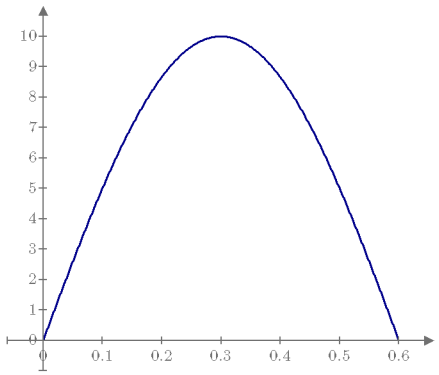
Example 5.1

$m := 0.2533 \ \frac{kip \cdot s^2}{in}$ $\quad k := 10 \ \frac{kip}{in}$ $\quad \omega_n := \sqrt{\frac{k}{m}} = 6.2832 \ \frac{rad}{s}$ $\quad T_n := \frac{2 \ \pi}{\omega_n} = 1 \ s$

$\xi := 5\%$ $\quad \Delta t := 0.1 \ s$ $\quad p(t) := 10 \sin\left(\frac{\pi \cdot t}{0.6 \ s}\right) kip$



$p(t \cdot s) \ (kip)$

$t \cdot s \ (s)$

Solution

$\omega_D := \omega_n \cdot \sqrt{1 - \xi^2} = 6.2754 \ \frac{rad}{s}$

$A := e^{-\xi \cdot \omega_n \cdot \Delta t} \left(\frac{\xi}{\sqrt{1 - \xi^2}} \sin(\omega_D \cdot \Delta t) + \cos(\omega_D \cdot \Delta t) \right) = 0.8129 \ rad$

$B := e^{-\xi \cdot \omega_n \cdot \Delta t} \left(\frac{1}{\omega_D} \sin(\omega_D \cdot \Delta t) \right) = 0.09067 \ s$

$C := \frac{1}{k} \left(\frac{2 \ \xi}{\omega_n \cdot \Delta t} + e^{-\xi \cdot \omega_n \cdot \Delta t} \left(\left(\frac{1 - 2 \ \xi^2}{\omega_D \cdot \Delta t} - \frac{\xi}{\sqrt{1 - \xi^2}} \right) \sin(\omega_D \cdot \Delta t) - \left(1 + \frac{2 \ \xi}{\omega_n \cdot \Delta t} \right) \cos(\omega_D \cdot \Delta t) \right) \right) = 0.01236 \ \frac{in}{kip}$

$D := \frac{1}{k} \left(1 - \frac{2 \ \xi}{\omega_n \cdot \Delta t} + e^{-\xi \cdot \omega_n \cdot \Delta t} \left(\frac{2 \ \xi^2 - 1}{\omega_D \cdot \Delta t} \sin(\omega_D \cdot \Delta t) + \frac{2 \ \xi}{\omega_n \cdot \Delta t} \cos(\omega_D \cdot \Delta t) \right) \right) = 0.006352 \ \frac{in}{kip}$

$A' := -e^{-\xi \cdot \omega_n \cdot \Delta t} \left(\frac{\omega_n}{\sqrt{1 - \xi^2}} \sin(\omega_D \cdot \Delta t) \right) = -3.5796 \ \frac{rad}{s}$

$B' := e^{-\xi \cdot \omega_n \cdot \Delta t} \left(\cos(\omega_D \cdot \Delta t) - \frac{\xi}{\sqrt{1 - \xi^2}} \sin(\omega_D \cdot \Delta t) \right) = 0.756 \ rad$

$C' := \frac{1}{k} \left(-\frac{1}{\Delta t} + e^{-\xi \cdot \omega_n \cdot \Delta t} \left(\left(\frac{\omega_n}{\sqrt{1 - \xi^2}} + \frac{\xi}{\Delta t \cdot \sqrt{1 - \xi^2}} \right) \sin(\omega_D \cdot \Delta t) + \frac{1}{\Delta t} \cos(\omega_D \cdot \Delta t) \right) \right) = 0.1709 \ \frac{in}{kip \cdot s}$

$D' := \frac{1}{k \cdot \Delta t} \left(1 - e^{-\xi \cdot \omega_n \cdot \Delta t} \left(\frac{\xi}{\sqrt{1 - \xi^2}} \sin(\omega_D \cdot \Delta t) + \cos(\omega_D \cdot \Delta t) \right) \right) = 0.1871 \ \frac{in}{kip \cdot s}$

$i := 0 .. 9$

$t_{i+1} := \left(1 \cdot \frac{i+1}{10}\right) s$

$$p\left(t_i\right) = \begin{bmatrix} 0 \\ 5 \\ 8.6603 \\ 10 \\ 8.6603 \\ 5 \\ 0 \\ -5 \\ -8.6603 \\ -10 \end{bmatrix} kip \quad C \cdot p\left(t_i\right) = \begin{bmatrix} 0 \\ 0.0618 \\ 0.107 \\ 0.1236 \\ 0.107 \\ 0.0618 \\ 0 \\ -0.0618 \\ -0.107 \\ -0.1236 \end{bmatrix} in \qquad D \cdot p\left(t_{i+1}\right) = \begin{bmatrix} 0.0318 \\ 0.055 \\ 0.0635 \\ 0.055 \\ 0.0318 \\ 0 \\ -0.0318 \\ -0.055 \\ -0.0635 \\ -0.055 \end{bmatrix} in$$

$$C' \cdot p\left(t_i\right) = \begin{bmatrix} 0 \\ 0.8544 \\ 1.4799 \\ 1.7089 \\ 1.4799 \\ 0.8544 \\ 0 \\ -0.8544 \\ -1.4799 \\ -1.7089 \end{bmatrix} \frac{in}{s} \qquad D' \cdot p\left(t_{i+1}\right) = \begin{bmatrix} 0.9354 \\ 1.6201 \\ 1.8707 \\ 1.6201 \\ 0.9354 \\ 0 \\ -0.9354 \\ -1.6201 \\ -1.8707 \\ -1.6201 \end{bmatrix} \frac{in}{s}$$

its necessary that results of ui and ui' are the same up to ti=0.6

$$u_{i+1} = Au_i + B\dot{u}_i + Cp_i + Dp_{i+1} \qquad (5.2.5a)$$

$$\dot{u}_{i+1} = A'u_i + B'\dot{u}_i + C'p_i + D'p_{i+1} \qquad (5.2.5b)$$

TABLE E5.1a NUMERICAL SOLUTION USING LINEAR INTERPOLATION OF EXCITATION

t_i	p_i	Cp_i	Dp_{i+1}	$B\dot{u}_i$	$\dot{u}_i$	Au_i	u_i	Theoretical u_i
0.0	0.0000	0.0000	0.0318	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	5.0000	0.0618	0.0550	0.0848	0.9354	0.0258	0.0318	0.0328
0.2	8.6602	0.1070	0.0635	0.2782	3.0679	0.1849	0.2274	0.2332
0.3	10.0000	0.1236	0.0550	0.4403	4.8558	0.5150	0.6336	0.6487
0.4	8.6603	0.1070	0.0318	0.4290	4.7318	0.9218	1.1339	1.1605
0.5	5.0000	0.0618	0.0000	0.1753	1.9336	1.2109	1.4896	1.5241
0.6	0.0000	0.0000	0.0000	−0.2735	−3.0159	1.1771	1.4480	1.4814
0.7	0.0000	0.0000	0.0000	−0.6767	−7.4631	0.7346	0.9037	0.9245
0.8	0.0000	0.0000	0.0000	−0.8048	−8.8765	0.0471	0.0579	0.0593
0.9	0.0000	0.0000	0.0000	−0.6272	−6.9177	−0.6160	−0.7577	−0.7751
1.0	0.0000				−2.5171		−1.2432	−1.2718

TABLE E5.1b NUMERICAL SOLUTION USING LINEAR INTERPOLATION OF EXCITATION

t_i	p_i	$C'p_i$	$D'p_{i+1}$	$A'u_i$	u_i	$B'\dot{u}_i$	$\dot{u}_i$	Theoretical $\dot{u}_i$
0.0	0.0000	0.0000	0.9354	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	5.0000	0.8544	1.6201	−0.1137	0.0318	0.7071	0.9354	0.9567
0.2	8.6602	1.4799	1.8707	−0.8140	0.2274	2.3192	3.0679	3.1383
0.3	10.0000	1.7088	1.6201	−2.2679	0.6336	3.6708	4.8558	4.9674
0.4	8.6603	1.4799	0.9354	−4.0588	1.1339	3.5771	4.7318	4.8408
0.5	5.0000	0.8544	0.0000	−5.3320	1.4896	1.4617	1.9336	1.9783
0.6	0.0000	0.0000	0.0000	−5.1832	1.4480	−2.2799	−3.0159	−3.0848
0.7	0.0000	0.0000	0.0000	−3.2347	0.9037	−5.6418	−7.4631	−7.6346
0.8	0.0000	0.0000	0.0000	−0.2074	0.0579	−6.7103	−8.8765	−9.0808
0.9	0.0000	0.0000	0.0000	2.7124	−0.7577	−5.2295	−6.9177	−7.0771
1.0	0.0000				−1.2432		−2.5171	−2.5754