there is no longitudinal stress from an interference fit and since the radial stress is negative, the strain from Eq. 51.18 is

$$\epsilon = \frac{\Delta d}{d} = \frac{\Delta C}{C} = \frac{\Delta r}{r} = \frac{\sigma_c + \nu\sigma_r}{E} \qquad \text{51.21}$$

compare w/ 51.15

Equation 51.22 applies to the general case where both cylinders are hollow and have different moduli of elasticity and Poisson's ratios. The outer cylinder is designated as the *hub*; the inner cylinder is designated as the *shaft*. If the shaft is solid, use $r_{i,\text{shaft}} = 0$ in Eq. 51.22.

$$\begin{aligned} I_{\text{diametral}} &= 2I_{\text{radial}} \\ &= \left(\frac{2pr_{o,\text{shaft}}}{E_{\text{hub}}}\right)\left(\frac{r_{o,\text{hub}}^2 + r_{o,\text{shaft}}^2}{r_{o,\text{hub}}^2 - r_{o,\text{shaft}}^2} + \nu_{\text{hub}}\right) \\ &\quad + \left(\frac{2pr_{o,\text{shaft}}}{E_{\text{shaft}}}\right)\left(\frac{r_{o,\text{shaft}}^2 + r_{i,\text{shaft}}^2}{r_{o,\text{shaft}}^2 - r_{i,\text{shaft}}^2} - \nu_{\text{shaft}}\right) \end{aligned} \qquad \text{51.22}$$

In the special case where the shaft is solid and is made from the same material as the hub, the diametral interference is given by Eq. 51.23.

$$\begin{aligned} I_{\text{diametral}} &= 2I_{\text{radial}} \\ &= \left(\frac{4pr_{\text{shaft}}}{E}\right)\left[\frac{1}{1 - \left(\frac{r_{\text{shaft}}}{r_{o,\text{hub}}}\right)^2}\right] \end{aligned} \qquad \text{51.23}$$

The maximum assembly force required to overcome friction during a press-fitting operation is given by Eq. 51.24. The coefficient of friction is highly variable. Values in the range of 0.03 to 0.33 have been reported. This relationship is approximate because the coefficient of friction is not known with certainty and the assembly force affects the pressure, p, through Poisson's ratio.

$$F_{\text{max}} = fN = 2\pi f p r_{\text{shaft}} L_{\text{interface}} \qquad \text{51.24}$$

The maximum torque that the press-fitted hub can withstand or transmit is given by Eq. 51.25. This can be greater or less than the shaft's torsional shear capacity. Both values should be calculated.

$$T_{\text{max}} = 2\pi f p r_{\text{shaft}}^2 L_{\text{interface}} \qquad \text{51.25}$$

Most interference fits are designed to keep the contact pressure or the stress below a given value. Designs of interference fits limited by strength generally use the distortion energy failure criterion. That is, the maximum shear stress is compared with the shear strength determined from the failure theory.

Example 51.3

A steel cylinder has inner and outer diameters of 1.0 in and 2.0 in (25 mm and 50 mm), respectively. The cylinder is pressurized internally to 10,000 lbf/in² (70 MPa). The modulus of elasticity is 2.9×10^7 lbf/in² (200 GPa), and Poisson's ratio is 0.3. What is the radial strain at the inside face?

Solution

The stresses at the inner face are found from Table 51.2.

$$\begin{aligned} \sigma_{c,i} &= \frac{(r_o^2 + r_i^2)p}{r_o^2 - r_i^2} \\ &= \frac{\left((1.0 \text{ in})^2 + (0.5 \text{ in})^2\right)\left(10{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)}{(1.0 \text{ in})^2 - (0.5 \text{ in})^2} \\ &= 16{,}667 \text{ lbf/in}^2 \\ \sigma_{r,i} &= -p = -10{,}000 \text{ lbf/in}^2 \\ \sigma_l &= \frac{F}{A} = \frac{p\pi r_i^2}{\pi(r_o^2 - r_i^2)} \\ &= \frac{\left(10{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)(0.5 \text{ in})^2}{(1.0 \text{ in})^2 - (0.5 \text{ in})^2} \\ &= 3333 \text{ lbf/in}^2 \end{aligned}$$

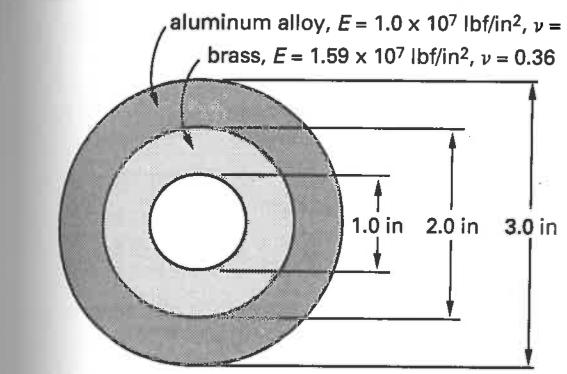
The circumferential stress increases the radial strain; the radial and longitudinal stresses decrease the radial strain. The radial strain is

$$\begin{aligned} \frac{\Delta r}{r} &= \frac{\sigma_{c,i} - \nu(\sigma_{r,i} + \sigma_l)}{E} \\ &= \frac{16{,}667 \ \frac{\text{lbf}}{\text{in}^2} - (0.3)\left(-10{,}000 \ \frac{\text{lbf}}{\text{in}^2} + 3333 \ \frac{\text{lbf}}{\text{in}^2}\right)}{2.9 \times 10^7 \ \frac{\text{lbf}}{\text{in}^2}} \\ &= 6.44 \times 10^{-4} \end{aligned}$$

DIAMETRAL

Example 51.4

A hollow aluminum cylinder is pressed over a hollow brass cylinder as shown. Both cylinders are 2 in long. The interference is 0.004 in. The average coefficient of friction during assembly is 0.25. (a) What is the maximum shear stress in the brass? (b) What initial disassembly force is required to separate the two cylinders?



Solution

(a) Work with the aluminum outer cylinder, which is under internal pressure.

$$\begin{aligned} \sigma_{c,i} &= \frac{(r_o^2 + r_i^2)p}{r_o^2 - r_i^2} \\ &= \frac{\left((1.5 \text{ in})^2 + (1.0 \text{ in})^2\right)p}{(1.5 \text{ in})^2 - (1.0 \text{ in})^2} \\ &= 2.6p \\ \sigma_{r,i} &= -p \end{aligned}$$

From Eq. 51.18, the diametral strain is

$$\begin{aligned} \epsilon &= \frac{\sigma_{c,i} - \nu(\sigma_{r,i} + \sigma_l)}{E} \\ &= \frac{2.6p - (0.33)(-p)}{1.0 \times 10^7 \ \frac{\text{lbf}}{\text{in}^2}} \\ &= 2.93 \times 10^{-7}p \\ \Delta d &= \epsilon_d = (2.93 \times 10^{-7}p)(2.0 \text{ in}) \\ &= 5.86 \times 10^{-7}p \end{aligned}$$

Now work with the brass inner cylinder, which is under external pressure.

$$\begin{aligned} \sigma_{c,o} &= \frac{-(r_o^2 + r_i^2)p}{r_o^2 - r_i^2} \\ &= \frac{-\left((1.0 \text{ in})^2 + (0.5 \text{ in})^2\right)p}{(1.0 \text{ in})^2 - (0.5 \text{ in})^2} \\ &= -1.667p \\ \sigma_{r,o} &= -p \end{aligned}$$

From Eq. 51.18, the diametral strain is

$$\begin{aligned} \epsilon &= \frac{\sigma_{c,o} - \nu(\sigma_{r,o} + \sigma_l)}{E} \\ &= \frac{-1.667p - (0.36)(-p)}{1.59 \times 10^7 \ \frac{\text{lbf}}{\text{in}^2}} \\ &= -0.822 \times 10^{-7}p \\ \Delta d &= \epsilon_d = (-0.822 \times 10^{-7}p)(2.0 \text{ in}) \\ &= -1.644 \times 10^{-7}p \end{aligned}$$

The diametral interference is known to be 0.004 in. From Eq. 51.20,

$$\begin{aligned} I_{\text{diametral}} &= |\Delta d_{o,\text{inner}}| + |\Delta d_{i,\text{outer}}| \\ 0.004 \text{ in} &= 5.86 \times 10^{-7}p + |-1.644 \times 10^{-7}p| \\ p &= 5330 \text{ lbf/in}^2 \end{aligned}$$

From Table 51.2, the circumferential stress at the inner face of the brass (under external pressure) is

$$\begin{aligned} \sigma_{c,i} &= \frac{-2r_o^2 p}{r_o^2 - r_i^2} \\ &= \frac{(-2)(1.0 \text{ in})^2\left(5330 \ \frac{\text{lbf}}{\text{in}^2}\right)}{(1.0 \text{ in})^2 - (0.5 \text{ in})^2} \\ &= -14{,}213 \text{ lbf/in}^2 \end{aligned}$$

Also from Table 51.2, the maximum shear stress is

$$\begin{aligned} \tau_{\text{max}} &= (0.5)\sigma_{c,i} = (0.5)\left(-14{,}213 \ \frac{\text{lbf}}{\text{in}^2}\right) \\ &= -7107 \text{ lbf/in}^2 \end{aligned}$$

(b) The initial force necessary to disassemble the two cylinders is the same as the maximum assembly force.

$$\begin{aligned} F_{\text{max}} &= 2\pi f p r_{\text{shaft}} t_{\text{hub}} \\ &= (2\pi)(0.25)\left(5330 \ \frac{\text{lbf}}{\text{in}^2}\right)(1 \text{ in})(2 \text{ in}) \\ &= 16{,}745 \text{ lbf} \end{aligned}$$

8. STRESS CONCENTRATIONS FOR PRESS-FITTED SHAFTS IN FLEXURE

When a shaft carrying a press-fitted hub (whose thickness is less than the shaft length) is loaded in flexure, there will be an increase in shaft bending stress in the vicinity of the inner hub edge. The fatigue life of the shaft can be seriously affected by this stress increase. The extent of the increase depends on the magnitude of the bending stress and the contact pressure, and can be as high as 2.0 or more.

Some designs attempt to reduce the increase in shaft stress by grooving the disk (to allow the disk to flex). Other designs rely on various treatments to increase the fatigue strength of the shaft. For an unmodified simple press-fit, the stress concentration factor (to be applied to the bending stress calculated from Mc/I) is given by Fig. 51.4.