Space-Time Simulation of Electrical Cable Heat up to Hot-Short Failure Temperature

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This MATHCAD 11 spread sheet calculator simulates the space-time heatup of an XLPE cable subject to an external fire. The model: (a) calculates initial temperature profile as a function of radius due to dissipation of electrical heat from continuous current flows, (b) the effects of both convective and radiative heat trasfer to the exterior of the cable, (c) effects of different material properties as a function of radius. Values hightlighted in GREY are adjusted to match different types of electrical cables.

Number of Conductor Strands: $Ns := 7$

Strand diameter in inches: $d := 0.1285$ in.

Insulation thickness in inches: $tins := 0.06$ in.

Steady State Pre-Fire Initial Conditions:

Calculation of Heat dissipation per foot:

Steady State RMS Current (Amps): $Iss := 0.0$

Cable Resistance per foot(Ohms/ft): $Rcable := \frac{0.67}{1000}$

$Rcable = 6.7 \times 10^{-4}$ (Ohms/ft)

RMS Power dissipation in Cable per foot (Watts/foot): $Prms := Iss^2 \cdot Rcable$

- this is the <u>volumetric heat genertion</u> rate and assumes current is distributed evenly in cable strands-

$Prms = 0$ Watts/ft

Cross-sectional area of Conductor in sq.ft.: $Acu := Ns \cdot \pi \cdot \left(\frac{\frac{d}{12}}{2}\right)^2$

$Acu = 6.304 \times 10^{-4}$ sq.ft.

Conversion of Watts/ft to BTU/sec.ft.via multiplying by $9.478 \cdot 10^{-4}$: $Qo := Prms \cdot \left(9.478 \cdot 10^{-4}\right)$

$Qo = 0$ BTU/sec.ft.

Conversion of heat dissipation in BTU/sec. ft to volumetric heat disspation in BTU/sec.cu.ft. via dividing by Volume/ft or: Acu $qo := \frac{Qo}{Acu}$

$qo = 0$ BTU/sec.cu.ft

Ambient air temperature "pre-fire" in deg. F: $Tair := 65$ deg. F:

XLPE Insulation Thermal and Materials Property Data:

Softening Temperature of XLPE 105 C - 115 C is conservatively used as material temperature limit for onset of electrical hot-shorting. $Ths := 221$ deg. F

Melting Point Temperature of XLPE 124 C - 131 C is an upper limit on insulation material integrity $Tmelt := 255$ deg. F

Using data from NUREG-1821 Vol. 6 Table 5-1:

Thermal conductivity of the Insulation layer: gives 0.00021 kW/m degK $ksh := 3.37 \cdot 10^{-5}$ Btu/sec.ft.deg F

Material density (same source) ρ = 1375 kg/cu.m. $\rho sh := 85.837$ lb./cu.ft.

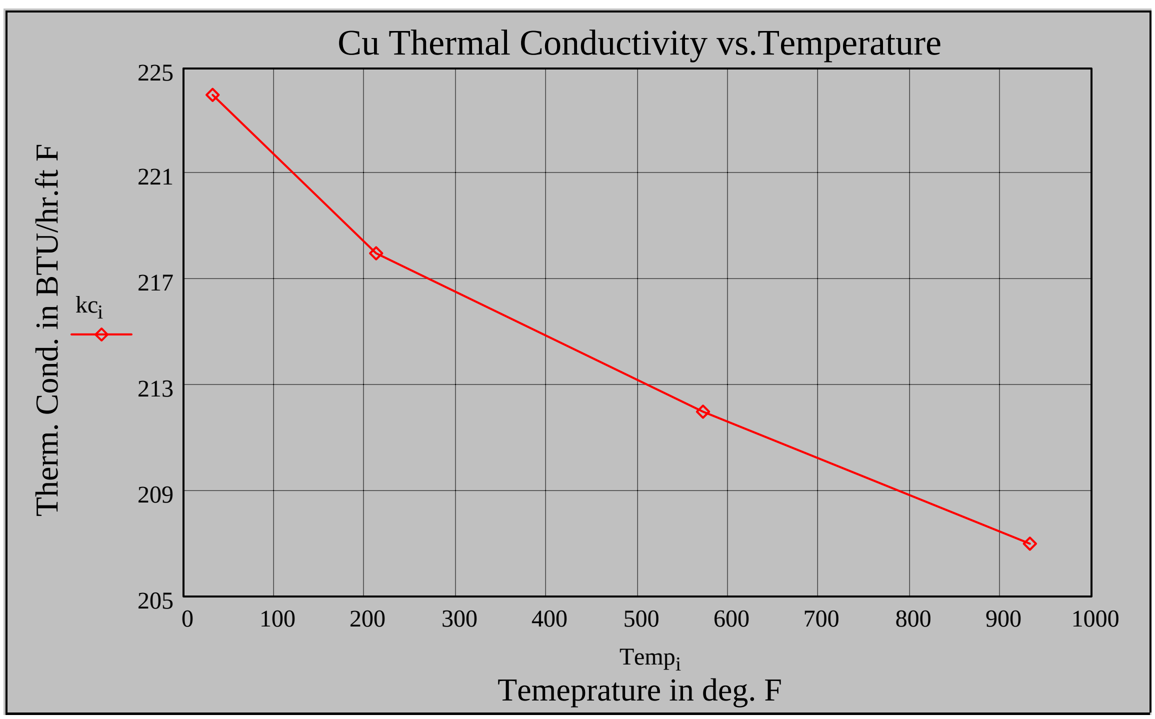
Heat Capacity (same source) Cp = 1.566 kJ/kg degK $Csh := 0.374$ BTU/lb.deg F

Copper Thermal and Materials Property Data
Source: Kreith: "*Principles of Heat Transfer*", p. 593

$i := 0 .. 3$

$$ThermalCondData := \begin{pmatrix} 32 & 224 \\ 212 & 218 \\ 572 & 212 \\ 932 & 207 \end{pmatrix}$$

$Temp_i := ThermalCondData_{i,0}$ $kc_i := ThermalCondData_{i,1}$



Copper Thermal Conductivity in Btu/sec.ft.deg F requires dividing by 3600sec/hr: $kcu := \frac{215}{3600}$

$kcu = 0.06$ Btu/sec.ft.deg F

Density of Copper Conductor (lbs/cu.ft.): $\rho cu := 556.85$ lbs/cu.ft.

Mass of Copper Conductor per foot: $Mcu := Acu \cdot \rho cu$

$Mcu = 0.351$ lbs/ft

Effective radius of Copper Conductor bundle (ft.) : $Ro := \sqrt{\frac{Acu}{\pi}}$

$Ro = 0.014$ ft.

Effective radius of Insulating sheath (ft.) : $Rs := Ro + \frac{tins}{12}$

$Rs = 0.019$ ft.

Heat Capacity of Copper Conductor: $Ccu := 0.091$ BTU/lb.deg F

Equation governing "pre-fire" steady state heat transfer from cable surface to free air:

Assume surface convection from cable surface to surrounding free air.

qo V = qo ($\pi \cdot Rs^2 \cdot L$) = 2π Rs L hc (Tins(Rs) - Tair) then: Tins(Rs) = Ts = Tair + qo Rs/ hc

$hc := \frac{5}{3600}$ $hc = 1.389 \times 10^{-3}$ BTU/sec sq.ft.F

$Ts := Tair + \frac{qo \cdot Rs}{hc}$ $Ts = 65$

Equation governing "pre-fire" steady state heat transfer in the Insulator layer:

0 = -ksh/r $\frac{d}{dr}\left[r \cdot \left(\frac{d}{dr} Tins(r, 0)\right)\right]$, has no internal volumetric heat source - only heat transfer

Integrating this expression yields: $\frac{d}{dr} Tins(r, 0) = \frac{-Co}{r}$, integrating again yields:

$Tins(r) = C_1 - Co \cdot \ln(r)$

Applying the boundary condition: Tins(Rs) = Ts and solving for C_1 yields:

C_1 = Ts + Co ln(Rs), thus: Tins(r) = Ts - Co ln(r/Rs)

To solve for Co, the boundary condition related to the temperature drop across a cylindrical shell from a heat source $qo \cdot \pi \cdot Ro^2 \cdot L$ (BTU/sec) is used:

$$qo \cdot \pi \cdot Ro^2 \cdot L = \frac{2 \cdot \pi \cdot L \cdot k_{sh} \cdot (Tins(Ro) - Tins(Rs))}{\ln\left(\frac{Rs}{Ro}\right)}$$

Simplifying the expression and solving for Tins(Rs) yields:

$$Tins(Ro) = Tins(Rs) + \frac{qo \cdot Ro^2}{2 \cdot ksh} \cdot \ln\left(\frac{Rs}{Ro}\right)$$

$$Tins(Rs) + \frac{qo \cdot Ro^2}{2 \cdot ksh} \cdot \ln\left(\frac{Rs}{Ro}\right) = Tins(Rs) - Co \cdot \ln\left(\frac{Ro}{Rs}\right)$$

Solving for Co then yields: $Co = \frac{qo \cdot Ro^2}{2 \cdot ksh}$, thus:

$$Tins(r) := Ts - \frac{qo \cdot Ro^2}{2 \cdot ksh} \cdot \ln\left(\frac{r}{Rs}\right)$$

$$Tins(r) := Tair + \frac{qo \cdot Rs}{hc} - \frac{qo \cdot Ro^2}{2 \cdot ksh} \cdot \ln\left(\frac{r}{Rs}\right)$$

$Tins(Ro) = 65$ $Ts = 65$

Equation governing "pre-fire" steady state heat transfer in the Conductor:

$q\,r = -kcu \frac{d}{dr}\left[r \cdot \left(\frac{d}{dr}T(r,0)\right)\right]$ for volumetric heat source q(BTU/sec cu.ft.) with boundary conditions:

$\left(\frac{d}{dr}T(r,t)\right) = 0$ when r = 0, and T(Ro,0) = To

Integrating this yields:

$$\frac{-q}{kcu} \cdot \int r\,dr + Co = r \cdot \left(\frac{d}{dr}T(r,0)\right)$$

$$\frac{-q \cdot r}{kcu} + \frac{Co}{r} = \frac{d}{dr}T(r,t)$$

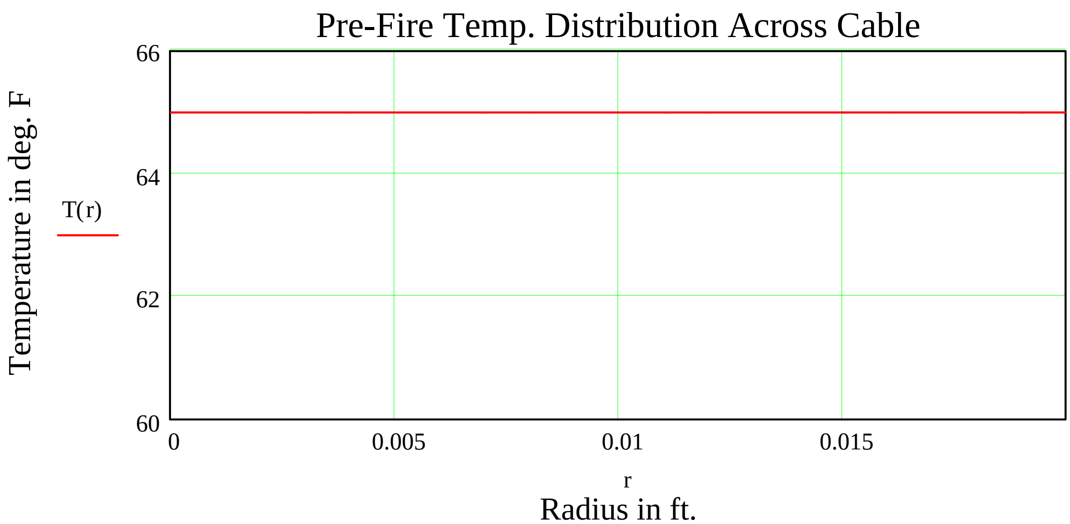
Using the boundary condition: $\left(\frac{d}{dr}T(r,0)\right) = 0$ when r = 0

Co must be Co = 0

Integrating again yields: $T(r,t) = C_1 - \frac{q \cdot r^2}{4kcu}$

Solving for C_1 using T(Ro,0) = To, yields: $C_1 = To + \frac{q \cdot Ro^2}{4 \cdot kcu}$ Thus:

$$T(r) := \begin{cases} Tair + \frac{qo \cdot Rs}{hc} - \frac{qo \cdot Ro^2}{2 \cdot ksh} \cdot \ln\left(\frac{Ro}{Rs}\right) + \frac{qo \cdot Ro^2}{4 \cdot kcu} \cdot \left(1 - \frac{r^2}{Ro^2}\right) & \text{if } r \le Ro \\ Tair + \frac{qo \cdot Rs}{hc} - \frac{qo \cdot Ro^2}{2 \cdot ksh} \cdot \ln\left(\frac{r}{Rs}\right) & \text{if } Ro < r \le Rs \\ Tair & \text{otherwise} \end{cases}$$



Specification of the material properties and Misc. constants as a function of region:

$$q(r,t) := \begin{cases} qo & \text{if } (r \le Ro) \wedge (t \ge 0) \\ 0 & \text{otherwise} \end{cases}$$

Heat Source (BTU/sec.cu.ft.) vs. r

$$To(r,t) := \begin{cases} Tair + \frac{qo \cdot Rs}{hc} - \frac{qo \cdot Ro^2}{2 \cdot ksh} \cdot \ln\left(\frac{Ro}{Rs}\right) + \frac{qo \cdot Ro^2}{4 \cdot kcu} \cdot \left(1 - \frac{r^2}{Ro^2}\right) & \text{if } r \le Ro \\ Tair + \frac{qo \cdot Rs}{hc} - \frac{qo \cdot Ro^2}{2 \cdot ksh} \cdot \ln\left(\frac{r}{Rs}\right) & \text{if } Ro < r \le Rs \\ Tair & \text{otherwise} \end{cases}$$

Temperature (deg F) distribution before fire.

$kair := \frac{0.02}{3600}$ $kair = 5.556 \times 10^{-6}$ BTU/sec.ft.F - based on kair at 400 F, Kreith, p.595

$$k(r) := \begin{cases} kcu & \text{if } r \le Ro \\ ksh & \text{if } Ro < r \le Rs \\ kair & \text{otherwise} \end{cases}$$

Conductance(Btu/sec.ft.deg F) vs. r
Inner region is copper conductor, outer region is insulator, followed by air.

$$\rho(r) := \begin{cases} 556.85 & \text{if } r \le Ro \\ 85.837 & \text{otherwise} \end{cases}$$

Mass Density (lb./cu.ft.) vs. r
Inner region is copper, outer is insulator

$$C(r) := \begin{cases} 0.091 & \text{if } r \le Ro \\ 0.374 & \text{otherwise} \end{cases}$$

Heat Capacity (BTU/lb.deg F) vs. r
Inner region is copper, outer is insulator

$$Tfire(t) := \begin{cases} 0.0014 \cdot t^2 + Tair & \text{if } 0 \le t \le 600 \\ 0.0014 \cdot (600)^2 + Tair & \text{otherwise} \end{cases}$$

This simplified model for fire-related heatup assume time-square fire growth rate which assumes heat conduction to an ultimate heat sink such as concrete walls and other surfaces.

$$\varepsilon := 0.9$$

Emissivity recommended value per NUREG-1821

$$\sigma := \frac{1.714 \cdot 10^{-9}}{3600}$$

$$\sigma = 4.761 \times 10^{-13}$$ BTU/sec.sq.ft.deg R

Stefan-Boltzman constant converted from BTU/hr.sq.ft.deg R to units of BTU/sec.sq.ft.deg R -- Source: Kreith, Principles of Heat Transfer, p.12

Partial Differential Equation of Space-Time Dependent Temperature:

$spacepts := 1000$ $timepts := 100$ $time := 1000$

Given

$$T_t(r,t) = \frac{k(r)}{\rho(r) \cdot C(r)} \cdot \left(T_{rr}(r,t) + \frac{1}{r} \cdot T_r(r,t) \right) + \frac{q(r,t)}{\rho(r) \cdot C(r)}$$

Space-Time equation for temperature distribution based upon energy balance

$$T(r,0) = To(r,0)$$

Sets the initial temperature ditribution to pre-fire values based upon internal heat generted by electrical cable resistive heat loss.

$$T_r(0.0001,t) = 0$$

This is the standard symmetry boundary condition. NOTE: r = 0 **cannot be used as this results in singularity** in PDE solver routine.

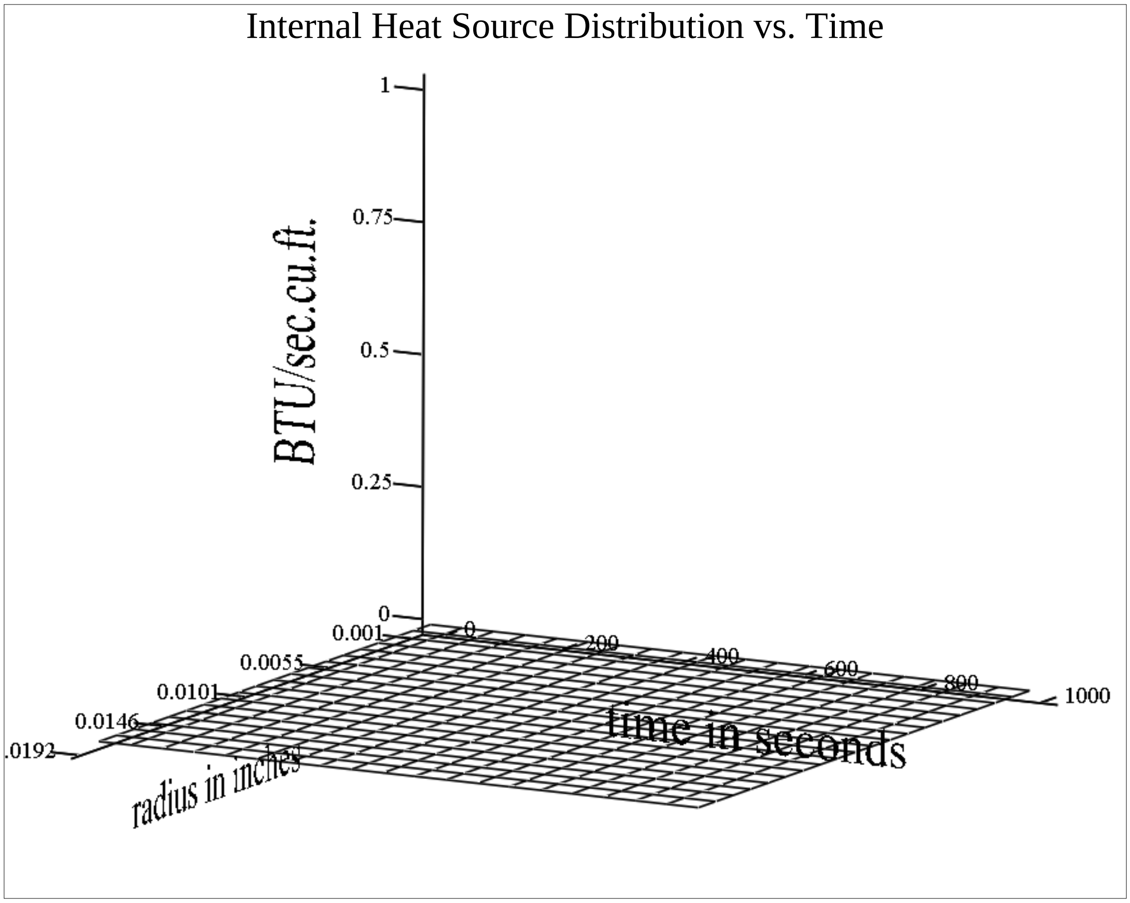
$$T_r(Rs,t) = \frac{-hc}{k(Rs)} \cdot (T(Rs,t) - Tfire(t)) - \varepsilon \cdot \frac{\sigma}{k(Rs)} \cdot \left[(T(Rs,t) + 459.67)^4 - (Tfire(t) + 459.67)^4\right]$$

This boundary condition incorporates the convective and radiative heat source terms.

$$T := \text{Pdesolve}\left[T, r, \begin{pmatrix} 0.0001 \\ Rs \end{pmatrix}, t, \begin{pmatrix} 0 \\ time \end{pmatrix}, spacepts, timepts\right]$$

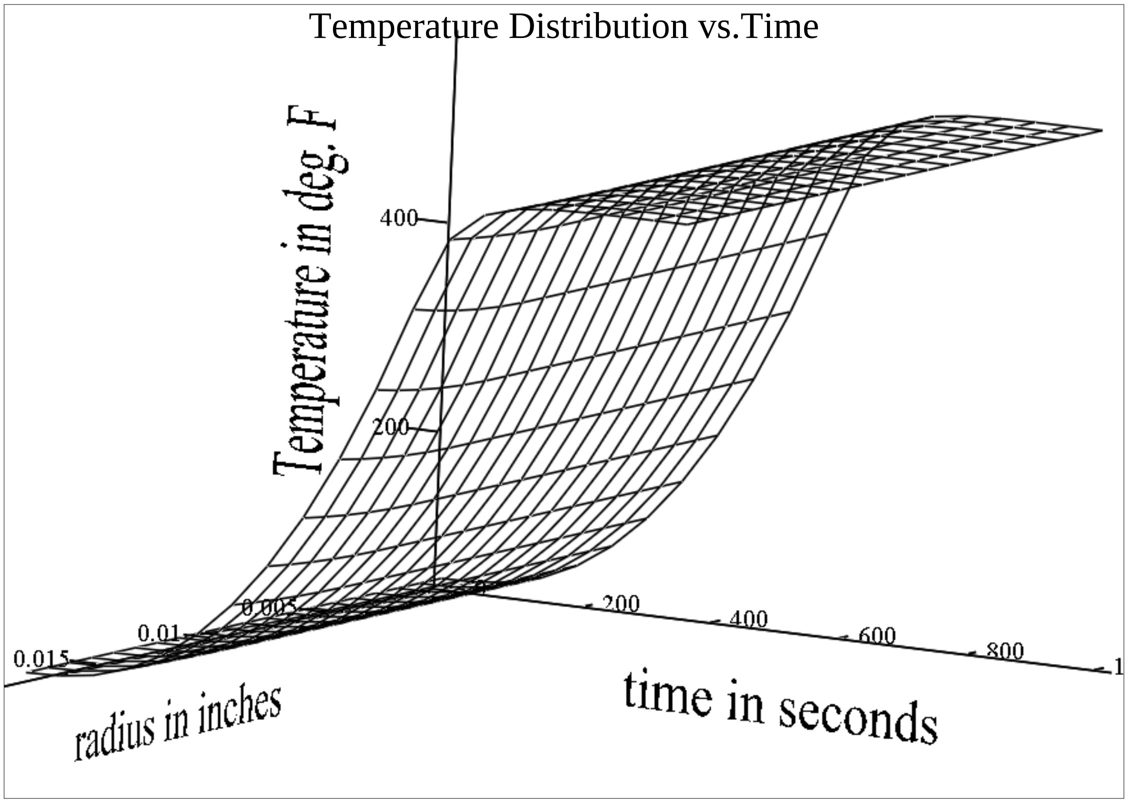
This expression executes the standard MATHCAD PDE solver routine.

$$Q := \text{CreateMesh}(q, 0.001, Rs, 0, time)$$



Q

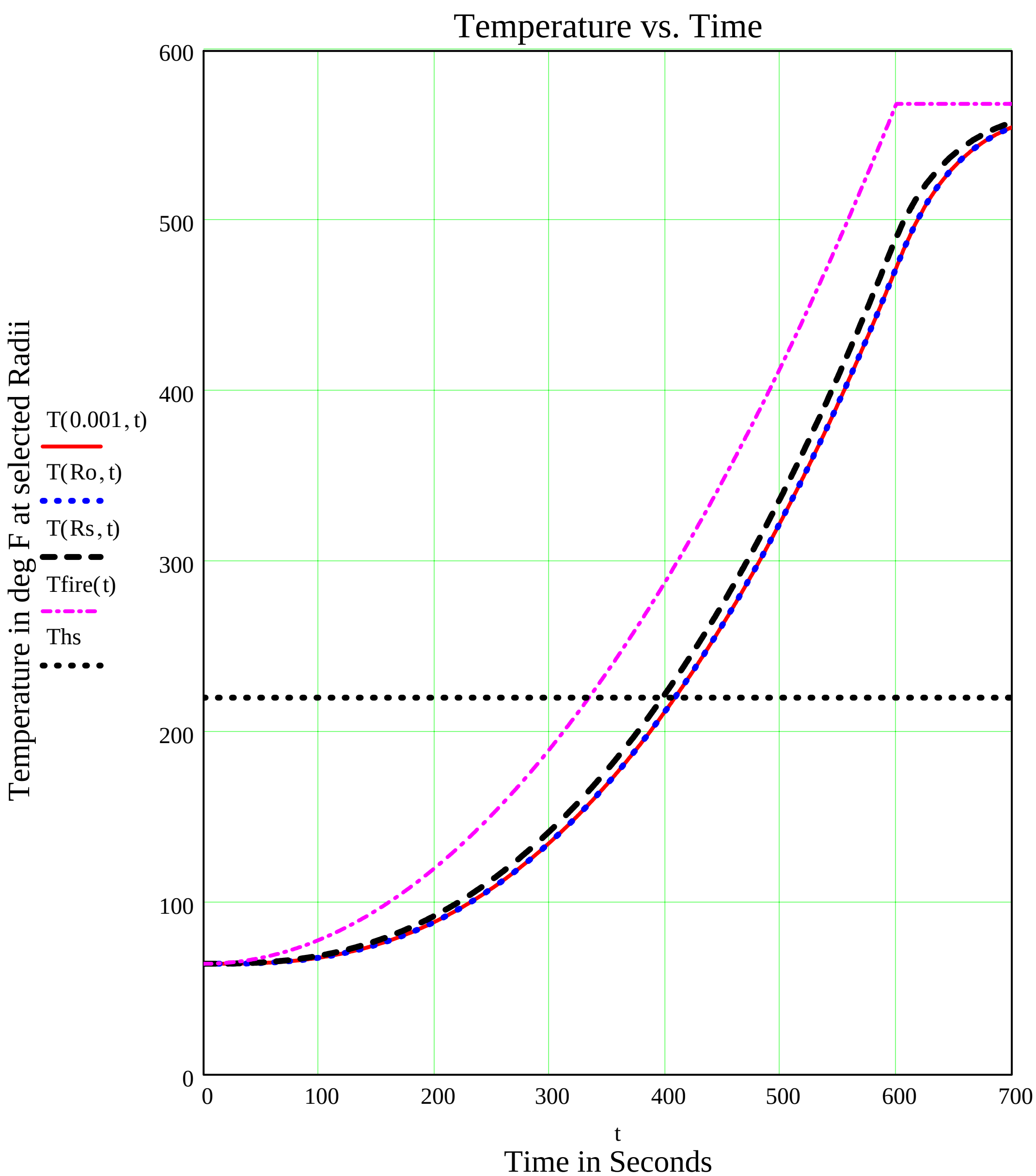
M := CreateMesh(T, 0.001, Rs, 0, time)



M

Comparison of Copper conductor temperatures: T(0.001,t) T(Ro,t),

XLPE insulator temperature: T(Rs,t) vs assumed hot-short failure temperature: Ths and ambient air temperature during fire: Tfire(t)



Comparison of Convective vs. Radiative Heat Flux Source Terms

$\text{Qconv}(t) := -hc\cdot(T(Rs,t) - \text{Tfire}(t))$ BTU/sec.sq.ft.

$\text{Qrad}(t) := -\varepsilon\cdot\sigma\cdot\left[(T(Rs,t) + 459.67)^4 - (\text{Tfire}(t) + 459.67)^4\right]$ BTU/sec.sq.ft.