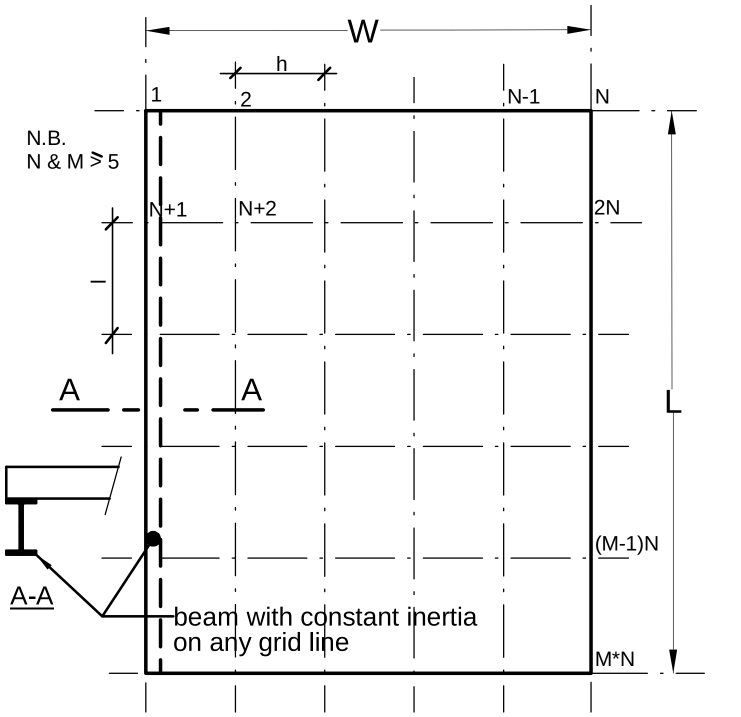
Rectangular solid slabs supported by beams, analysis by Finite Differences

This elastic analysis of rectangular slabs allows for any loading pattern and support arrangement which can be described by referring to the uniform rectangular grid shown below, beams may be present on any grid line. This example describes a composite steel beam bridge deck, 6 beams spanning parallel to L with a 152mm deep concrete deck slab.



RECTANGULAR MESH, nomenclature

SLAB DATA						
Width W mm	Length L mm	N	M	slab depth d mm	Young's Mod kN/mm^2	Poisson's ratio, 'v'
8000	17070	6	11	152	28	0.15

BEAM DATA				
Beam No.	Start node	End node	Intertia cm^4	Young's Mod kN/mm^2
1	1	61	4.90E+05	210
2	2	62	4.90E+05	210
3	3	63	4.90E+05	210
4	4	64	4.90E+05	210
5	5	65	4.90E+05	210
6	6	66	4.90E+05	210
-1				

data extraction

slab distributed inertia......... $D := \frac{d^3}{12\cdot\left(1 - v^2\right)}\cdot E$

Enter the Support Stiffness at each node, (10^10 kN/mm for a rigid support) in the Excel tables below:-
(re-adjust the size to suit the problem in hand)

SUPPORT STIFFNESSES at NODES in kN/mm					
1.00E+11	1.00E+11	1.00E+11	1.00E+11	1.00E+11	1.00E+11
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
1.00E+11	1.00E+11	1.00E+11	1.00E+11	1.00E+11	1.00E+11

$\text{Support} := \text{Ex}(\text{Supp}, N, M)\cdot\frac{\text{kN}}{\text{mm}}$

Enter the loads coexistent with the abnormal vehicle at each node, in the Excel tables below:-
(re-adjust the size to suit the problem in hand)

LOADS at NODES in kN					
4.575	4.575	4.575	0	0	0
9.15	9.15	9.15	0	0	0
9.15	9.15	9.15	0	0	0
9.15	9.15	9.15	0	0	0
9.15	9.15	9.15	0	0	0
29.15	29.15	29.15	0	0	0
9.15	9.15	9.15	0	0	0
9.15	9.15	9.15	0	0	0
9.15	9.15	9.15	0	0	0
9.15	9.15	9.15	0	0	0
4.575	4.575	4.575	0	0	0

(In this case, 1/3 rd. HA loading on the left hand lane.)

$\text{Load} := \text{Ex}(\text{Ld}, N, M)\cdot\text{kN}$

ENTER the abnormal vehicle details:-

No. of units of HB loading.......... $N\_HB := 45$

Enter x-, y-coords to leading wheel $x\_veh := 14.85ft$ $y\_veh := 17ft$

$T := 2240lbf$

wheel load........... $Wheel := \frac{45 \cdot T}{4}$ $Wheel = 112.1 \cdot kN$

In the area 'workings' the HB wheel loads are apportioned to the grid, the finite difference equations are calculated, the slab stiffness matrix is assembled and modified to include the beams and the supports.

workings

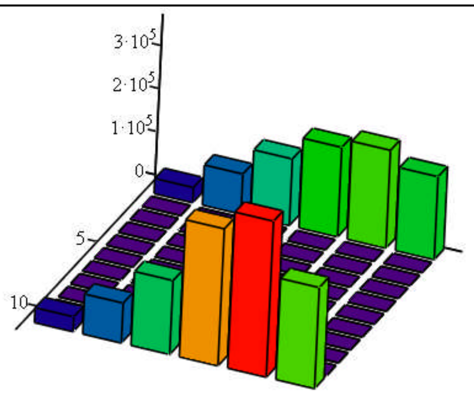
$Deflexions := lsolve(A, Load)$

reactions at the supported nodes:- $Rn := \overrightarrow{(Deflexions \cdot Support)}$

Check that sum of reactions is equal to sum of applied loads:

$\sum Rn = 2128.02 \cdot kN$ $\sum Load = 2128.02 \cdot kN$

(If these two are equal we have a balance of vertical forces, if not, we have an error!)



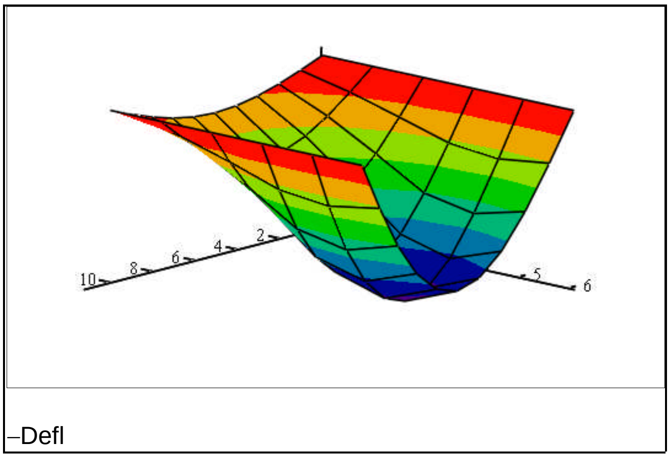
Rect(Rn)

$$Rect(Rn) = \begin{pmatrix} 35.28 & 93.26 & 154.72 & 208.07 & 227.81 & 192.88 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 34.69 & 93 & 171.89 & 318.54 & 362.84 & 235.05 \end{pmatrix} \cdot kN$$

Rearrange deflexions into a rectangular array, (for plotting).

$Defl := Rect(Deflexions)$

$$Defl = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 2.02 & 4.4 & 7.41 & 10.57 & 11.64 & 10.22 \\ 3.83 & 8.37 & 14.1 & 20.2 & 22.27 & 19.47 \\ 5.26 & 11.51 & 19.42 & 27.93 & 30.8 & 26.85 \\ 6.19 & 13.54 & 22.83 & 32.71 & 36.05 & 31.52 \\ 6.52 & 14.26 & 23.99 & 34.16 & 37.62 & 33.07 \\ 6.19 & 13.55 & 22.83 & 32.54 & 35.83 & 31.49 \\ 5.25 & 11.52 & 19.49 & 28.05 & 30.93 & 26.95 \\ 3.82 & 8.37 & 14.24 & 20.81 & 23 & 19.76 \\ 2.01 & 4.41 & 7.53 & 11.13 & 12.32 & 10.47 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot mm$$



Beam Moments:

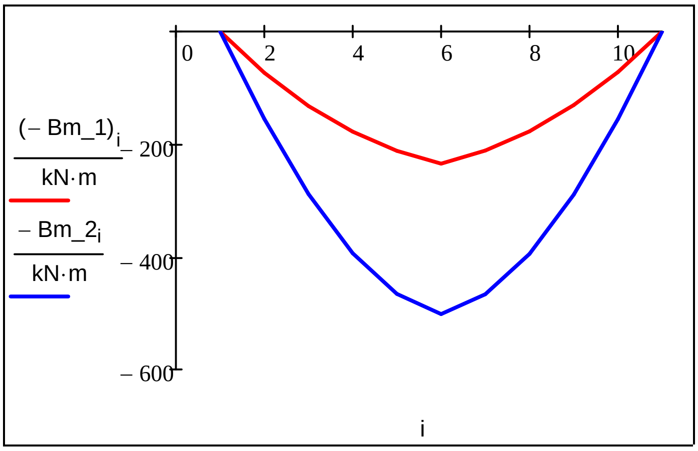
The beam moments are calculated in 'workings' from the deflexions using finite differences

$Bm\_1 := Mom\_beam(1, Deflexions)$ $Bm\_2 := Mom\_beam(2, Deflexions)$

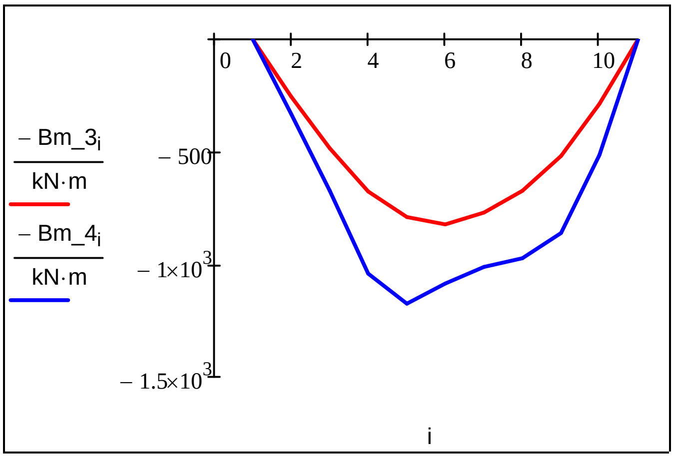
$Bm\_3 := Mom\_beam(3, Deflexions)$ $Bm\_4 := Mom\_beam(4, Deflexions)$

$Bm\_5 := Mom\_beam(5, Deflexions)$ $Bm\_6 := Mom\_beam(6, Deflexions)$

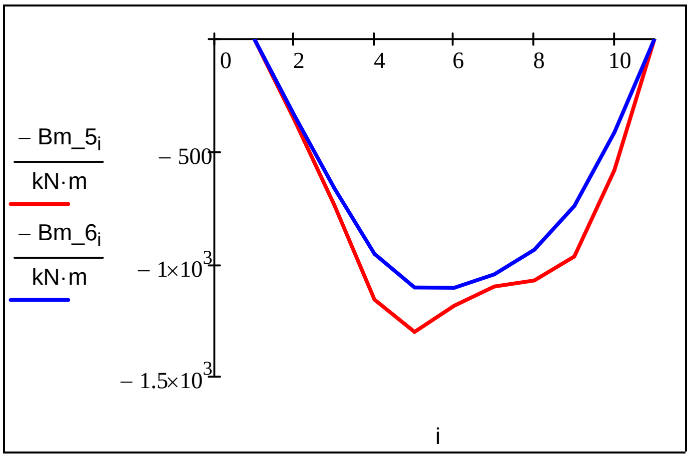
$$Bm\_1 = \begin{pmatrix} 0 \\ 73.21 \\ 132.17 \\ 178.11 \\ 212.08 \\ 234.56 \\ 211.64 \\ 177.15 \\ 130.91 \\ 72.29 \\ 0 \end{pmatrix} \cdot kN \cdot m \quad Bm\_2 = \begin{pmatrix} 0 \\ 155.38 \\ 288.6 \\ 393.67 \\ 465.76 \\ 501.68 \\ 466.44 \\ 394.69 \\ 289.55 \\ 155.91 \\ 0 \end{pmatrix} \cdot kN \cdot m$$



$$Bm\_3 = \begin{pmatrix} 0 \\ 254.44 \\ 483.74 \\ 676.71 \\ 790.24 \\ 821.81 \\ 770.2 \\ 673.63 \\ 517.92 \\ 286.58 \\ 0 \end{pmatrix} \cdot kN \cdot m \quad Bm\_4 = \begin{pmatrix} 0 \\ 330.65 \\ 673.4 \\ 1040.84 \\ 1175 \\ 1085.93 \\ 1010.85 \\ 973.16 \\ 860.51 \\ 513.64 \\ 0 \end{pmatrix} \cdot kN \cdot m$$



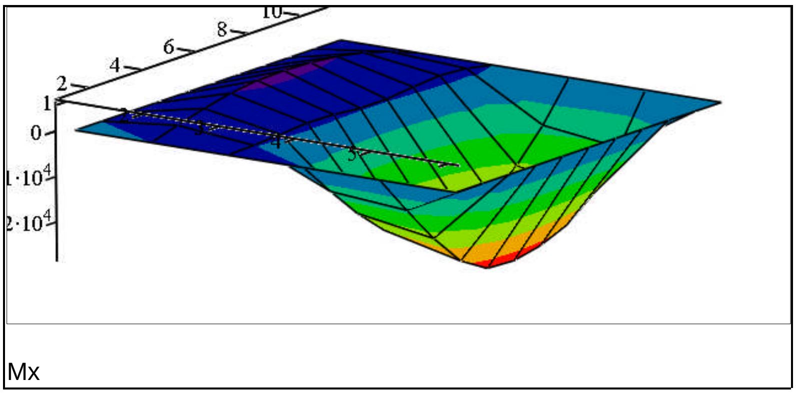
$$Bm\_5 = \begin{pmatrix} 0 \\ 359.93 \\ 739.23 \\ 1157.6 \\ 1301.36 \\ 1184.64 \\ 1099.35 \\ 1072.79 \\ 966.16 \\ 582.25 \\ 0 \end{pmatrix} \cdot kN \cdot m \quad Bm\_6 = \begin{pmatrix} 0 \\ 339.79 \\ 663.47 \\ 954.81 \\ 1103.37 \\ 1104.59 \\ 1044.86 \\ 936.05 \\ 741.95 \\ 415.11 \\ 0 \end{pmatrix} \cdot kN \cdot m$$



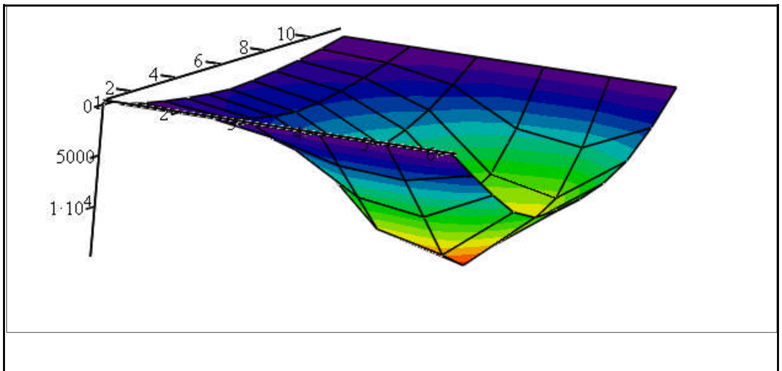
SLAB MOMENTS:-

$Mx := MX(Defl)$ $My := MY(Defl)$ $Mxy := MXY(Defl)$

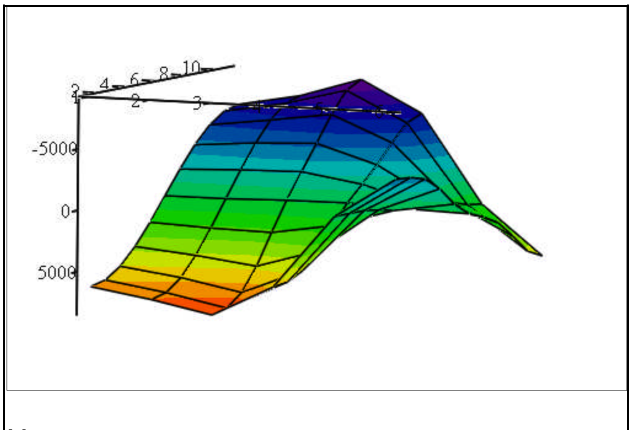
$$Mx = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.84 & -0.18 & 7.23 & 8.63 & 0 \\ 0 & -3.55 & -0.61 & 14.04 & 16.82 & 0 \\ 0 & -4.94 & -1.13 & 19.73 & 23.75 & 0 \\ 0 & -5.74 & -1 & 22.85 & 27.38 & 0 \\ 0 & -5.93 & -0.41 & 23.29 & 27.69 & 0 \\ 0 & -5.72 & -0.44 & 22.22 & 26.37 & 0 \\ 0 & -5.09 & -1.11 & 19.78 & 23.8 & 0 \\ 0 & -3.94 & -1.66 & 15.4 & 18.94 & 0 \\ 0 & -2.17 & -1.24 & 8.54 & 10.69 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot kN$$



$$My = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.58 & 0.96 & 2 & 3.72 & 4.16 & 2.71 \\ 1.05 & 1.77 & 3.76 & 7.47 & 8.41 & 5.28 \\ 1.42 & 2.39 & 5.22 & 11.25 & 12.78 & 7.6 \\ 1.69 & 2.85 & 6.14 & 12.78 & 14.47 & 8.79 \\ 1.87 & 3.11 & 6.48 & 12.14 & 13.59 & 8.8 \\ 1.69 & 2.86 & 6.07 & 11.38 & 12.71 & 8.32 \\ 1.41 & 2.38 & 5.2 & 10.72 & 12.11 & 7.45 \\ 1.04 & 1.71 & 3.88 & 9.16 & 10.54 & 5.91 \\ 0.58 & 0.92 & 2.1 & 5.37 & 6.24 & 3.31 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot kN$$



$$Mxy = \begin{pmatrix} 6.23 & 7.04 & 8.04 & 5.52 & -0.46 & -3.72 \\ 5.92 & 6.7 & 7.72 & 5.33 & -0.48 & -3.65 \\ 5.04 & 5.72 & 6.69 & 4.66 & -0.48 & -3.29 \\ 3.67 & 4.15 & 4.78 & 3.3 & -0.3 & -2.26 \\ 1.94 & 2.16 & 2.27 & 1.47 & -0.01 & -0.78 \\ 0.01 & 0.01 & -0.11 & -0.15 & 0.09 & 0.24 \\ -1.92 & -2.11 & -2.2 & -1.42 & -0.01 & 0.74 \\ -3.66 & -4.06 & -4.27 & -2.77 & -0 & 1.45 \\ -5.05 & -5.69 & -6.4 & -4.34 & 0.29 & 2.78 \\ -5.95 & -6.8 & -8.11 & -5.71 & 0.69 & 4.22 \\ -6.25 & -7.2 & -8.77 & -6.26 & 0.87 & 4.84 \end{pmatrix} \cdot kN$$



The reactions are calculated by multiplying the deflexion at a supported node by the support stiffness. To demonstrate that this procedure gives the correct result the reaction at Node 2, for example, is calculated by finite differences from the deflexions allowing for contirbutions from beam bending, slab bending, slab twisting and any load applied at that node, (a tedious business by comparison).

From the deflexion at Node 2 times the support stiffness $Rn2a := Deflexions_2 \cdot Support_2$ $Rn2a = 93.26\,kN$

Calculate the reaction at node 2 from the rate of change of the bending and twisting moments:

$Mxy\_pan1 := Mxy_{1,1}$ $Mxy\_pan2 := \frac{D \cdot (1 - v)}{h \cdot l} \cdot \left[\left(Defl_{1,2} - Defl_{1,3} \right) - \left(Defl_{2,2} - Defl_{2,3} \right) \right]$

$Rn2b := \frac{\left(My_{2,2} \cdot h + Bm\_2_2 \right)}{l} + 2 \cdot Mxy\_pan1 - 2 \cdot Mxy\_pan2 + Load_2$ $Rn2b = 93.26\,kN$