_i \\ y0_i \\ z0_i \end{pmatrix} - \begin{pmatrix} \cos(\varphi_{\text{quad}}(\text{hitq}(1, \varphi_{0_i}))) \cdot d_l \\ \sin(\varphi_{\text{quad}}(\text{hitq}(1, \varphi_{0_i}))) \cdot d_l \\ 0 \end{pmatrix}$$

Coordinates in 3D in the Silicon layers

$$\text{Si1\_3D}(1, i) := \text{first}(\text{Isolve}(\text{M1}(1, i), \text{v1}(1, i))_2, i)$$

1<sup>st</sup> Layer (0)

2<sup>nd</sup> Layer (1)

$$\text{Si1\_3D}(0, 19) = \begin{pmatrix} 8.631 \\ -76.222 \\ 45.5 \end{pmatrix} \begin{matrix} x \\ y \\ z \end{matrix} \quad \text{Si1\_3D}(1, 19) = \begin{pmatrix} 11.694 \\ -101.443 \\ 111.886 \end{pmatrix}$$

Determine intercept of second prong and detectors

$$\text{M2}(1, i) := \begin{pmatrix} -\cos(\varphi_{\text{quad}}(\text{hitq}(1, \varphi_{0_{r_i}}))) & 0 & -A2_i \\ \sin(\varphi_{\text{quad}}(\text{hitq}(1, \varphi_{0_{r_i}}))) & 0 & -B2_i \\ 0 & 1 & -C2_i \end{pmatrix} \quad \text{v2}(1, i) := \begin{pmatrix} x0_i \\ y0_i \\ z0_i \end{pmatrix} - \begin{pmatrix} \cos(\varphi_{\text{quad}}(\text{hitq}(1, \varphi_{0_{r_i}}))) \cdot d_l \\ \sin(\varphi_{\text{quad}}(\text{hitq}(1, \varphi_{0_{r_i}}))) \cdot d_l \\ 0 \end{pmatrix}$$

Coordinates in 3D in the Silicon layers

$$\text{Si2\_3D}(1, i) := \text{second}(\text{Isolve}(\text{M2}(1, i), \text{v2}(1, i))_2, i)$$

1<sup>st</sup> Layer (0)

2<sup>nd</sup> Layer (1)

$$\text{Si2\_3D}(0, 19) = \begin{pmatrix} -9.743 \\ 75.109 \\ -133.031 \end{pmatrix} \begin{matrix} x \\ y \\ z \end{matrix} \quad \text{Si2\_3D}(1, 19) = \begin{pmatrix} 11.694 \\ -101.443 \\ -202.73 \end{pmatrix}$$

Function to convert the 3D coordinates of the first track into silicon wires along **sw** and **sz**. We start counting the strips along **w** from smaller values of  $\varphi$ .

$$\text{Si1\_2D}(1, i) := \sqrt{\left( \frac{d_l}{\cos(\varphi_{\text{quad}}(\text{hitq}(1, \varphi_{0_i})) - \varphi_{\text{acc}}(1, \text{hitq}(1, \varphi_{0_i})))_0} \cdot \cos(\varphi_{\text{acc}}(1, \text{hitq}(1, \varphi_{0_i})))_0 - \text{Si1\_3D}(1, i)_0 \right)^2 + \left( \frac{d_l}{\cos(\varphi_{\text{quad}}(\text{hitq}(1, \varphi_{0_i})) - \varphi_{\text{acc}}(1, \text{hitq}(1, \varphi_{0_i})))_0} \cdot \sin(\varphi_{\text{acc}}(1, \text{hitq}(1, \varphi_{0_i})))_0 \right)^2} + |\text{Si1\_3D}(1, i)_2 + |z0_{\text{min}}|$$

$$\text{Si1\_2D}(0, 5) = \begin{pmatrix} 57.469 \\ 303.863 \end{pmatrix} \quad \text{Si1\_2D}(1, 5) = \begin{pmatrix} 59.863 \\ 380.683 \end{pmatrix}$$

Function to convert the 3D coordinates of the recoil track into silicon wires along **sw** and **sz**.

$$Si2\_2D(1,i) := \sqrt{\left( \frac{d_1}{\cos(\varphi_{quad}(\text{hitq}(1, \varphi_{0r_i})) - \varphi_{acc}(1, \text{hitq}(1, \varphi_{0r_i})))_0} \cdot \cos(\varphi_{acc}(1, \text{hitq}(1, \varphi_{0r_i})))_0 - Si2\_3D(1,i)_0 \right)^2 + \left( \frac{d_1}{\cos(\varphi_{quad}(\text{hitq}(1, \varphi_{0r_i})) - \varphi_{acc}(1, \text{hitq}(1, \varphi_{0r_i})))_0} \cdot \sin(\varphi_{acc}(1, \text{hitq}(1, \varphi_{0r_i})))_0 + Si2\_3D(1,i)_2 + |z0_{min}| \right)^2}$$

$$Si2\_2D(0,0) = \begin{pmatrix} 91.892 \\ 68.939 \end{pmatrix} \quad Si2\_2D(1,0) = \begin{pmatrix} 110.175 \\ 15.053 \end{pmatrix}$$

	0	1	2	3	4	5
0	0.295	6.128	-1.668	1.775	-153.218	
1	0.55	0.794	-3.946	1.792	50.103	
2	1.064	4.594	-3.296	-2.364	-36.239	
3	0.12	3.637	0.033	1.087	-128.419	
4	0.738	4.533	0.907	-4.008	55.571	
5	0.256	2.475	-0.287	-0.135	-126.184	
6	0.498	5.465	1.679	-0.953	19.844	
7	1.475	1.845	2.102	1.422	...	

X =

quad #, layer 0

quad #, layer 1

first track

$$\text{hit10}_i := \text{hitq}(0, \varphi_{0i})$$

$$\text{hit11}_i := \text{hitq}(1, \varphi_{0i})$$

3D positions

$$\text{hit10x}_1 := Si1\_3D(0,i)_0$$

$$\text{hit11x}_1 := Si1\_3D(1,i)_0$$

$$\text{hit10y}_1 := Si1\_3D(0,i)_1$$

$$\text{hit11y}_1 := Si1\_3D(1,i)_1$$

$$\text{hit10z}_1 := Si1\_3D(0,i)_2$$

$$\text{hit11z}_1 := Si1\_3D(1,i)_2$$

quad #, layer 0

quad #, layer 1

second track

$$\text{hit20}_i := \text{hitq}(0, \varphi_{0r_i})$$

$$\text{hit21}_i := \text{hitq}(1, \varphi_{0r_i})$$

$$\text{hit20x}_1 := Si2\_3D(0,i)_0$$

$$\text{hit21x}_1 := Si2\_3D(1,i)_0$$

$$\text{hit20y}_1 := Si2\_3D(0,i)_1$$

$$\text{hit21y}_1 := Si2\_3D(1,i)_1$$

$$\text{hit20z}_1 := Si2\_3D(0,i)_2$$

$$\text{hit21z}_1 := Si2\_3D(1,i)_2$$

Append hits to event parameter vector  $X := \text{augment}(X, \text{hit10}, \text{hit11}, \text{hit20}, \text{hit21}, \text{hit10x}, \text{hit10y}, \text{hit10z}, \text{hit11x}, \text{hit11y}, \text{hit11z}, \text{hit20x}, \text{hit20y}, \text{hit20z}, \text{hit21x}, \text{hit21y}, \text{hit21z})$

Include energy of both particles  $TT1_i := TT_p(\theta0_i)$   $TT2_i := TT_p(\theta0_{r_i})$  in MeV  $X := \text{augment}(X, TT1, TT2)$

Velocities of both particles  $\beta1_i := \beta(TT1_i)$   $\beta2_i := \beta(TT2_i)$   $c_{light} := 3 \cdot 10^8 \cdot 1000$  mm/s

calculate time of impact in layers with respect to collision

for particle 1

for particle 2

$$t10_i := \frac{\sqrt{(X_{i,2} - X_{i,9})^2 + (X_{i,3} - X_{i,10})^2 + (X_{i,4} - X_{i,11})^2}}{\beta1_i \cdot c_{light}}$$

$$t20_i := \frac{\sqrt{(X_{i,2} - X_{i,15})^2 + (X_{i,3} - X_{i,16})^2 + (X_{i,4} - X_{i,17})^2}}{\beta1_i \cdot c_{light}}$$

$$t11_i := \frac{\sqrt{(X_{i,2} - X_{i,12})^2 + (X_{i,3} - X_{i,13})^2 + (X_{i,4} - X_{i,14})^2}}{\beta1_i \cdot c_{light}}$$

$$t21_i := \frac{\sqrt{(X_{i,2} - X_{i,18})^2 + (X_{i,3} - X_{i,19})^2 + (X_{i,4} - X_{i,20})^2}}{\beta1_i \cdot c_{light}}$$

Include time of impact into array

$X := \text{augment}(X, t10, t11, t20, t21)$

	0	1	2	3	4	5	6	7	8	9
0	0.295	6.128	-1.668	1.775	-153.218	4	0	2	0	74.666
1	0.55	0.794	-3.946	1.792	50.103	1	1	3	3	39.168
2	1.064	4.594	-3.296	-2.364	-36.239	3	0	1	0	-11.744
3	0.12	3.637	0.033	1.087	-128.419	3	3	1	1	-55.788
4	0.738	4.533	0.907	-4.008	55.571	3	0	1	0	-11.639
5	0.256	2.475	-0.287	-0.135	-126.184	2	2	4	4	-47.707
6	0.498	5.465	1.679	-0.953	19.844	4	4	2	2	41.434
7	1.475	1.845	2.102	1.422	83.998	2	2	4	4	...

$$\text{Extract}\varphi(A) := \begin{array}{l} i \leftarrow 0 \\ U \leftarrow 0 \\ \text{for } j \in 0.. \text{rows}(A) - 1 \\ \quad \text{if } (A_{j,5} = A_{j,6} \neq 0) \wedge (A_{j,7} = A_{j,8} \neq 0) \\ \quad \quad \text{for } k \in 0..26 \\ \quad \quad \quad U_{i,k} \leftarrow A_{j,k} \\ \quad \quad \quad i \leftarrow i + 1 \\ U \end{array}$$

$$X\varphi := \text{Extract}\varphi(X) \quad \frac{\text{rows}(X\varphi)}{\text{rows}(X)} = 0.713$$

$X\varphi =$

	0	1	2	3	4	5	6	7	8
0	0.55	0.794	-3.946	1.792	50.103	1	1	3	3
1	0.12	3.637	0.033	1.087	-128.419	3	3	1	1
2	0.256	2.475	-0.287	-0.135	-126.184	2	2	4	4
3	0.498	5.465	1.679	-0.953	19.844	4	4	2	2
4	1.475	1.845	2.102	1.422	83.998	2	2	4	4
5	0.66	2.335	-4.658	-2.15	12.473	2	2	4	4
6	1.211	2.135	3.363	-2.39	35.11	2	2	4	4
7	1.183	2.801	4.08	-3.945	3.241	2	2	4	...

The detectors shall extend in z to +/- 150 mm, remove events outside the phi acceptance and outside this range

$zdet_{\min} := -50 \text{ mm}$        $zdet_{\max} := 250 \text{ mm}$

$$\text{Extract}z(A) := \begin{array}{l} i \leftarrow 0 \\ U \leftarrow 0 \\ \text{for } j \in 0.. \text{rows}(A) - 1 \\ \quad \text{if } (zdet_{\min} \leq A_{j,11} \leq zdet_{\max}) \wedge (zdet_{\min} \leq A_{j,14} \leq zdet_{\max}) \wedge (zdet_{\min} \leq A_{j,17} \leq zdet_{\max}) \wedge (zdet_{\min} \leq A_{j,20} \leq zdet_{\max}) \\ \quad \quad \text{for } k \in 0..26 \\ \quad \quad \quad U_{i,k} \leftarrow A_{j,k} \\ \quad \quad \quad i \leftarrow i + 1 \\ U \end{array}$$

Now throw away those events that do not hit the detector in z

$Xz := \text{Extractz}(X\varphi)$

	0	1	2	3	4	5
0	0.55	0.794	-3.946	1.792	50.103	1
1	0.498	5.465	1.679	-0.953	19.844	4
2	0.66	2.335	-4.658	-2.15	12.473	2
3	1.211	2.135	3.363	-2.39	35.11	2
4	1.183	2.801	4.08	-3.945	3.241	2
5	0.776	3.439	0.28	3.289	-104.627	3
6	0.637	2.7	0.277	0.216	-77.369	2
7	0.733	5.27	-3.46	-4.786	55.601	4
8	0.872	5.058	-1.473	-3.865	28.459	4
9	0.405	4.38	-3.016	4.302	7.96	3
10	0.642	3.87	-4.323	4.648	-55.693	3
11	0.385	5.74	-3.198	-1.923	17.46	...

$Xz =$

Histogram the final result:

$HX\theta_0_f := \text{histogram}(\text{nbin}, Xz^{(0)})$

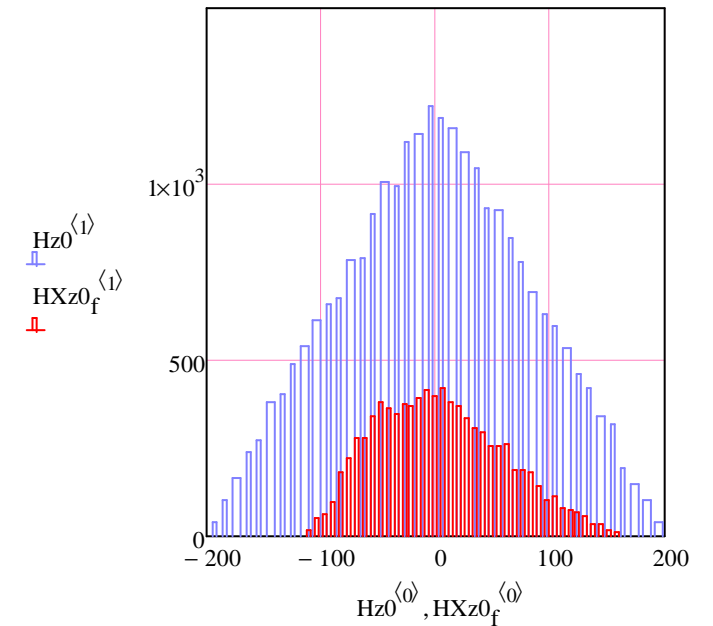
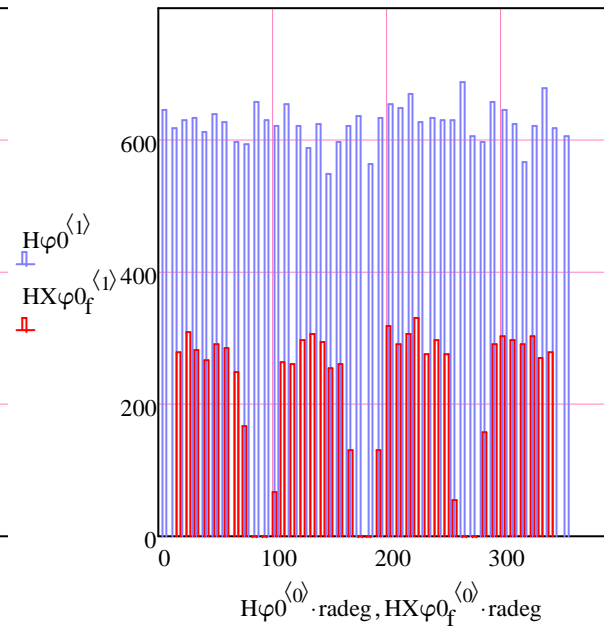
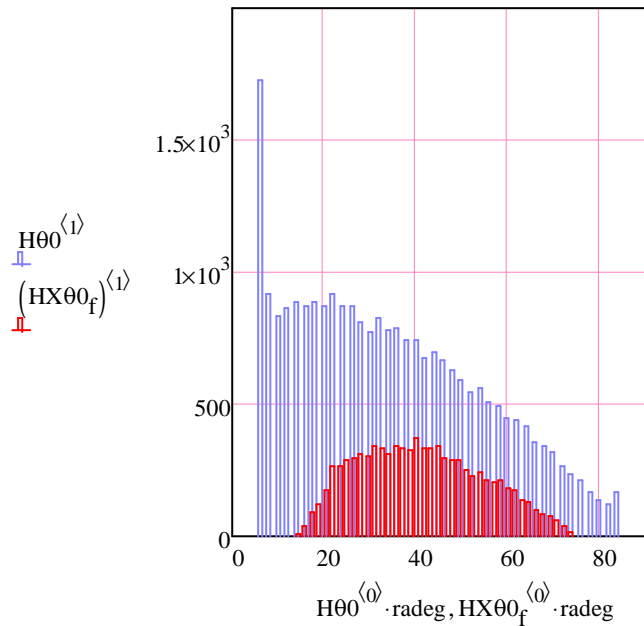
$HX\varphi_0_f := \text{histogram}(\text{nbin}, Xz^{(1)})$

$HXz_0_f := \text{histogram}(\text{nbin}, Xz^{(4)})$

Distribution of Scattering Angles

Distribution of Azimuths

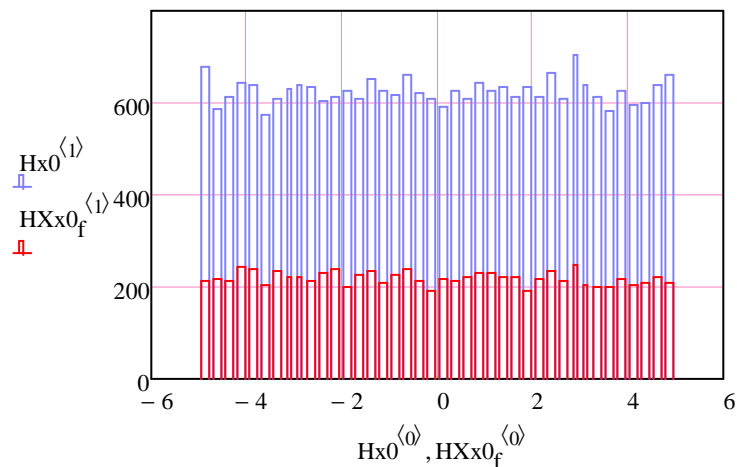
Distribution of Vertices



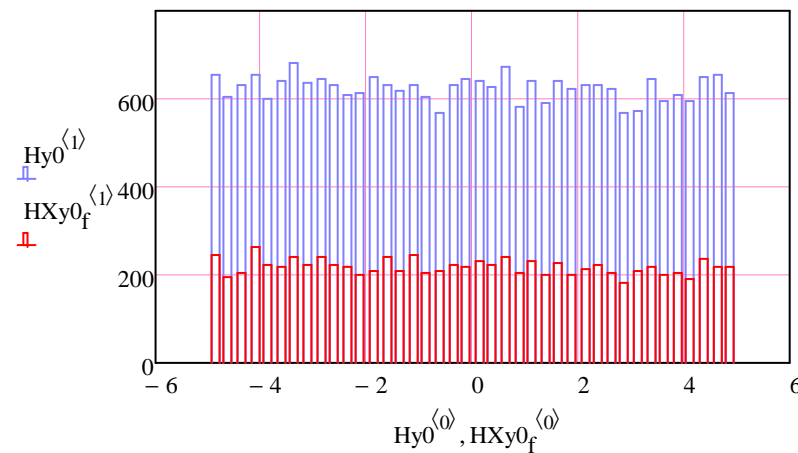
$$HXx0_f := \text{histogram}(nbin, Xz^{(2)})$$

$$HXy0_f := \text{histogram}(nbin, Xz^{(3)})$$

Distribution of Horizontal Transverse Vertex Offset



Distribution of Vertical Transverse Vertex Offset



$$HT1 := \text{histogram}(nbin, X^{(21)})$$

$$HT2 := \text{histogram}(nbin, X^{(22)})$$

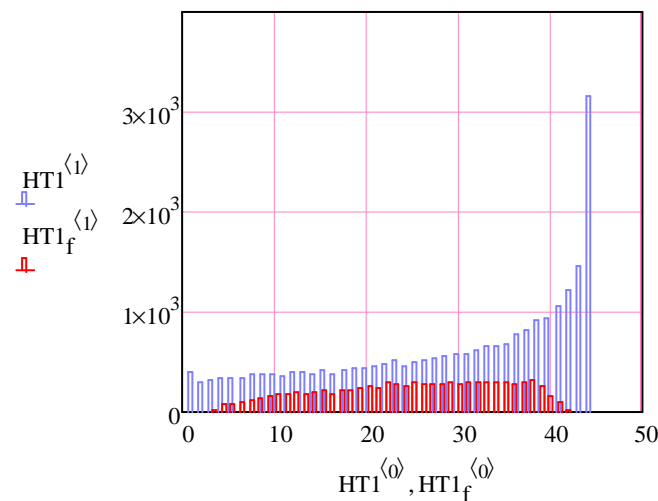
Total acceptance

$$\frac{\text{rows}(Xz)}{\text{rows}(X)} = 0.349$$

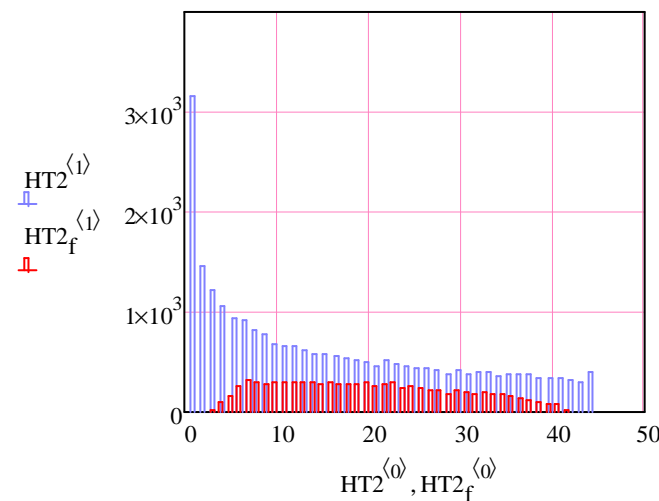
$$HT1_f := \text{histogram}(nbin, Xz^{(21)})$$

$$HT2_f := \text{histogram}(nbin, Xz^{(22)})$$

Distribution of Recoil Energy (particle 1)



Distribution of Recoil Energy (particle 2)

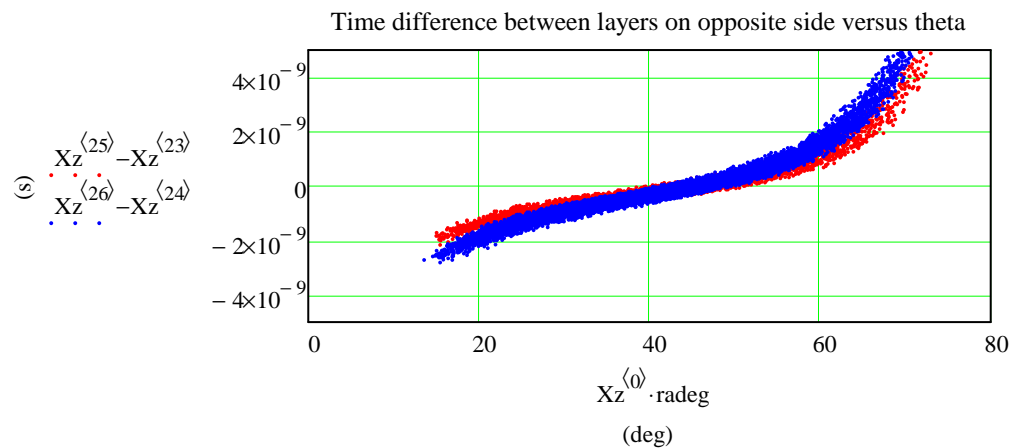
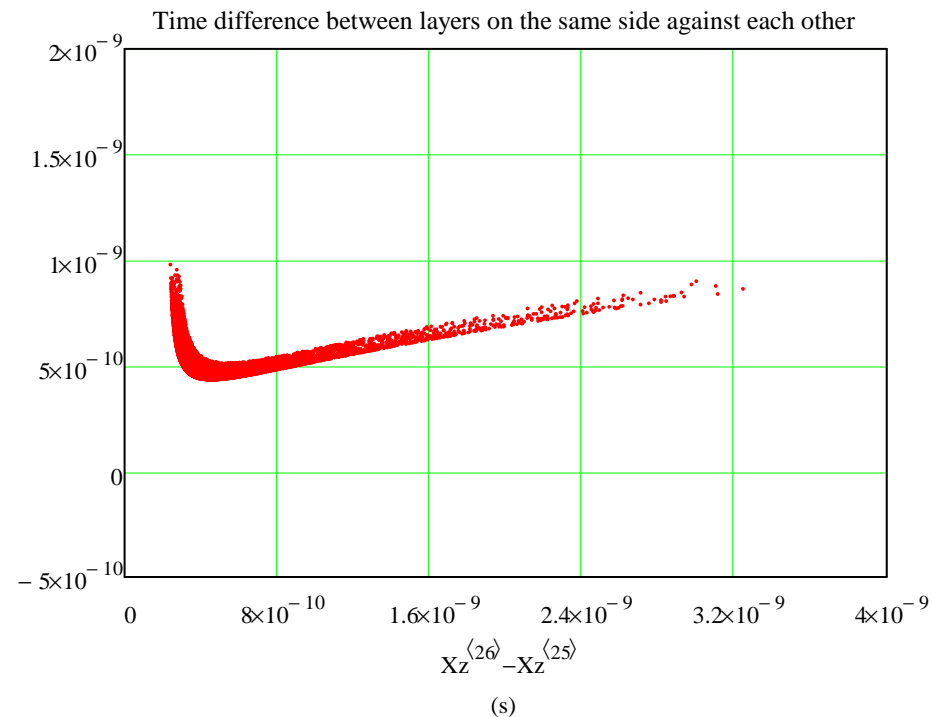
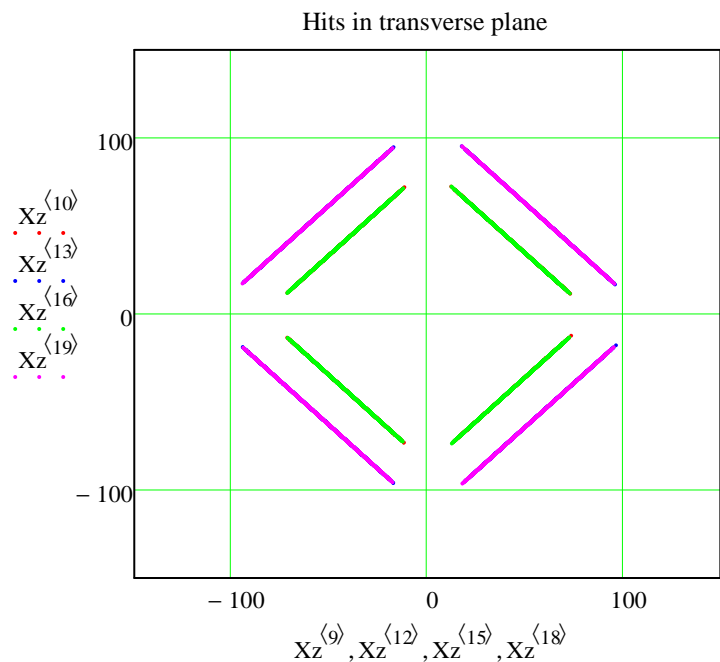


$$\max(Xz^{(21)}) = 42.517 \text{ MeV}$$

$$\min(Xz^{(21)}) = 3.078 \text{ MeV}$$

$$HI0x_f := \text{histogram}(\text{nbin}, Xz^{(9)}) \quad HI0y_f := \text{histogram}(\text{nbin}, Xz^{(10)})$$

Plot distribution of time difference between layers



- • • First Layer (red)
- • • Second Layer (blue)

$$\left[ \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right) - \text{Si1\_3D}(1,i)_1 \right]^2$$



$$\left. \begin{array}{l} \text{tq}(1, \varphi_{r_i}^0) - \text{Si2\_3D}(1, i) \\ \phantom{\text{tq}(1, \varphi_{r_i}^0)} \phantom{- \text{Si2\_3D}(1, i)} \phantom{1} \phantom{0} \end{array} \right\}^2$$