Ex.10 Convolution (Response of input waves)

when the impulse response is also input function

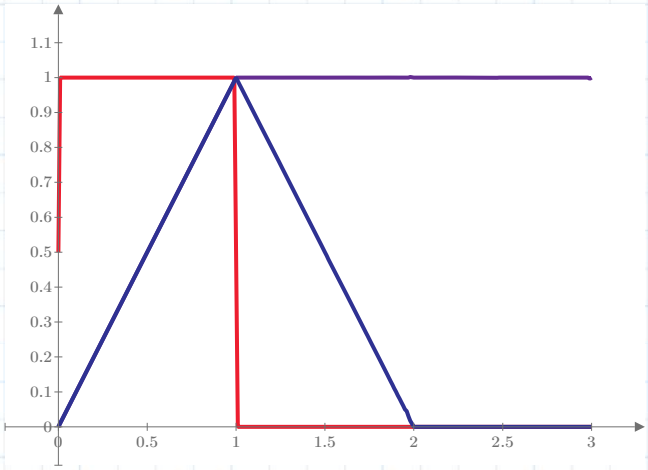
$t := 0, 0.01 .. 4$

$rec(t) := \Phi(t) - \Phi(t-1)$

$x(t) := rec(t)$

$h(t) := rec(t)$

$y(t) := \int_0^t x(\tau) \cdot h(t-\tau)\, d\tau$



t

$x(t)$ *input*

$\int_0^t h(t)\, dt$ impulse response

indicial response

$y(t)$ *output*

$y(t) := t \cdot \Phi(t) + 2 \cdot \Phi(t-1) - 2 \cdot \Phi(t-2) - 2 \cdot t \cdot \Phi(t-1) + t \cdot \Phi(t-2)$

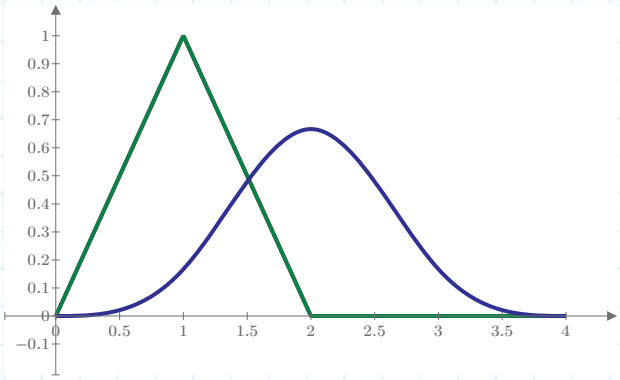
The one of triangular function input.

$$x(t) := t \cdot (\Phi(t) - \Phi(t-1)) + (2-t) \cdot (\Phi(t-1) - \Phi(t-2))$$

$$h(t) := t \cdot (\Phi(t) - \Phi(t-1)) + (2-t) \cdot (\Phi(t-1) - \Phi(t-2))$$

$$y(t) := \int_0^t h(\tau) \cdot x(t-\tau)\, d\tau$$

$$y(t) := \frac{t^3}{6} \cdot \Phi(t) - \frac{2 \cdot \Phi(t-1) \cdot (t-1)^3}{3} + \Phi(t-2) \cdot (t-2)^3 - \frac{2 \cdot \Phi(t-3) \cdot (t-3)^3}{3} + \frac{\Phi(t-4) \cdot (t-4)^3}{6}$$



$x(t)$ *input*

$h(t)$ impulse response

$y(t)$ *output*

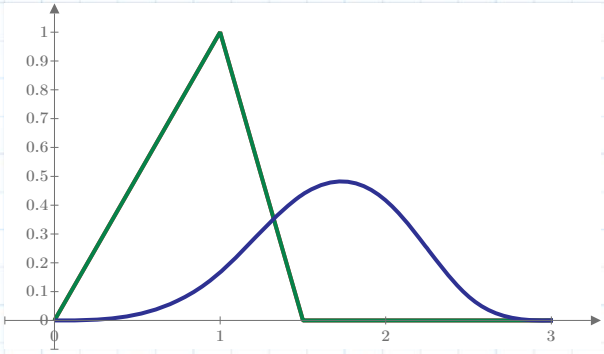
t

$$x(t) := t \cdot (\Phi(t) - \Phi(t-1)) + (3-2 \cdot t) \cdot \left(\Phi(t-1) - \Phi\left(t - \frac{3}{2}\right)\right)$$

$$h(t) := t \cdot (\Phi(t) - \Phi(t-1)) + (3-2 \cdot t) \cdot \left(\Phi(t-1) - \Phi\left(t - \frac{3}{2}\right)\right)$$

$$y(t) := \int_0^t h(\tau) \cdot x(t-\tau)\, d\tau$$

$$y_1(t) := \frac{t^2}{2} \cdot \Phi(t) - 2 \cdot \Phi(t-1) \cdot (t-1)^2 + \frac{3 \cdot \Phi(t-2) \cdot (t-2)^2}{2} + \frac{\Phi\left(t - \frac{3}{2}\right) \cdot (2 \cdot t - 3)^2}{4} - \frac{\Phi\left(t - \frac{5}{2}\right) \cdot (2 \cdot t - 5)^2}{4}$$



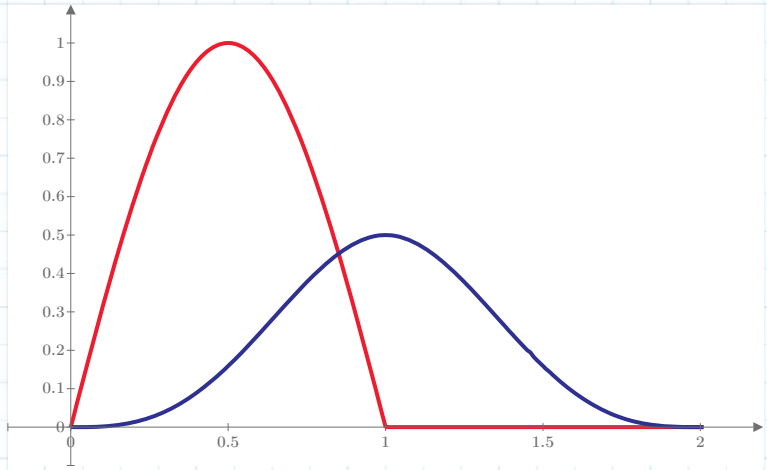
$x(t)$ *input*

$h(t)$ impulse response

$y(t)$ *output*

t

$$\Pi(t) := \sin(\pi \cdot t) \cdot (\Phi(t+0) - \Phi(t-1)) \qquad y(t) := \int_0^t \Pi(\tau) \cdot \Pi(t-\tau)\, d\tau$$



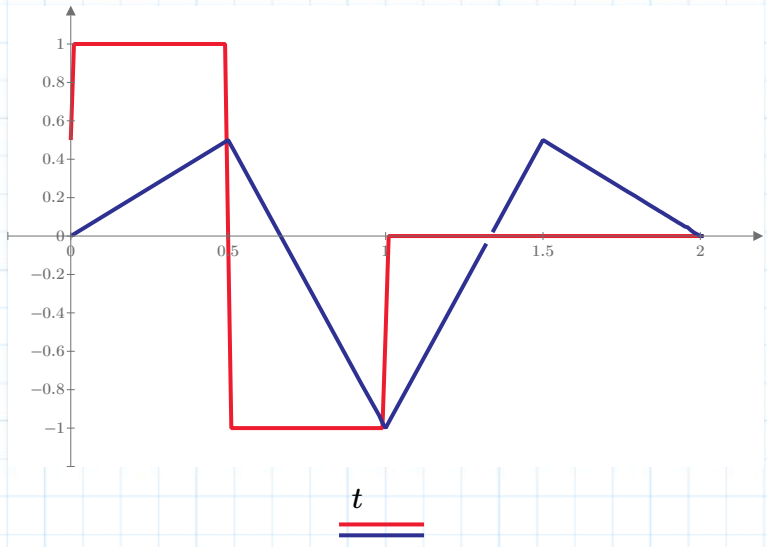
impulse response

$\Pi(t)$ *input*

$y(t)$ *output*

$$y(t) := \frac{\sin(\pi \cdot t) \cdot \Phi(t) - 2 \cdot \sin(\pi \cdot t) \cdot \Phi(t-1) + \sin(\pi \cdot t) \cdot \Phi(t-2) - 2 \cdot \pi \cdot \cos(\pi \cdot t) \cdot \Phi(t-1) + 2 \cdot \pi \cdot \cos(\pi \cdot t) \cdot \Phi(t-2) - \pi \cdot t \cdot \cos(\pi \cdot t) + 2 \cdot \pi \cdot t \cdot \cos(\pi \cdot t) \cdot \Phi(t-1) - \pi \cdot t \cdot \cos(\pi \cdot t) \cdot \Phi(t-2)}{2 \cdot \pi}$$

$$R(t) := \Phi(t) - 2 \cdot \Phi(t-0.5) + \Phi(t-1) \qquad y(t) := \int_0^t R(\tau) \cdot R(t-\tau)\, d\tau$$



impulse response

$R(t)$ *input*

$y(t)$ *output*

$$y(t) := t \cdot \Phi(t) - 2.0 \cdot \Phi(t-2.0) - 6.0 \cdot \Phi(t-1.0) + 2.0 \cdot \Phi(t-0.5) + 6.0 \cdot \Phi(t-1.5) + t \cdot \Phi(t-2.0) + 6.0 \cdot t \cdot \Phi(t-1.0) - 4.0 \cdot t \cdot \Phi(t-0.5) - 4.0 \cdot t \cdot \Phi(t-1.5)$$