1- Constant α Effect

Original Values

$X0 := \text{stack}(3,10,20,60,100,200,400,600)$

$R := \text{stack}(2,4,8,10,100,200)$

$m = 4 \qquad x0 := X0_m \qquad x0 = 100$

$n = 3 \qquad \rho := R_n \qquad \rho = 10$

Original Equations

$$\kappa(\alpha) := \frac{[1 + (\alpha - 1)\sin(\pi\cdot\alpha)]}{2\alpha\cdot(1-\alpha)}$$

$$p(\alpha) := 4\cdot\pi^{\frac{(1-4\alpha)}{2\alpha}}\left(\alpha\cdot\frac{\Gamma(\alpha)}{\Gamma(.5+\alpha)}\right)^{\frac{1}{\alpha}}$$

$$N(X,Y,\alpha) := \left[\left(\frac{2}{\pi}\right)\left(2\frac{Y}{X}\right)^{.5}\right]^{\frac{(1-2\alpha)}{\alpha}}$$

$$N0(X0,\alpha) := \left[\left(\frac{2}{\pi}\right)\left(2\frac{1}{X0}\right)^{.5}\right]^{\frac{(1-2\alpha)}{\alpha}}$$

$Mmin(\rho) := .5 + .5\ln(4\rho)$

$MMmin(\rho) := .5 + .5\ln(4\rho)$

$M(\rho) := 1.2\cdot MMmin(\rho)$

$$Mmin(\rho,\alpha,X0) := .5 + .5\ln\left[4\rho\cdot p(\alpha)\left[\left(\frac{2}{\pi}\right)\left(\frac{2}{X0}\right)^{.5}\right]^{\frac{(1-2\alpha)}{\alpha}}\right] - \frac{(1-2\alpha)\cdot p(\alpha)\cdot\left[\left(\frac{2}{\pi}\right)\left(\frac{2}{X0}\right)^{.5}\right]^{\frac{(1-2\alpha)}{\alpha}}}{2\alpha\cdot X0}$$

$Mf(\rho,\alpha,x0) := 1.2\cdot Mmin(\rho,\alpha,x0)$

===================== First and only Differential Equation =====================

Given

$$f'(x) = \frac{2\cdot\alpha}{p(\alpha)\cdot\left(\frac{2}{\pi}\sqrt{\frac{2\cdot f(x)}{x}}\right)^{\frac{1-2\cdot\alpha}{\alpha}}}\left[Mf(\rho,\alpha,x0) - \frac{1}{2} - \frac{1}{2}\ln\left[4\cdot\rho\cdot p(\alpha)\cdot f(x)\cdot\left(\frac{2}{\pi}\sqrt{\frac{2\,f(x)}{x}}\right)^{\frac{1}{\alpha}-2}\right]\right] + (1-2\alpha)\frac{f(x)}{x}$$

$f(100) = 1$

$f(\alpha) := \text{Odesolve}(x,400)$

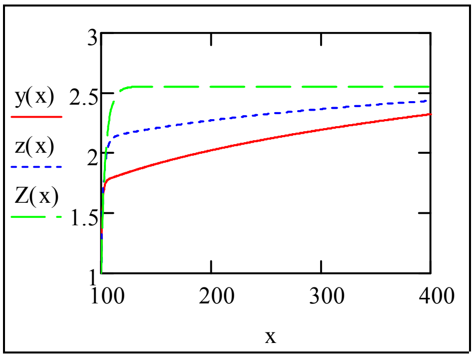
$$F(x,\rho,\alpha,x0,f) := \frac{2\cdot\alpha}{p(\alpha)\cdot\left(\frac{2}{\pi}\sqrt{\frac{2\cdot f(x)}{x}}\right)^{\frac{1-2\cdot\alpha}{\alpha}}}\left[Mf(\rho,\alpha,x0) - \frac{1}{2} - \frac{1}{2}\ln\left[4\cdot\rho\cdot p(\alpha)\cdot f(x)\cdot\left(\frac{2}{\pi}\sqrt{\frac{2\,f(x)}{x}}\right)^{\frac{1}{\alpha}-2}\right]\right] + (1-2\alpha)\frac{f(x)}{x}$$

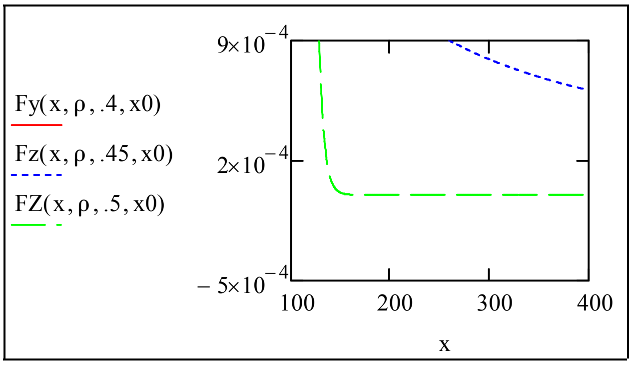
$y := f(0.4)$ $\quad Fy(x,\rho,\alpha,x0) := F(x,\rho,\alpha,x0,y)$

$z := f(0.45)$ $\quad Fz(x,\rho,\alpha,x0) := F(x,\rho,\alpha,x0,z)$

$Z := f(0.5)$ $\quad FZ(x,\rho,\alpha,x0) := F(x,\rho,\alpha,x0,Z)$

====================== Differential Equation and Solution Plots ======================





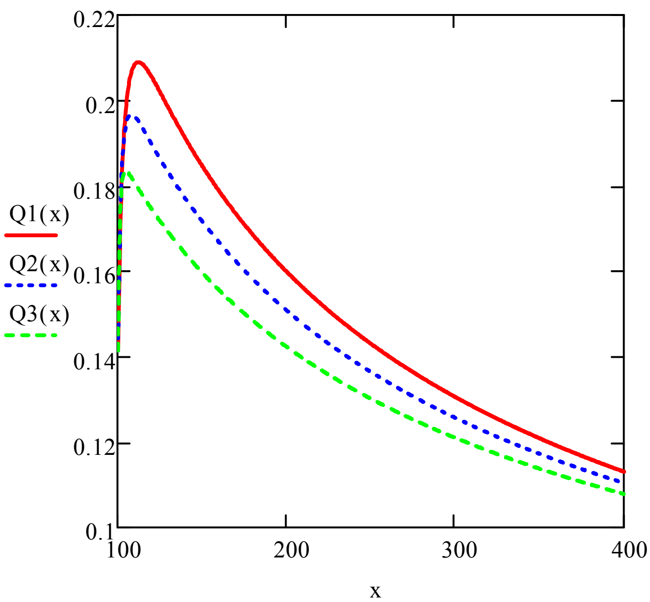
====================== End of Differential Equation Definitions ======================

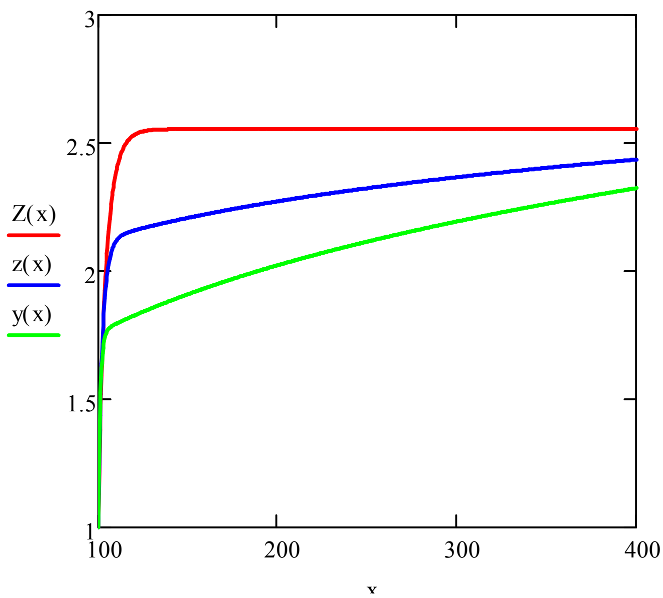
Q definitions

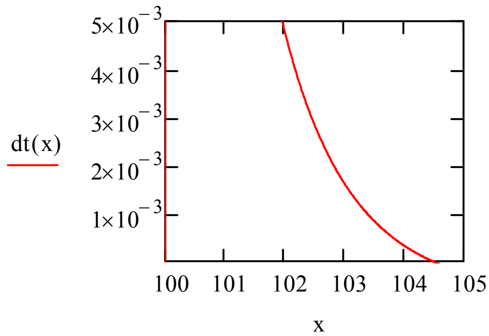
$$t(x) := \left(2\,\frac{y(x)}{x}\right)^{.5}$$

$$Q1(x) := \left(2\,\frac{Z(x)}{x}\right)^{.5} \qquad Q2(x) := \left(2\,\frac{z(x)}{x}\right)^{.5} \qquad Q3(x) := \left(2\,\frac{y(x)}{x}\right)^{.5}$$

$$dt(x) := \frac{d}{dx}t(x)$$







$X0^T = (3 \;\; 10 \;\; 20 \;\; 60 \;\; 100 \;\; 200 \;\; 400 \;\; 600)$

$R^T = (2 \;\; 4 \;\; 8 \;\; 10 \;\; 100 \;\; 200)$

$m \equiv 4$ $\qquad X0_m = 100$

$n \equiv 3$ $\qquad R_n = 10$

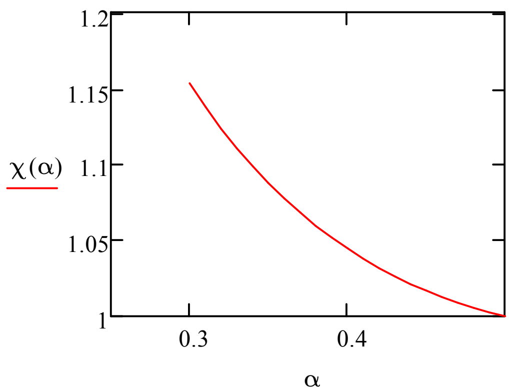
2- Variable α Effect

Please help me to do the following:
I want to focus on the range of α from 0.3 to 0.5, and would like to approximate the function - shown below - in this range. Please see the last graph. It is very rapidly changing near α=0.5.
Once such function α=α(ρ) is established, I would like to substitute the NEW function for α=α(x) into my governing ODEs above (considering α to be VARIABLE depending on x).
NOTE: to change ρ to x, simply multiply it by a large number, e.g. 1000 or 10000. The last graph shows the range of the function needed to describe the dependence of α on x.

$\alpha := 0.30, 0.31 .. 0.50$

$$\chi(\alpha) := \frac{\pi^{\alpha-1}\,\Gamma(\alpha)}{\Gamma\left(\alpha + \frac{1}{2}\right)}$$

$\chi(0.5) = 1$

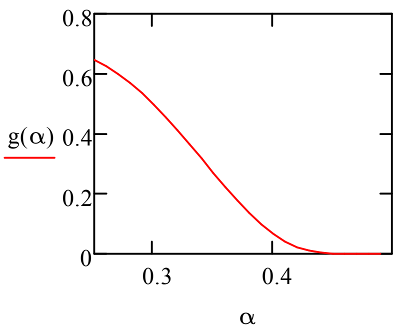


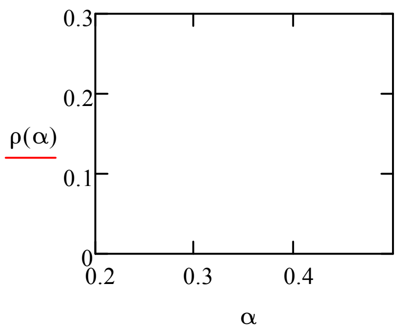
USING THIS EQUATION TO PLOT ρ vs. α - or better yet the inverse function - as shown below

$$g(\alpha) := \left[\frac{\chi(\alpha)}{2^{1+\alpha}\cdot(.05)^{\alpha}}\right]^{\frac{2}{2\alpha-1}} \qquad \rho(\alpha) := \left(\frac{1}{\pi}\right) g(\alpha)$$

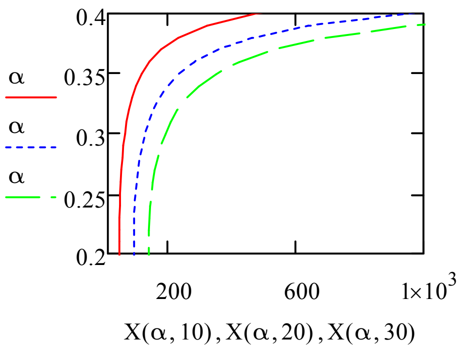
$\rho(.499999) = 0$

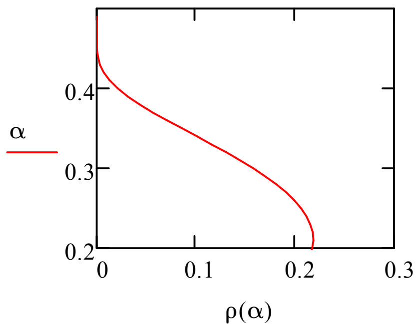
$\alpha := 0, .01 .. .5 \qquad g(.499) = 0$

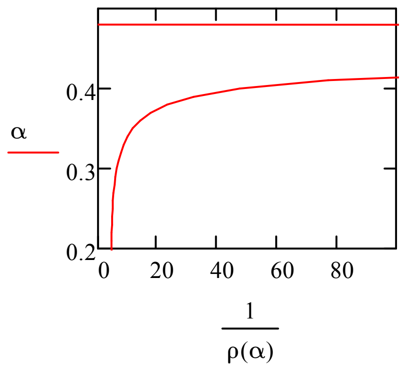


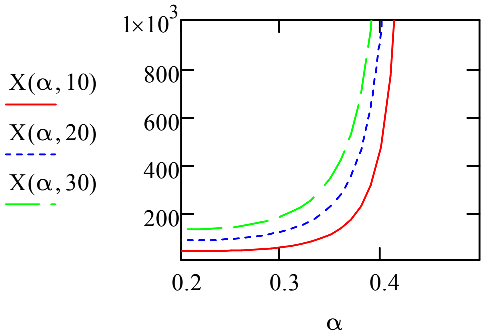


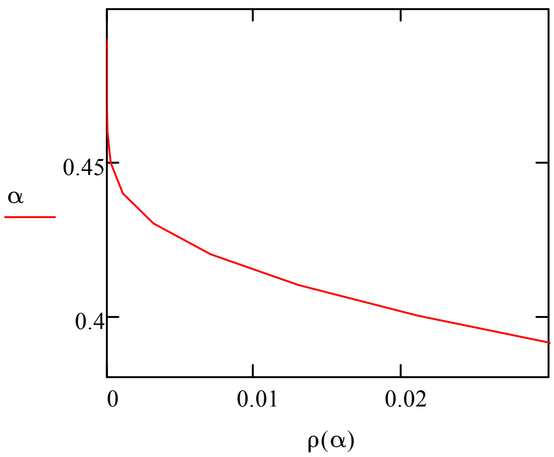
$$X(\alpha, C) := \pi \frac{C}{g(\alpha)}$$

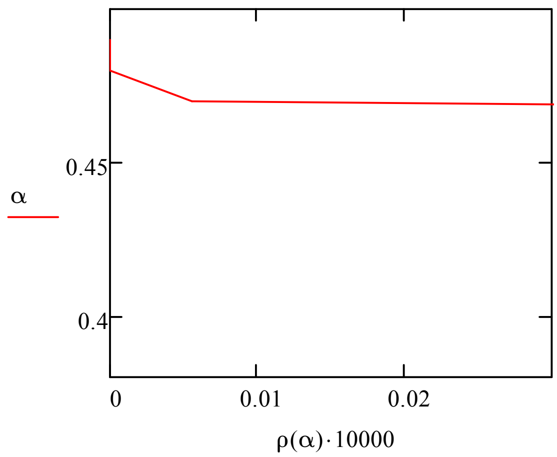












Given

$\alpha := 0.4$

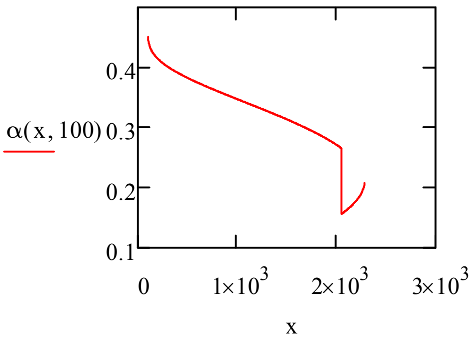
$$x = \frac{x0^2}{\pi}\cdot\left[\frac{\dfrac{\pi^{\alpha-1}\Gamma(\alpha)}{\Gamma\left(\alpha+\frac{1}{2}\right)}}{2^{1+\alpha}\cdot(.05)^{\alpha}}\right]^{\frac{2}{2\alpha-1}} + x0$$

To avoid confusion I would rename "α" in the solveblock to something else, maybe "a"

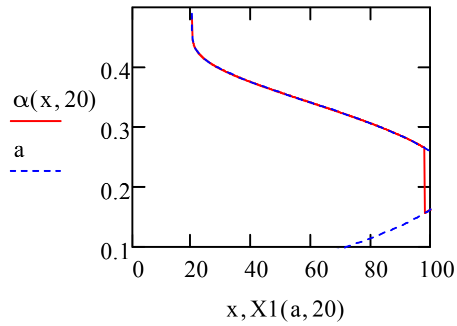
$\alpha(x, x0) := \text{Find}(\alpha)$

$\alpha\left(10^3, 100\right) = 0.347$

$a := 0, 0.01 .. 0.5$



comparison



$X1(0.49, 100) = 100$

$\alpha(100, 100) = 0.486$ The maximum!

OR: Given

$a := 0.35$

$$x = \frac{x0^2}{\pi}\cdot g(a) + x0$$

$\alpha(x, x0) := \text{Find}(a)$

$\alpha\left(10^3, 100\right) = 0.347$

$\rho := 10$

====================== First and only Differential Equation ======================

Given

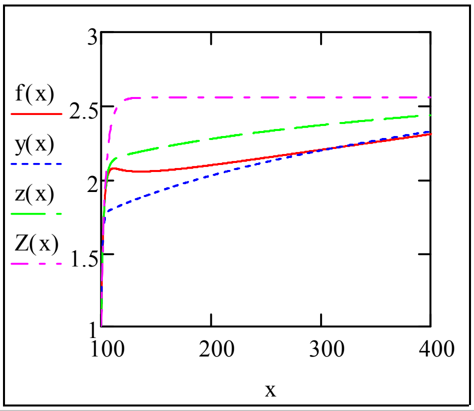
$$f'(x) = \left|\; \begin{aligned} &\alpha0 \leftarrow \alpha(x, x0) \\ &\frac{2\cdot\alpha0}{p(\alpha0)\cdot\left(\frac{2}{\pi}\sqrt{\frac{2\cdot f(x)}{x}}\right)^{\frac{1}{\alpha0}-2}}\left[Mf(\rho, \alpha0, x0) - \frac{1}{2} - \frac{1}{2}\ln\left[4\cdot\rho\cdot p(\alpha0)\cdot f(x)\cdot\left(\frac{2}{\pi}\sqrt{\frac{2\,f(x)}{x}}\right)^{\frac{1}{\alpha0}-2}\right]\right] + (1-2\alpha0)\frac{f(x)}{x} \end{aligned}\right.$$

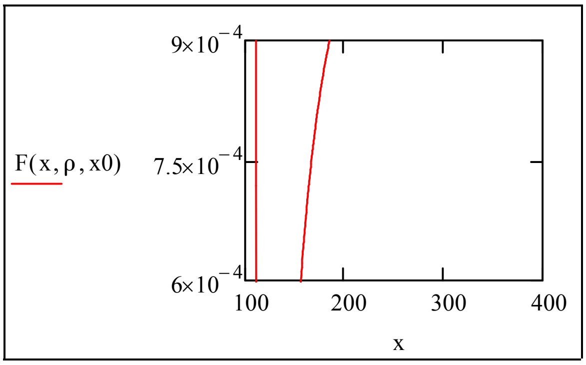
$f(100) = 1$

$f := \mathrm{Odesolve}(x, 400)$

$$F(x,\rho,x0) := \begin{array}{|l} \alpha0 \leftarrow \alpha(x,x0) \\ \dfrac{2\cdot\alpha0}{p(\alpha0)\cdot\left(\dfrac{2}{\pi}\sqrt{\dfrac{2\cdot f(x)}{x}}\right)^{\frac{1}{\alpha0}-2}}\left[Mf(\rho,\alpha0,x0) - \dfrac{1}{2} - \dfrac{1}{2}\ln\left[4\cdot\rho\cdot p(\alpha0)\cdot f(x)\cdot\left(\dfrac{2}{\pi}\sqrt{\dfrac{2\,f(x)}{x}}\right)^{\frac{1}{\alpha0}-2}\right]\right] + (1-2\alpha0)\dfrac{f(x)}{x} \end{array}$$

====================== Differential Equation and Solution Plots ======================





====================== End of Differential Equation Definitions ======================

Range of x0

$X0 := \mathrm{stack}(2, 10, 20, 60, 100, 200, 400, 600)$

Range if x is dependent on x0

$$X1(0.4999999, X0) = \begin{pmatrix} 2 \\ 10 \\ 20 \\ 60 \\ 100 \\ 200 \\ 400 \\ 600 \end{pmatrix} \qquad X1(0.3, X0) = \begin{pmatrix} 2.632 \\ 25.802 \\ 83.21 \\ 628.886 \\ 1680.24 \\ 6520.958 \\ 25683.834 \\ 57488.626 \end{pmatrix} \qquad X1(0.25, X0) = \begin{pmatrix} 2.824 \\ 30.608 \\ 102.431 \\ 801.879 \\ 2160.775 \\ 8443.101 \\ 33372.402 \\ 74787.906 \end{pmatrix}$$

$$X1(0.48, X0) = \begin{pmatrix} 2 \\ 10 \\ 20 \\ 60 \\ 100 \\ 200 \\ 400 \\ 600 \end{pmatrix} \qquad X1(0.45, X0) = \begin{pmatrix} 2.001 \\ 10.024 \\ 20.096 \\ 60.864 \\ 102.399 \\ 209.595 \\ 438.38 \\ 686.355 \end{pmatrix}$$

Construction of matrix for polyfit(c)

Vector of values for x0: $X0 := \text{stack}(2,10,20,35,60,100,150,200,300,400,500,600)$

Number of points in the appropriate vectors for x: $NrPts := 50$

$$\begin{pmatrix} Xmatrix \\ Yvector \end{pmatrix} := \left| \begin{array}{l} X \leftarrow 0 \\ Y \leftarrow 0 \\ \text{for } x_0 \in X0 \\ \quad \left| \begin{array}{l} x_{end} \leftarrow X1(0.25, x_0) \\ \text{for } x \in x_0, x_0 + \dfrac{x_{end} - x_0}{NrPts} \,..\, x_{end} \\ \quad \left| \begin{array}{l} i \leftarrow \text{rows}(Y) \\ X_{i,0} \leftarrow x \\ X_{i,1} \leftarrow x_0 \\ Y_i \leftarrow \alpha(x, x_0) \end{array} \right. \end{array} \right. \\ \text{return } \begin{pmatrix} X \\ Y \end{pmatrix} \end{array} \right.$$

$MC := \text{polyfitc}(Xmatrix, Yvector, 3)$

MC =

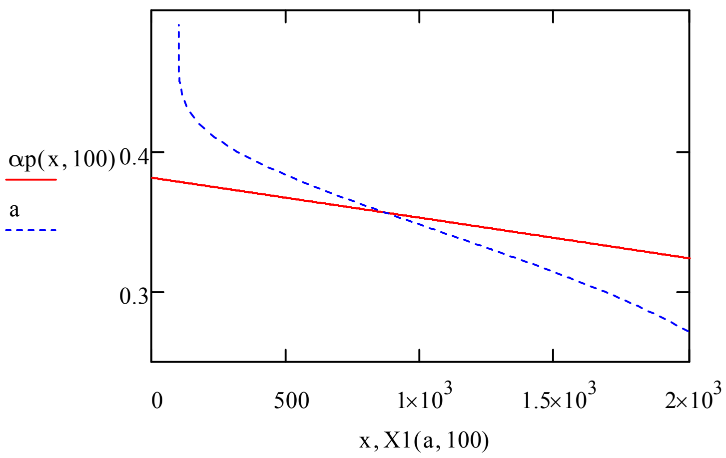
	0	1	2
0	"Term"	"Coefficient"	"Std Error"
1	"Intercept"	0.33	NaN
2	"A"	$-4.235\cdot10^{-5}$	NaN
3	"B"	0.001	NaN
4	"AB"	$1.559\cdot10^{-7}$	NaN
5	"AA"	$-1.862\cdot10^{-10}$	NaN
6	"BB"	$-1.29\cdot10^{-6}$	NaN
7	"AAB"	$5.407\cdot10^{-13}$	NaN
8	"ABB"	$-1.61\cdot10^{-10}$	NaN
9	"AAA"	$-1.124\cdot10^{-15}$	NaN
10	"BBB"	$9.449\cdot10^{-10}$	...

$coeffs := \text{submatrix}(MC, 1, 10, 1, 1) =$

	0
0	0.33
1	$-4.235\cdot10^{-5}$
2	0.001
3	$1.559\cdot10^{-7}$
4	$-1.862\cdot10^{-10}$
5	$-1.29\cdot10^{-6}$
6	$5.407\cdot10^{-13}$
7	$-1.61\cdot10^{-10}$
8	$-1.124\cdot10^{-15}$
9	$9.449\cdot10^{-10}$

$\alpha p(x, x0) := coeffs^T \cdot stack(1, x, x0, x \cdot x0, x^2, x0^2, x^2 \cdot x0, x \cdot x0^2, x^3, x0^3) \rightarrow -4.235 \times 10^{-5} \cdot x + 0.001 \cdot x0 + -1.61 \times 10^{-10} \cdot x \cdot x0^2 + 5.4($

$a := 0.25, 0.26 .. 0.5$



Thats a very bad fit, probably unusable. Increasing the number of points for x does not do anything better.

Lets try it with the older function "regress"

$MV := regress(Xmatrix, Yvector, 3)$

MV =

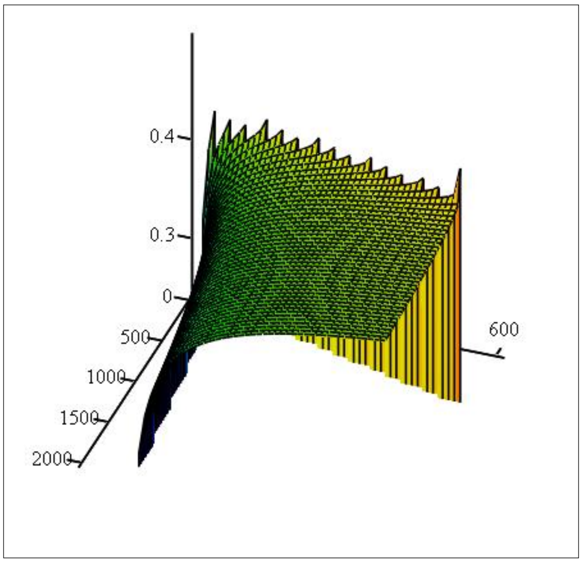
	0
0	3
1	3
2	3
3	$-1.61 \cdot 10^{-10}$
4	$9.449 \cdot 10^{-10}$
5	$-1.29 \cdot 10^{-6}$
6	0.001
7	$1.559 \cdot 10^{-7}$
8	$5.407 \cdot 10^{-13}$
9	0.33
10	$-4.235 \cdot 10^{-5}$
11	$-1.862 \cdot 10^{-10}$
12	$-1.124 \cdot 10^{-15}$

The coefficients begin with index 3 and are identical to the output of polyfitc.
The order of the coeffs is very strange (you may lookup the appropriate quicksheet) but we will get the very same function as above.

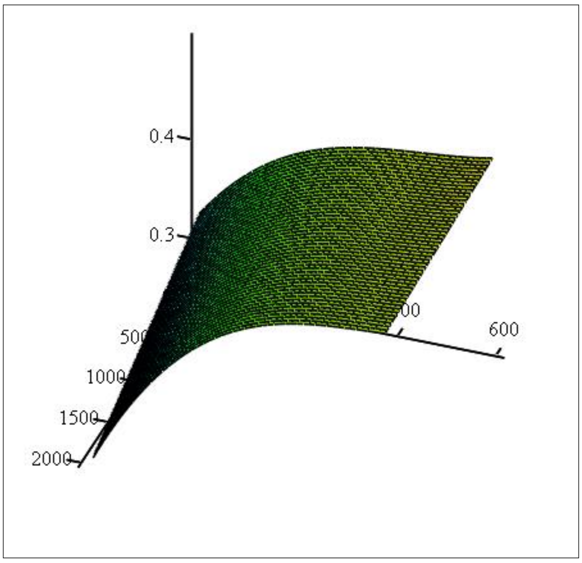
Lets do a quick hack and look at the 3D-surfaces. As many combinations of x/x0 do not evaluate and throw an error we define auxiliary functions to cope with. Unfortunately 3D-plots won't accept NaN so in case auf an error we set the return value to someithing outside of the plot area (-10) but we will see this as nasty vertical planes.

the "correct" function $\alpha1(x, x0) := -10 \ \text{on error} \ \alpha(x, x0)$

x should go from 0 to 2000, x0 from 3 to 600, α is set from 0.25 to 0.5



α1



αp

Hmmm, function α does not look so bad behaved, so I would had expected a better fit.
I am not sure if I had setup everything right as I do not have not much experience with those numerical approximations.

I won't do it but one thing you could try is doing a polynomial fit of higher degree than 3, but I'm not sure if this would help.