Flutter Analysis with Damping

1. Input

$I_{xx} = 4.70\ in^2\text{-}lbm$

$I_{yy} = 2.00\ in^2\text{-}lbm$

$I_{xy} = 0.57\ in^2\text{-}lbm$

$\Delta = 10\ \frac{in\text{-}lbf}{r}$

$A = 65.4\ in^2$

$C_{\ell\alpha} = 2.065\ in$

$X_{ac} = -0.020\ in$

$Y_{ac} = 2.065\ in$

$M = 3.0$

$g = 386.4\ \frac{in\text{-}lbm}{lbf\text{-}s^2}$

$R = 639.6\ \frac{in}{°R}$

$\gamma = 1.4$

$P_a = 14.7\ psia$

$T = 70°F$

$Z_\theta = 0.90$ Bending Damping Factor

$Z_\alpha = 0.90$ Torsion Damping Factor

$\eta_{xx} = 0$ Aerodynamic Damping

$\eta_{yy} = 0$

$\eta_{xy} = 0$

2. Initial Calculations

$I_{xx} = \frac{I_{xx}}{g}\ in\text{-}lbf\text{-}s^2$

$I_{yy} = \frac{I_{yy}}{g}\ in\text{-}lbf\text{-}s^2$

$I_{xy} = \frac{I_{xy}}{g}\ in\text{-}lbf\text{-}s^2$

$Q = \frac{1}{2} \gamma P_a M^2\ psia$

$V = \sqrt{\gamma R (T + 459.67)}\ \frac{in}{s}$

3. Initial k_θ & k_α Values (No Damping)

$$k_{\theta i} = k_\theta = \frac{I_{xx}}{I_{xy}} Q A C_{L\alpha} Y_{ac}$$

$$k_{\alpha i} = k_\alpha = \frac{I_{yy}}{I_{xy}} Q A C_{L\alpha} Y_{ac} - Q A C_{L\alpha} X_{ac}$$

$J = 1$

4. Parameters

$$C_\theta = 2 \zeta_\theta \sqrt{I_{xx} k_\theta}$$

$$C_\alpha = 2 \zeta_\alpha \sqrt{I_{yy} k_\alpha}$$

$$C'_\theta = C_\theta + \frac{Q}{V A C_{L\alpha}} \eta_{yy}$$

$$C'_\alpha = C_\alpha + \frac{Q}{V A C_{L\alpha}} \eta_{xx}$$

$$C'_{\alpha\theta} = \frac{Q}{V A C_{L\alpha}} \eta_{xy}$$

$$k_{\alpha\theta} = Q A C_{L\alpha} Y_{ac}$$

$$k_{\alpha\alpha} = k_\alpha + Q A C_{L\alpha} X_{ac}$$

5. Coefficients of Characteristic Equation

$$a = I_{xx} I_{yy} - I_{xy}^2$$

$$b = I_{xx} C'_\alpha + I_{yy} C'_\theta$$

$$c = I_{xx} k_{\alpha\alpha} + C'_\alpha C'_\theta + I_{yy} k_\theta - C'^2_{\alpha\theta} - I_{xy} k_{\alpha\theta}$$

$$d = k_{\alpha\alpha} C'_\theta + k_\theta C'_\alpha - k_{\alpha\theta} C'_{\alpha\theta}$$

$$e = k_\theta k_{\alpha\alpha}$$

6. Solve for roots of characteristic equation

$$a s^4 + b s^3 + c s^2 + d s + e = 0$$

Roots s_1, s_2, s_3, s_4

7. Evaluate Roots (Positive Root Results in Flutter)

If Real Part of S_1, S_2, S_3, or S_4 is ≥ 0

Then $R(j) = 1$, Otherwise $R(j) = -1$

If $R(j) + R(j-1) = 0$, then Print K_α and K_θ

$K_\theta = K_\theta - \Delta$

$J = J + 1$

If $K_\theta > 0$ Then Go To 4

8. $K_\alpha = K_\alpha - \Delta$

$K_\theta = K_{\theta i}$

If $K_\alpha > 0$ Then Go To 4