

## Initial Assumptions & Constants

$$V_{ru} := 500000 \text{ m}^3$$

Upper Reservoir Volume

$$V_{rl} := V_{ru}$$

Lower Reservoir Volume

$$g := 9.81 \frac{\text{m}}{\text{s}^2}$$

Acceleration Due to Gravity

$$\mu := 8.9 \cdot 10^{-4} \text{ Pa} \cdot \text{s}$$

Fluid Viscosity

$$\rho := 1000 \frac{\text{kg}}{\text{m}^3}$$

Fluid Density

$$t_g := 4 \text{ hr}$$

Time to Empty Reservoir

$$t_p := 6 \text{ hr}$$

Time to Fill Reservoir

$$\eta_t := 0.9$$

Efficiency of Turbine & Generator

$$\eta_p := 0.85$$

Efficiency of Pump

$$A := 0.9$$

Plant Availability

$$\Delta h := 500 \text{ m}$$

Elevation Difference

$$\phi := 20^\circ$$

Bedding angle of mine seam

	"hours"	"SMPg"	"SMPp"
2	66.892	35.071	
3	65.592	35.246	
4	63.747	35.536	
5	61.989	35.874	
6	60.158	36.322	
7	58.644	36.811	
8	57.286	37.416	

PriceData :=

$$L(\Delta h) := \left( \left( 300 \text{ m} + \left( \frac{\Delta h - 300 \text{ m}}{\sin(\phi)} \right) \right) \cdot 1.1 \right) \quad \text{Pipe Length}$$

$$L(\Delta h) = 973 \text{ m}$$

$$D_p := 2 \text{ m} \quad \text{Pipe Diameter}$$

$$A_s(D_p) := \pi \cdot \frac{D_p^2}{4} \quad \text{Pipe Cross Sectional Area}$$

$$A_s(D_p) = 3.14 \text{ m}^2$$

$$\varepsilon_p := 0.25 \text{ mm} \quad \text{Pipe Surface Roughness}$$

$$\frac{\varepsilon_p}{D_p} = 1.25 \cdot 10^{-4} \quad \text{Pipe Relative Roughness}$$

$$m_g(V_{ru}, t_g) := \frac{V_{ru}}{t_g \cdot 3600 \frac{s}{hr}}$$

Volume Flow Rate (Generating)

$$m_g(V_{ru}, t_g) = 34.72 \frac{m^3}{s}$$

$$m_p(V_{rl}, t_p) := \frac{V_{rl}}{t_p \cdot 3600 \frac{s}{hr}}$$

Volume Flow Rate (Pumping)

$$m_p(V_{rl}, t_p) = 23.15 \frac{m^3}{s}$$

$$V_g(V_{ru}, t_g, D_p) := \frac{m_g(V_{ru}, t_g)}{A_s(D_p)}$$

Average Fluid Velocity (Generating)

$$V_g(V_{ru}, t_g, D_p) = 11.052 \frac{m}{s}$$

$$V_p(V_{rl}, t_p, D_p) := \frac{m_p(V_{rl}, t_p)}{A_s(D_p)}$$

Average Fluid Velocity (Pumping)

$$V_p(V_{rl}, t_p, D_p) = 7.37 \frac{m}{s}$$

## Fluid Dynamics Calculations

$$Re_g(V_{ru}, t_g, D_p) := \frac{\rho \cdot V_g(V_{ru}, t_g, D_p) \cdot D_p}{\mu}$$

Reynolds Number (Generating)

$$Re_g(V_{ru}, t_g, D_p) = 2.48 \cdot 10^7$$

$$Re_p(V_{rl}, t_p, D_p) := \frac{\rho \cdot V_p(V_{rl}, t_p, D_p) \cdot D_p}{\mu}$$

Reynolds Number (Pumping)

$$Re_p(V_{rl}, t_p, D_p) = 1.66 \cdot 10^7$$

Solver Constraints Guess Values

$$f := 1$$

$$\frac{1}{\sqrt{f}} = -2 \cdot \log \left( \frac{\varepsilon_p \cdot D_p^{-1}}{3.7} + \frac{2.51}{x \cdot \sqrt{f}} \right)$$

$$ff(x, D_p) := find(f)$$

Friction Factors According to Colebrook Equation

$$h_g(V_{ru}, t_g, D_p, \Delta h) := ff(Re_g(V_{ru}, t_g, D_p), D_p) \cdot \frac{L(\Delta h) \cdot V_g(V_{ru}, t_g, D_p)^2}{2 D_p \cdot g}$$

$$h_g(V_{ru}, t_g, D_p, \Delta h) = 38.08 \text{ m}$$

Head Loss (Generating)

$$h_p(V_{rl}, t_p, D_p, \Delta h) := ff(Re_p(V_{rl}, t_p, D_p), D_p) \cdot \frac{L(\Delta h) \cdot V_p(V_{rl}, t_p, D_p)^2}{2 D_p \cdot g}$$

$$h_p(V_{rl}, t_p, D_p, \Delta h) = 16.97 \text{ m}$$

Head Gain (Pumping)

$$H_g(V_{ru}, t_g, D_p, \Delta h) := \Delta h - h_g(V_{ru}, t_g, D_p, \Delta h)$$

$$H_g(V_{ru}, t_g, D_p, \Delta h) = 461.92 \text{ m}$$

Effective Head (Generating)

$$H_p(V_{rl}, t_p, D_p, \Delta h) := \Delta h + h_p(V_{rl}, t_p, D_p, \Delta h)$$

$$H_p(V_{rl}, t_p, D_p, \Delta h) = 516.97 \text{ m}$$

Effective Head (Pumping)

$$P_t(V_{ru}, t_g, D_p, \Delta h) := \rho \cdot g \cdot H_g(V_{ru}, t_g, D_p, \Delta h) \cdot m_g(V_{ru}, t_g) \cdot \eta_t$$

$$P_t(V_{ru}, t_g, D_p, \Delta h) = 141.61 \text{ MW}$$

Turbine Power (Generating)

$$P_p(V_{rl}, t_p, D_p, \Delta h) := \frac{\rho \cdot g \cdot H_p(V_{rl}, t_p, D_p, \Delta h) \cdot m_p(V_{rl}, t_p)}{\eta_p}$$

$$P_p(V_{rl}, t_p, D_p, \Delta h) = 138.11 \text{ MW}$$

Pump Power (Pumping)

$$\eta_{rt}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) := \frac{P_t(V_{ru}, t_g, D_p, \Delta h) \cdot t_g}{P_p(V_{rl}, t_p, D_p, \Delta h) \cdot t_p}$$

$$\eta_{rt}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) = 68.4\%$$

Round Trip Efficiency

$$hr_g := 365 \cdot A \cdot t_g$$

Maximum Annual Running Hours  
(Generating)

$$hr_g = 1314 \text{ hr}$$

$$hr_p := 365 \cdot A \cdot t_p$$

Maximum Annual Running Hours  
(Pumping)

$$hr_p = 1971 \text{ hr}$$

$$P_c(V_{rl}, V_{ru}, t_p, t_g, D_p, \Delta h) := P_p(V_{rl}, t_p, D_p, \Delta h) \cdot hr_p - P_t(V_{ru}, t_g, D_p, \Delta h) \cdot hr_g$$

$$P_c(V_{rl}, V_{ru}, t_p, t_g, D_p, \Delta h) = 86146 \text{ MW} \cdot \text{hr}$$

Energy Consumed Per Annum

$$C_p := \frac{t_g}{24 \text{ hr}}$$

Plant Generating Capacity Factor

$$C_p = 16.7\%$$

**Price Rates**

$$P_{shaft} := 50 \frac{\text{₹}}{m^3}$$

Cost of Shaft Excavations per m3

$$P_{res} := 10 \frac{\text{₹}}{m^3}$$

Cost of Reservoir Excavations per m3

$$AC_{dw}(\Delta h) := 100 \frac{\text{₹}}{m} \cdot L(\Delta h)$$

Annual Dewatering Cost

$$AC_{dw}(\Delta h) = 97324 \text{ ₹}$$

## Civil Works Calculations

$$GH_l(V_{ru}, t_g, D_p, \Delta h) := 80 \text{ m} + 0.05 \frac{m}{MW} \cdot P_t(V_{ru}, t_g, D_p, \Delta h)$$

Generator Hall Length

$$GH_l(V_{ru}, t_g, D_p, \Delta h) = 87.1 \text{ m}$$

$$GH_w(V_{ru}, t_g, D_p, \Delta h) := 20 \text{ m} + 0.05 \frac{m}{MW} \cdot P_t(V_{ru}, t_g, D_p, \Delta h)$$

Generator Hall Width

$$GH_w(V_{ru}, t_g, D_p, \Delta h) = 27.1 \text{ m}$$

$$GH_h := 40 \text{ m}$$

Generator Hall Height

$$Vol_{gh}(V_{ru}, t_g, D_p, \Delta h) := GH_l(V_{ru}, t_g, D_p, \Delta h) \cdot GH_w(V_{ru}, t_g, D_p, \Delta h) \cdot GH_h$$

$$Vol_{gh}(V_{ru}, t_g, D_p, \Delta h) = 94327 \text{ m}^3$$

Generator Hall Volume

$$Vol_{res\_ex} := 1000 \text{ m}^3$$

Volume of Existing Reservoirs at Various Depths

$$Vol_{res\_new}(V_{ru}, t_g, D_p, \Delta h) := (V_{ru} + V_{rl} + Vol_{gh}(V_{ru}, t_g, D_p, \Delta h)) - Vol_{res\_ex}$$

$$Vol_{res\_new}(V_{ru}, t_g, D_p, \Delta h) = 1093327 \text{ m}^3$$

Volume of Excavations For Reservoirs & Generator Hall

$$L_{shaft} := 326 \text{ m}$$

$$D_{shaft} := 3.5 \text{ m}$$

Dimensions of Existing Shaft

$$V_{shaft\_u}(D_p) := if\left(D_p > D_{shaft}, \left(L_{shaft} \cdot \frac{\pi}{4} \cdot (D_p - D_{shaft})^2\right), 0 \text{ m}^3\right)$$

$$V_{shaft\_u}(D_p) = 0 \text{ m}^3$$

Excavations Required for Upper Shaft

$$V_{shaft\_l}(D_p, \Delta h) := if\left(L(\Delta h) > L_{shaft}, (L(\Delta h) - L_{shaft}) \cdot \frac{\pi}{4} \cdot D_p^2, 0 \text{ m}^3\right)$$

$$V_{shaft\_l}(D_p, \Delta h) = 2033 \text{ m}^3$$

Excavations Required for Lower Shaft

$$V_{shaft\_t}(\Delta h, D_p) := V_{shaft\_u}(D_p) + V_{shaft\_l}(D_p, \Delta h)$$

Total Shaft Excavations

$$V_{shaft\_t}(\Delta h, D_p) = 2033 \text{ m}^3$$

$$C_{shaft}(\Delta h, D_p) := P_{shaft} \cdot V_{shaft\_t}(\Delta h, D_p)$$

Capital Cost of Shaft Excavations

$$C_{shaft}(\Delta h, D_p) = 101668 \text{ } \alpha$$

$$C_{res}(V_{ru}, t_g, D_p, \Delta h) := P_{res} \cdot Vol_{res\_new}(V_{ru}, t_g, D_p, \Delta h)$$

Capital Cost of Reservoir & Generator Hall Excavations

$$C_{res}(V_{ru}, t_g, D_p, \Delta h) = 10933267 \text{ } \alpha$$

$$C_{ex}(V_{ru}, t_g, D_p, \Delta h) := C_{shaft}(\Delta h, D_p) + C_{res}(V_{ru}, t_g, D_p, \Delta h)$$

$$C_{ex}(V_{ru}, t_g, D_p, \Delta h) = 11034935 \text{ \AA}$$

Total Capital Cost of Excavations

$$C_{civil}(V_{ru}, t_g, D_p, \Delta h) := C_{ex}(V_{ru}, t_g, D_p, \Delta h) \cdot 1.5$$

Total Civil Costs

$$C_{civil}(V_{ru}, t_g, D_p, \Delta h) = 16552402 \text{ \AA}$$

$$C_t(V_{ru}, t_g, D_p, \Delta h) := 500000 \frac{\text{ \AA}}{MW} \cdot P_t(V_{ru}, t_g, D_p, \Delta h)$$

Total Turbine Costs

$$C_t(V_{ru}, t_g, D_p, \Delta h) = 70803590 \text{ \AA}$$

$$C_{bop}(V_{ru}, t_g, D_p, \Delta h) := 0.5 \cdot C_t(V_{ru}, t_g, D_p, \Delta h)$$

Total Balance of Plant Costs

$$C_{bop}(V_{ru}, t_g, D_p, \Delta h) = 35401795 \text{ \AA}$$

$$R_g := 0 \text{ \AA}$$

Total Grants & Subsidies at Start of Project

$$ACP(V_{ru}, t_g, D_p, \Delta h) := 78131 \frac{\alpha}{MW} \cdot P_t(V_{ru}, t_g, D_p, \Delta h)$$

$$ACP(V_{ru}, t_g, D_p, \Delta h) = 11063911 \alpha$$

Annual Capacity Payment

## Financial Calculations

$$Elec_{sell} := 60 \frac{\alpha}{MW \cdot hr}$$

Average Price at which Electricity is Sold (Generating)

$$Elec_{buy} := 36 \frac{\alpha}{MW \cdot hr}$$

Average Price at which Electricity is Bought (Pumping)

$$R_{elec}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) := Elec_{sell} \cdot hr_g \cdot P_t(V_{ru}, t_g, D_p, \Delta h) - Elec_{buy} \cdot hr_p \cdot P_p(V_{rl}, t_p, D_p, \Delta h) \quad R_{elec}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) = 1364483 \alpha$$

Annual Revenue From Electricity Sales

$$\alpha := 0.03$$

Annual Rate of Inflation

$$i_n := 0.05$$

Annual Rate of Interest

$$\beta := 0.01$$

Annual Electricity Price Inflation Rate

$$K := 25$$

Years Until Major Plant Replacement

$$N := 100$$

Lifetime of Project

$$\delta := \frac{1 + \alpha}{1 + i_n}$$

$$\delta = 0.981$$

$$\varepsilon := \frac{1 + \beta}{1 + i_n}$$

$$\varepsilon = 0.962$$

$$AC_{semo}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) := \left( 107 \frac{\text{¤}}{MW} \cdot P_t(V_{ru}, t_g, D_p, \Delta h) \right) + \left( 413 \frac{\text{¤}}{MW} \cdot P_p(V_{rl}, t_p, D_p, \Delta h) \right) + \left( 0.686 \frac{\text{¤}}{MW \cdot hr} \cdot P_t(V_{ru}, t_g, D_p, \Delta h) \cdot hr_g \right)$$

Annual Costs Charged by SEMO

$$AC_{semo}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) = 199837 \text{ ¤}$$

$$C_{semo} := 1115 \text{ ¤} + 2788 \text{ ¤}$$

Upfront Costs Charged by SEMO

$$MP_{pvf}(K) := \delta^K$$

Present Value Factor of Major Plant Replacement

$$MP_{pvf}(K) = 0.618$$

$$F_t := 0.8$$

Factor of Turbine Cost Incurred on Major Replacement

$$C_{mp}(V_{ru}, t_g, D_p, \Delta h) := C_t(V_{ru}, t_g, D_p, \Delta h) \cdot F_t$$

Capital Cost of Major Plant Replacement

$$C_{mp}(V_{ru}, t_g, D_p, \Delta h) = 56642872 \text{ ¤}$$

$$MP_{pv}(V_{ru}, t_g, D_p, K, \Delta h) := C_{mp}(V_{ru}, t_g, D_p, \Delta h) \cdot MP_{pvf}(K)$$

$$MP_{pv}(V_{ru}, t_g, D_p, K, \Delta h) = 35022199 \text{ ☒}$$

Present Value of Major Plant Replacement

$$C_{cap}(V_{ru}, t_g, D_p, \Delta h) := (C_{civil}(V_{ru}, t_g, D_p, \Delta h) + C_t(V_{ru}, t_g, D_p, \Delta h) + C_{bop}(V_{ru}, t_g, D_p, \Delta h) + C_{semo}) - R_g$$

Capital Cost

$$C_{cap}(V_{ru}, t_g, D_p, \Delta h) = 122761690 \text{ ☒}$$

$$C_{cap}(V_{ru}, t_g, D_p, \Delta h, N) := \frac{i_n \cdot (1 + i_n)^N}{(1 + i_n)^N - 1} \cdot C_{cap}(V_{ru}, t_g, D_p, \Delta h)$$

Annual Cost of Capital

$$C_{cap}(V_{ru}, t_g, D_p, \Delta h, N) = 6185119 \text{ ☒}$$

$$AC_{o\&m}(V_{ru}, t_g, D_p, \Delta h) := 36000 \frac{\alpha}{MW} \cdot P_t(V_{ru}, t_g, D_p, \Delta h)$$

Annual Operation &amp; Maintenance Cost

$$AC_{o\&m}(V_{ru}, t_g, D_p, \Delta h) = 5097858 \alpha$$

$$R_t(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) := R_{elec}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) + ACP(V_{ru}, t_g, D_p, \Delta h)$$

Total Annual Revenues

$$R_t(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) = (1.243 \cdot 10^7) \alpha$$

$$R_{pv}(V_{ru}, V_{rl}, t_g, t_p, D_p, N, \Delta h) := \varepsilon \cdot \frac{(1-\varepsilon^N)}{(1-\varepsilon)} \cdot (R_t(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h))$$

$$R_{pv}(V_{ru}, V_{rl}, t_g, t_p, D_p, N, \Delta h) = (3.074 \cdot 10^8) \alpha$$

Present Value of Lifetime Revenues

$$AC_t(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N) := (AC_{o\&m}(V_{ru}, t_g, D_p, \Delta h) + AC_{semo}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) + (AC_{dw}(\Delta h)))$$

Total Annual Costs

$$AC_t(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N) = (5.395 \cdot 10^6) \text{ \AA}$$

$$AC_{pv}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N) := AC_t(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N) \left( \frac{(\delta - \delta^{(N+1)})}{(1-\delta)} \right)$$

$$AC_{pv}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N) = (2.372 \cdot 10^8) \text{ \AA}$$

Present Value of Lifetime  
Annual Costs

$$AP := R_t(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h) - AC_t(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N)$$

$$AP = (7.033 \cdot 10^6) \text{ \AA}$$

Annual Profit

$$NPV(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N, K) := -(C_{cap}(V_{ru}, t_g, D_p, \Delta h, N) + AC_{pv}(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N) + MP_{pv}(V_{ru}, t_g, D_p, K, \Delta h)) + R_{pv}(V_{ru}, V_{rl}, t_g, t_p, D_p, N, K)$$

$$\text{Net Present Value of Investment } NPV(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N, K) = 28917542 \text{ \AA}$$

## Internal Rate of Return (IRR) Calculation

$$A_r := \sum_{N=1}^N AP$$

Sum of Inflows

$$A_r = (7.033 \cdot 10^8) \text{ } \square$$

$$C_0 := C_{cap}(V_{ru}, t_g, D_p, \Delta h, N)$$

Capital Cost

$$C_0 = (6.185 \cdot 10^6) \text{ } \square$$

$$NPV_{n\_in} := \varepsilon \cdot \frac{(1-\varepsilon^N)}{(1-\varepsilon)} \cdot AP$$

Net Present Value of Inflows  
Only For (N) Periods

$$NPV_{n\_in} = (1.739 \cdot 10^8) \text{ } \square$$

$$NPV_{n\_1\_in} := \varepsilon \cdot \frac{(1-\varepsilon^{(N-1)})}{(1-\varepsilon)} \cdot AP$$

Net Present Value of Inflows  
Only For (N-1) Periods

$$NPV_{n\_1\_in} = (1.738 \cdot 10^8) \text{ } \square$$

$$NPV_{1\_in} := \varepsilon \cdot AP$$

Net Present Value of Inflows  
Only Where N=1

$$NPV_{1\_in} = (6.765 \cdot 10^6) \text{ } \square$$

$$p := \frac{\log\left(\left(\frac{A_r}{|C_0|}\right), 10\right)}{\log\left(\left(\frac{A_r}{NPV_{1\_in}}\right), 10\right)}$$

Exponent in r2 equation

$$p = 1.019$$

$$r_1 := \left(\frac{A_r}{|C_0|}\right)^{\frac{2}{(N+1)}} + 1$$

$$r_1 = 2.098$$

$$r_2 := (1+r_1)^p - 1$$

$$r_2 = 2.167$$

$$P := \frac{\log\left(\left(\frac{NPV_{n\_in}}{|C_0|}\right), 10\right)}{\log\left(\left(\frac{NPV_{n\_in}}{NPV_{n\_1\_in}}\right), 10\right)}$$

exponent in rn1 equation

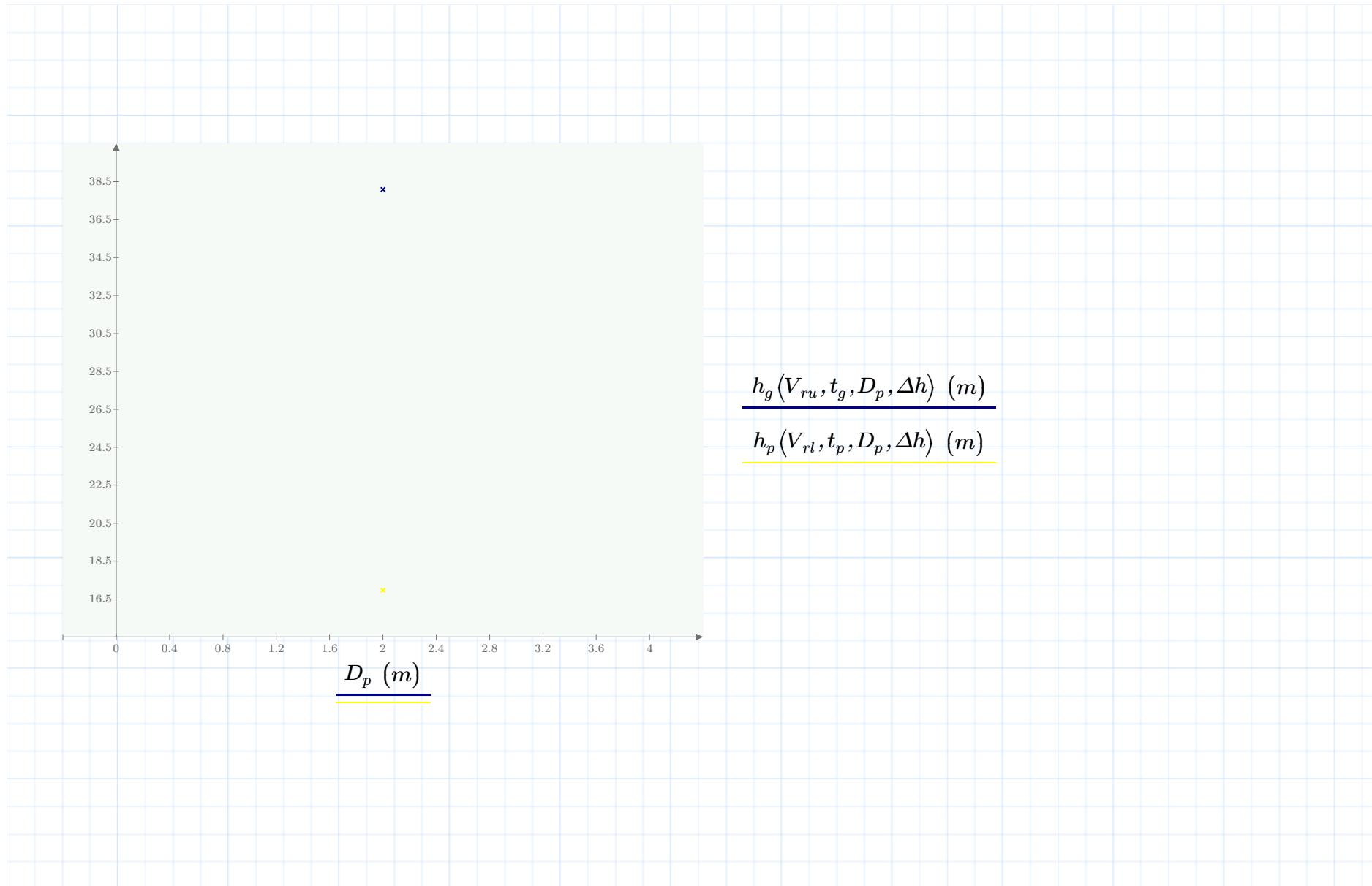
$$P = 4.01 \cdot 10^3$$

$$r_n := r_2$$

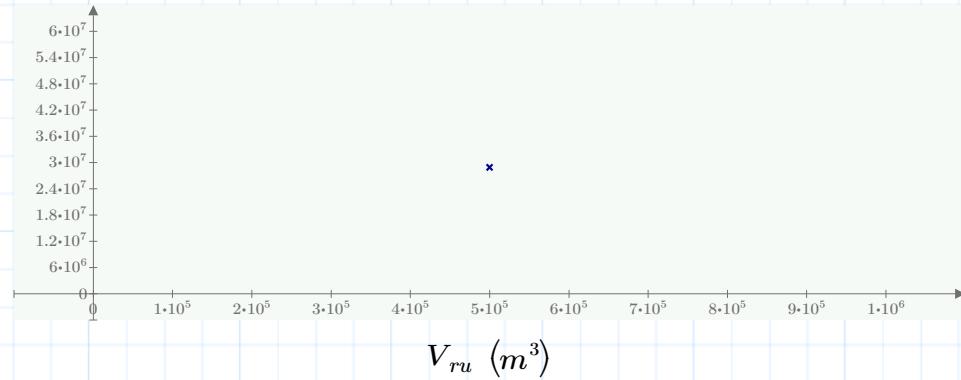
$$r_{n\_1} := r_1$$

$$r_{n1} := \left( (1 + r_n) \cdot \left( \left( \frac{1 + r_{n\_1}}{1 + r_n} \right)^P \right) \right) - 1$$

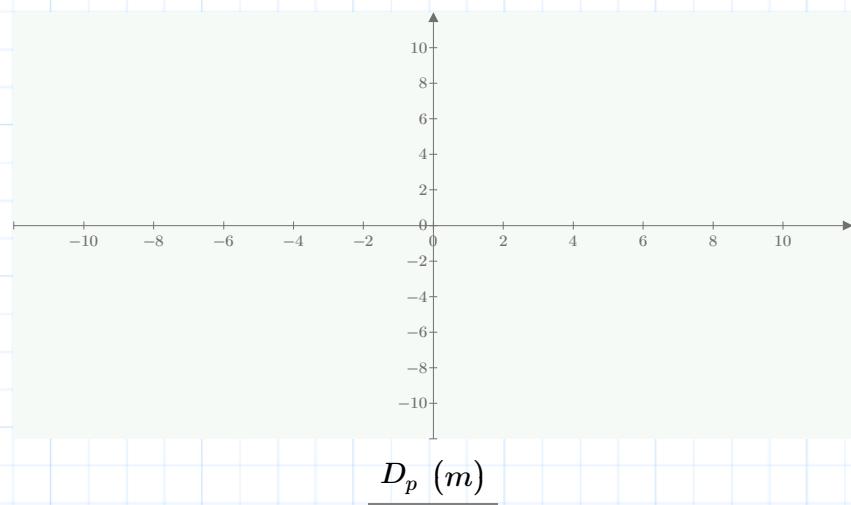
$$r_{n1} = -1$$



## Net Present Value



$$\underline{NPV(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N, K) \text{ } (\alpha)}$$



$$\underline{NPV(V_{ru}, V_{rl}, t_g, t_p, D_p, \Delta h, N, K) \text{ } (\alpha)}$$

## Turbine Power

