Geometry and Knowns

$D2 := 9.75\ in$ $\quad r2 := \frac{D2}{2}$ $\quad h_{cyl1} := 23\ in$

$D5 := 2\ in$ $\quad r5 := \frac{D5}{2}$ $\quad h_{cyl4} := 4\ in$

$A1 := 2 \cdot \pi \cdot r2 \cdot h_{cyl1}$

$A2 := \pi \cdot (r2)^2$

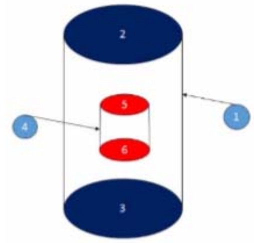
$A3 := A2$

$A4 := 2 \cdot \pi \cdot r5 \cdot h_{cyl4}$

$A5 := \pi \cdot (r5)^2$

$A6 := A5$

Surface Definitions



Nitrogen Properties [1]

$V_{nit} := 1704\ in^3$

$\rho_{nit} := 1.165\ \frac{kg}{m^3}$

$Cp_{nit} := 1040\ \frac{J}{kg \cdot K}$

Aluminum Properties [2]

$\rho_{al} := 2700 \cdot \frac{kg}{m^3}$

$cp_{al} := 893 \cdot \frac{J}{kg \cdot K}$

$V_{al1} := (\pi \cdot (5.3^2\ in^2) \cdot 23\ in) - (\pi \cdot (4.875^2\ in^2) \cdot 22\ in)$

$V_{al2} := \pi \cdot 4.875^2\ in^2 \cdot 0.5\ in$ $\quad V_{al3} := V_{al2}$

$V_{al4} := \pi \cdot 1^2\ in^2 \cdot 4\ in$ $\quad V_{al5} := V_{al4}$ $\quad V_{al6} := V_{al4}$

$vel := 4\ \frac{m}{s}$

$T_{init} := 300\ K$

$ORIGIN := 1$

View Factor Calculations

View Factor Calculations:
1 = Cylindrical Shell
2 = Payload Top
3 = Payload Bottom

4 = Expiriment Shell
5 = Expiriment Top
6 = Expiriment Bottom

7 = Gas

View Factors To Find:

1: F_{1-1} F_{1-2} F_{1-3} F_{1-4} F_{1-5} F_{1-6}

2: F_{2-1} F_{2-3} F_{2-4} F_{2-5}

3: F_{3-1} F_{3-2} F_{3-4} F_{3-6}

4: F_{4-1} F_{4-2} F_{4-3}

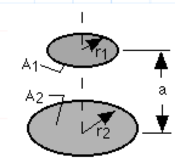
5: F_{5-1} F_{5-2}

6: F_{6-1} F_{6-3}

7: Transparent gas, negligible radiation interchange ([3] pg. 896)

Surface 2

F_{25-52} **= Disk to parallel coaxial disk of unequal radius ([3] pg. 867)**



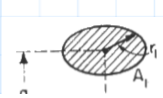
$$r_{251} := \frac{9.75\ in}{2} \qquad r_{252} := \frac{2\ in}{2} \qquad a_{25} := \frac{23\ in}{2} - \frac{4\ in}{2} \qquad R_{251} := \frac{r_{251}}{a_{25}} \qquad R_{252} := \frac{r_{252}}{a_{25}}$$

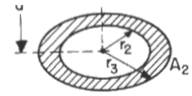
$$X := 1 + \frac{\left(1 + R_{252}{}^{2}\right)}{R_{251}{}^{2}} \qquad F_{25} := (0.5) \cdot \left(X - \left(X^{2} - 4 \cdot \left(\frac{R_{252}}{R_{251}}\right)^{2}\right)^{0.5}\right) \qquad F_{25} = 0.0087$$

Reciprocity

$$F_{52} := F_{25} \cdot \left(\frac{A2}{A5}\right) \qquad F_{52} = 0.207$$

F_{23} **=Disk to coaxial annular ring on parallel disk ([4])**

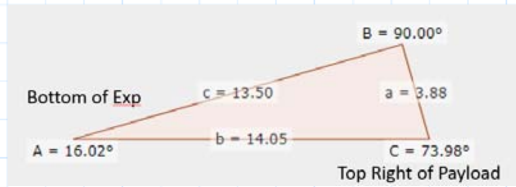




To project shadow onto bottom disc and create annuli, first place node at bottom corner of expiriment from a side view and calculate angles. We use aT, cT, and angle BT to find bT, and angles AT (CT is not needed)/

$aT := 3.875\ in$ $\quad BT := 90\ deg$

$cT := 13.50\ in$



$$bT := \sqrt{(aT)^2 + (cT)^2}$$

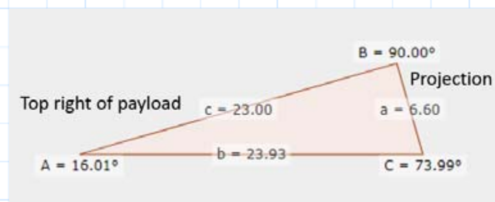
$$AT := \left(\operatorname{acos}\left(\frac{(bT)^2 + (cT)^2 - (aT)^2}{2 \cdot bT \cdot cT}\right)\right) \qquad AT = 16.0154\ deg$$

After calculating AT we now have the angle we need to determine the projection of the payload top onto the payload bottom. Again we have three knowns which in this case are AT2, BT2, and bT2. We are mainly concerned with the calculation here of aT2, but a diagram is shown for reference here as well.

$AT2 := AT$ $\quad cT2 := 23\ in$

$BT2 := BT$

$CT2 := 180° - AT2 - BT2$



$$aT2 := \frac{\sin(AT2) \cdot cT2}{\sin(CT2)} \qquad aT2 = 6.6019\ in$$

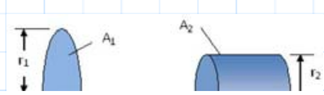
The shadow projects from each side of the payload 6.602 in along the bottom surface. This gives the shadow a radius of 1.727 inches, or a circular projection of 3.545 inches on the bottom surface. We can now compute the view factor.

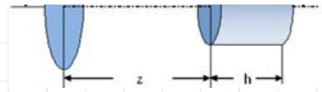
$a_{23} := 23\ in \quad r_{231} := 4.875\ in \quad r_{232} := 1.727\ in \quad r_{233} := 4.875\ in$

$$H_{23} := \frac{a_{23}}{r_{231}} \quad R_{232} := \frac{r_{232}}{r_{231}} \quad R_{233} := \frac{r_{233}}{r_{231}}$$

$$F_{23} := 0.5 \cdot \left(R_{233}^{\ 2} - R_{232}^{\ 2} - \left(\left(1 + R_{233}^{\ 2} + H_{23}^{\ 2}\right)^2 - 4 \cdot R_{233}^{\ 2}\right)^{0.5} + \left(\left(1 + R_{232}^{\ 2} + H_{23}^{\ 2}\right)^2 - 4 \cdot R_{232}^{\ 2}\right)^{0.5}\right) \qquad F_{23} = 0.0359$$

F_{24} = Disk to outer surface of separated coaxial cylinder [5]





$r_{241} := 4.875\ in$ $r_{242} := 1\ in$ $A_{241} := A2$ $A_{242} := A5$ $h_{24} := 4\ in$ $z_{24} := 9.5\ in$

$R_{241} := \frac{r_{241}}{z_{24}}$ $R_{242} := \frac{r_{242}}{z_{24}}$ $H_{24} := \frac{h_{24}}{z_{24}}$

$$F_{24} := \left(\left(\frac{R_{242} \cdot H_{24}}{\pi \cdot R_{241}^{2}}\right) \cdot \left(\operatorname{acos}\left(\frac{\left(1+H_{24}\right)^{2} - R_{241}^{2} + R_{242}^{2}}{\left(1+H_{24}\right)^{2} + R_{241}^{2} - R_{242}^{2}}\right) - \operatorname{acos}\left(\frac{1 - R_{241}^{2} + R_{242}^{2}}{1 + R_{241}^{2} - R_{242}^{2}}\right)\right)\right.$$
$$- \left(\left(\frac{H_{24}}{\pi \cdot R_{241}^{2}}\right) \cdot \left(\left(\frac{\left(\left(\left(1+H_{24}\right)^{2} + R_{241}^{2} + R_{242}^{2}\right)^{2} - 4 \cdot R_{241}^{2} \cdot R_{242}^{2}\right)^{0.5}}{2 \cdot \left(1+H_{24}\right)}\right) \cdot \operatorname{acos}\left(\left(\frac{R_{242}}{R_{241}}\right) \cdot \left(\frac{\left(1+H_{24}\right)^{2} - R_{241}^{2} + R_{242}^{2}}{1 + R_{241}^{2} - R_{242}^{2}}\right)\right)\right)\right)$$
$$+ \left(\left(\frac{H_{24}}{\pi \cdot R_{241}^{2}}\right) \cdot \left(\left(\frac{\left(\left(1 + R_{241}^{2} + R_{242}^{2}\right)^{2} - 4 \cdot R_{241}^{2} \cdot R_{242}^{2}\right)^{0.5}}{2}\right) \cdot \operatorname{acos}\left(\left(\frac{R_{242}}{R_{241}}\right) \cdot \left(\frac{1 - R_{241}^{2} + R_{242}^{2}}{1 + R_{241}^{2} - R_{242}^{2}}\right)\right)\right)\right)$$
$$\left. + \left(\left(\frac{H_{24}^{2}}{2 \cdot \pi \cdot R_{241}^{2}}\right) \cdot \left(\operatorname{acos}\left(\frac{R_{242}}{R_{241}}\right) - \left(\frac{\left(R_{241}^{2} - R_{242}^{2}\right)}{1 + H_{24}}\right) \cdot \left(\frac{\pi}{2} + \operatorname{asin}\left(\frac{R_{242}}{R_{241}}\right)\right)\right)\right)\right)$$

$F_{24} = 0.0428$

F_{24}, F_{23}, and F_{25} are solved for, therefor by summation

$F_{21} := 1 - F_{23} - F_{24} - F_{25}$ $F_{21} = 0.9126$

Sanity Check on summation
Base of right circular cylinder to inside surface of cylinder = 0.959 [6] (calculation not shown), our answer is 0.9126 which is just below that which is due to shadowing.

Surface 3

Identical Geometries

$F_{36} := F_{25}$ $F_{63} := F_{52}$ $F_{32} := F_{23}$ $F_{34} := F_{24}$ $F_{31} := F_{21}$

Surface 4

View Factors solved by reciprocity with system 2, and then summation.

Reciprocity and identical geometry $F_{42} := F_{24} \cdot \left(\frac{A2}{A4}\right)$ $F_{43} := F_{42}$

Summation $F_{41} := 1 - F_{42} - F_{43}$

Surface 5

Summation $F_{51} := 1 - F_{52}$

Surface 6

Summation $F_{61} := 1 - F_{63}$

Surface 1

$A_{15} := 2 \cdot \pi \cdot 4.875\ in \cdot 9.5\ in$ Exposed Cylinder seen by expiriment top

Reciprocity and identical gemoetries $F_{15} := F_{51} \cdot \left(\frac{A5}{A1}\right)$ $F_{16} := F_{15}$

Reciprocity $F_{14} := F_{41} \cdot \left(\frac{A4}{A1}\right)$

Reciprocity and identical geometries $F_{13} := F_{31} \cdot \left(\frac{A3}{A1}\right)$ $F_{12} := F_{13}$

Summation $F_{11} := 1 - F_{12} - F_{13} - F_{14} - F_{15} - F_{16}$

View Factor Summary: All should be populated (non zero)

$F_{11} = 0.7729$ $F_{12} = 0.0967$ $F_{13} = 0.0967$ $F_{14} = 0.0266$ $F_{15} = 0.0035$ $F_{16} = 0.0035$

$F_{21} = 0.9126$ $F_{23} = 0.0359$ $F_{24} = 0.0428$ $F_{25} = 0.0087$

$F_{31} = 0.9126$ $F_{32} = 0.0359$ $F_{34} = 0.0428$ $F_{36} = 0.0087$

$F_{41} = 0.7458$ $F_{42} = 0.1271$ $F_{43} = 0.1271$

$F_{51} = 0.7930$ $F_{52} = 0.2070$

$F_{61} = 0.7930$ $F_{63} = 0.2070$

Summary of Methodology:
F25, F24, and F23 were calculated using view factor integration
Rest were solved using reciprocity, summation, and identical geometry

Air Properties

$airdata := \text{READEXCEL}(\text{“.\textbackslash External Air Properties International Standard Atmosphere Values.xlsx”}, \text{“Sh}$

$$airdata = \begin{bmatrix} 0 & 288.15 & 1.0133 \cdot 10^{5} & 1.2251 & 1.007 & 1.7899 & 1.461 & 2.534 \\ 500 & 284.9 & 9.546 \cdot 10^{4} & 1.1674 & 1.007 & 1.7744 & 1.52 & 2.509 \\ 1 \cdot 10^{3} & 281.7 & 8.988 \cdot 10^{4} & 1.1116 & 1.007 & 1.7575 & 1.581 & 2.483 \\ 1.5 \cdot 10^{3} & 278.4 & 8.456 \cdot 10^{4} & 1.0582 & 1.007 & 1.7419 & 1.646 & 2.457 \\ 2 \cdot 10^{3} & 275.2 & 7.95 \cdot 10^{4} & 1.0065 & 1.007 & 1.7261 & 1.715 & 2.431 \\ 2.5 \cdot 10^{3} & 271.9 & 7.469 \cdot 10^{4} & 0.9571 & 1.007 & 1.7103 & 1.787 & 2.405 \\ & & & & & & & & \ddots \end{bmatrix}$$

$elev := airdata^{\langle 1 \rangle}\ m \quad temp := airdata^{\langle 2 \rangle}\ K \quad p := airdata^{\langle 3 \rangle} \cdot 10^{-5}\ Pa \quad \rho := airdata^{\langle 4 \rangle}\ \frac{kg}{m^3}$

$mu := airdata^{\langle 6 \rangle} \cdot 10^{-5}\ \frac{N \cdot s}{m^2} \quad nu := airdata^{\langle 7 \rangle} \cdot 10^{-5}\ \frac{m^2}{s} \quad k := airdata^{\langle 8 \rangle} \cdot 10^{-2}\ \frac{W}{m \cdot K}$

$alpha := airdata^{\langle 9 \rangle}\ 10^{-5}\ \frac{m^2}{s} \quad Pr := airdata^{\langle 10 \rangle}$

$Cp_{air} := 1.007 \cdot 10^{-3}\ \frac{J}{kg \cdot K} \quad time := \frac{elev}{4\ m}\ s$

$t := 0\ s, 125\ s..7500\ s$

$elev(t) := 4\ \frac{m}{s}\ (t) \qquad elev(7500\ s) = (3 \cdot 10^{4})\ m$

$T_{inf}(t) := \text{linterp}(time, temp, t) \qquad T_{inf}(7500\ s) = 226.5\ K$

$Tf_{top}(t) := \frac{T_{init} + T_{inf}(t)}{2} \qquad Tf_{top}(7500\ s) = 263.25\ K$

$p(t) := \text{linterp}(time, p, t) \qquad p(7500\ s) = 0.012\ \frac{kg}{m \cdot s^2}$

$\rho(t) := \text{linterp}(time, \rho, t) \qquad \rho(7500\ s) = 0.0184\ \frac{kg}{m^3}$

$mu(t) := \text{linterp}(time, mu, t) \qquad mu(7500\ s) = (1.4755 \cdot 10^{-5})\ \frac{kg}{m \cdot s}$

$nu(t) := \text{linterp}(time, nu, t) \qquad nu(7500\ s) = 0.0008\ \frac{m^2}{s}$

$k(t) := \text{linterp}(time, k, t) \qquad k(7500\ s) = 0.0203\ \frac{kg \cdot m}{s^3 \cdot K}$

$Pr(t) := \text{linterp}(time, Pr, t) \qquad Pr(7500\ s) = 0.7305$

$Re_{top}(t) := \frac{vel \cdot D2}{nu(t)}$ $Re_{side}(t) := \frac{vel \cdot h_{cyl1}}{nu(t)}$

$Re_{top}(7500\ s) = 1.2362 \cdot 10^{3}$

$constant := 0.667$ $constantm := 0.5$

$Nu_{top}(t) := constant \cdot \left(Re_{top}(t)\right)^{constantm} \cdot Pr(t)^{\frac{1}{3}}$ $Nu_{top}(0\ s) = 155.0371$

$Nu_{side}(t) := 0.664 \cdot \left(Re_{side}(t)\right)^{\frac{1}{2}} \cdot Pr(t)^{\frac{1}{3}}$ $Nu_{side}(0\ s) = 237.0498$

$h_{top}(t) := \frac{Nu_{top}(t) \cdot k(t)}{D2}$ $h_{top}(0\ s) = 15.8637\ \frac{kg}{s^{3} \cdot K}$

$h_{side}(t) := \frac{Nu_{side}(t) \cdot k(t)}{h_{cyl1}}$ $h_{side}(0\ s) = 10.2822\ \frac{kg}{s^{3} \cdot K}$

Radiation Circuit Equations

$\varepsilon := 0.8$

$Eb1 := \varepsilon \cdot \sigma \cdot T_{init}{}^4$ $\sigma = \left(5.6704 \cdot 10^{-8}\right) \frac{kg}{s^3 \cdot K^4}$

$Eb2 := \varepsilon \cdot \sigma \cdot T_{init}{}^4$

$Eb3 := \varepsilon \cdot \sigma \cdot T_{init}{}^4$

$Eb4 := \varepsilon \cdot \sigma \cdot T_{init}{}^4$

$Eb5 := \varepsilon \cdot \sigma \cdot T_{init}{}^4$

$Eb6 := \varepsilon \cdot \sigma \cdot T_{init}{}^4$

Guess Values

$J1 := 1 \frac{W}{m^2}$ $J2 := 2 \frac{W}{m^2}$ $J3 := 3 \frac{W}{m^2}$ $J4 := 4 \frac{W}{m^2}$ $J5 := 5 \frac{W}{m^2}$ $J6 := 6 \frac{W}{m^2}$

Constraints

$$Eb1 = \left(\frac{J1 - J2}{\left(A1 \cdot F_{12}\right)^{-1}} + \frac{J1 - J3}{\left(A1 \cdot F_{13}\right)^{-1}} + \frac{J1 - J4}{\left(A1 \cdot F_{14}\right)^{-1}} + \frac{J1 - J5}{\left(A1 \cdot F_{15}\right)^{-1}} + \frac{J1 - J6}{\left(A1 \cdot F_{16}\right)^{-1}}\right) \cdot \left(\frac{1 - \varepsilon}{\varepsilon \cdot A1}\right) + J1$$

$$Eb2 = \left(\frac{J2 - J1}{\left(A2 \cdot F_{21}\right)^{-1}} + \frac{J2 - J3}{\left(A2 \cdot F_{23}\right)^{-1}} + \frac{J2 - J4}{\left(A2 \cdot F_{24}\right)^{-1}} + \frac{J2 - J5}{\left(A2 \cdot F_{25}\right)^{-1}}\right) \cdot \left(\frac{1 - \varepsilon}{\varepsilon \cdot A2}\right) + J2$$

$$Eb3 = \left(\frac{J3 - J1}{\left(A3 \cdot F_{31}\right)^{-1}} + \frac{J3 - J2}{\left(A3 \cdot F_{32}\right)^{-1}} + \frac{J3 - J4}{\left(A3 \cdot F_{34}\right)^{-1}} + \frac{J3 - J6}{\left(A3 \cdot F_{36}\right)^{-1}}\right) \cdot \left(\frac{1 - \varepsilon}{\varepsilon \cdot A3}\right) + J3$$

$$Eb4 = \left(\frac{J4 - J1}{\left(A4 \cdot F_{41}\right)^{-1}} + \frac{J4 - J2}{\left(A4 \cdot F_{42}\right)^{-1}} + \frac{J4 - J3}{\left(A4 \cdot F_{43}\right)^{-1}}\right) \cdot \left(\frac{1 - \varepsilon}{\varepsilon \cdot A4}\right) + J4$$

$$Eb5 = \left(\frac{J5 - J1}{\left(A5 \cdot F_{51}\right)^{-1}} + \frac{J5 - J2}{\left(A5 \cdot F_{52}\right)^{-1}}\right) \cdot \left(\frac{1 - \varepsilon}{\varepsilon \cdot A5}\right) + J5$$

$$Eb6 = \left(\frac{J6 - J1}{\left(A6 \cdot F_{61}\right)^{-1}} + \frac{J6 - J3}{\left(A6 \cdot F_{63}\right)^{-1}} \right) \cdot \left(\frac{1 - \varepsilon}{\varepsilon \cdot A6} \right) + J6$$

Solver

$$\begin{bmatrix} J1 \\ J2 \\ J3 \\ J4 \\ J5 \\ J6 \end{bmatrix} := \textbf{Find}\left(J1, J2, J3, J4, J5, J6\right)$$

$J1 = 367.4419 \ \frac{kg}{s^3}$ $J2 = 367.4419 \ \frac{kg}{s^3}$ $J3 = 367.4419 \ \frac{kg}{s^3}$

$J4 = 367.4419 \ \frac{kg}{s^3}$ $J5 = 367.4419 \ \frac{kg}{s^3}$ $J6 = 367.4419 \ \frac{kg}{s^3}$

Energy Balance Equations

$G_{solar} := 1000 \ \frac{W}{m^2}$ $G_{albedosun} := 1300 \ \frac{W}{m^2}$ $\alpha := 0.2$ $\varepsilon_{out} := 0.91$ $\rho := 1 - \alpha$

$Eb_{top} := \varepsilon_{out} \cdot \sigma \cdot T_{init}{}^4$ $h_{inside} := 5 \ \frac{W}{m^2 \cdot K}$ $q_{dot} := 10 \ W$

$Eb_{side} := \varepsilon_{out} \cdot \sigma \cdot T_{init}{}^4$

Surface 1:

$Radiosity_{side} := Eb_{top}$ $Qrad_{side} := \alpha \cdot G_{albedosun}$

Surface 2:

$Qrad_{top} := \alpha \cdot G_{solar}$ $Radiosity_{top} := Eb_{side}$

$$\left[\frac{1}{\rho_{al} \cdot cp_{al} \cdot V_{al1}} \cdot \left(Qrad_{side} - Radiosity_{side} + \frac{Eb1 - J1}{\frac{1 - \varepsilon}{\varepsilon \cdot A1}} + h_{inside} \cdot A2 \cdot \left(Tt_1 - Tt_5\right) + \left(h_{side}(t) \cdot A1 \cdot \left(Tt_1 - T_{inf}(t)\right)\right) \right) \right.$$

$$
Dt(t,Tt) := \begin{bmatrix}
\frac{1}{\rho_{al}\cdot cp_{al}\cdot V_{al2}}\cdot\left(Qrad_{top}-Radiosity_{top}+\frac{Eb2-J2}{\frac{1-\varepsilon}{\varepsilon\cdot A2}} + \left(h_{inside}\cdot A2\cdot\left(Tt_2-Tt_5\right)\right) + \left(h_{top}(t)\cdot A2\cdot\left(Tt_2-T_{inf}(t)\right)\right)\right) \\
\frac{1}{\rho_{al}\cdot cp_{al}\cdot V_{al3}}\cdot\left(\frac{Eb3-J3}{\frac{1-\varepsilon}{\varepsilon\cdot A3}}+\left(h_{inside}\cdot A3\cdot\left(Tt_3-Tt_5\right)\right)\right) \\
\frac{1}{\rho_{al}\cdot cp_{al}\cdot V_{al4}}\cdot\left(\frac{Eb4-J4}{\frac{1-\varepsilon}{\varepsilon\cdot A4}}+\frac{Eb5-J5}{\frac{1-\varepsilon}{\varepsilon\cdot A5}}+\frac{Eb6-J6}{\frac{1-\varepsilon}{\varepsilon\cdot A6}} + \left(h_{inside}\cdot A5\cdot\left(Tt_4-Tt_5\right)\right) + \left(h_{inside}\cdot A4\cdot\left(Tt_4-Tt_5\right)\right)+\left(h_{inside}\cdot A6\cdot\left(Tt_4-Tt_5\right)\right) + q_{dot}\right) \\
\frac{1}{\rho_{nit}\cdot Cp_{nit}\cdot V_{nit}}\cdot\left(\left(\left(h_{inside}\cdot A4\cdot\left(Tt_4-Tt_5\right)\right)+\left(h_{inside}\cdot A5\cdot\left(Tt_5-Tt_5\right)\right) + \left(h_{inside}\cdot A6\cdot\left(Tt_6-Tt_5\right)\right)\right) + \left(h_{inside}\cdot A1\cdot\left(Tt_1-Tt_5\right)\right)+\left(h_{inside}\cdot A2\cdot\left(Tt_2-Tt_5\right)\right) + \left(h_{inside}\cdot A3\cdot\left(Tt_3-Tt_5\right)\right)\right)
\end{bmatrix}
$$

Payload

Payload

Payload

Expiring

Gas

$t_0 := 0$ $t_f := 7500$ $intvl := 1000$ $Tt := \begin{bmatrix} 300 \\ 300 \\ 300 \\ 300 \\ 300 \end{bmatrix}$

$yay := rkfixed\left(T, t_0, t_f, intvl, Dt\right)$