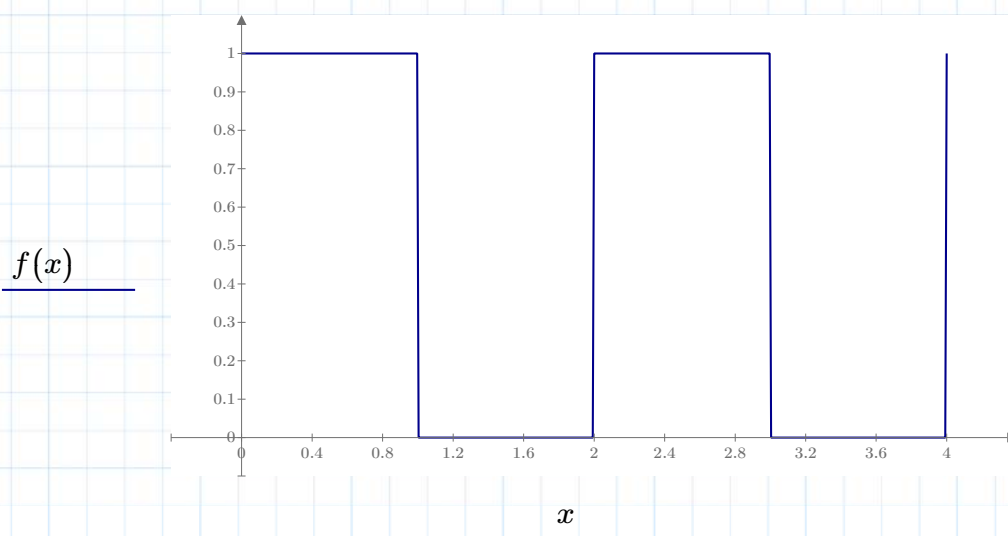


Ex.F8 Fourier-Analysis (Programing of Fourier Analysis)

$$f(x) := \begin{cases} \text{if } (0 \leq x) \wedge (x \leq 1) \\ \quad \| 1 \\ \text{else if } (-1 \leq x) \wedge (x \leq 0) \\ \quad \| 0 \\ \text{else if } (x > 1) \\ \quad \| f(x-2) \end{cases}$$



$$L := 1 \quad Nt := 20$$

$$FC(f, N, L) := \begin{cases} Z^{(0)} \leftarrow \left[\left(\frac{1}{2 \cdot L} \cdot \int_{-L}^L f(x) dx \right) \right] \\ \text{for } n \in 1..N \\ \quad \| Z_{n,0} \leftarrow \frac{1}{L} \cdot \int_{-L}^L f(x) \cdot \cos\left(\frac{n \cdot \pi \cdot x}{L}\right) dx \\ \quad \| Z_{n,1} \leftarrow \frac{1}{L} \cdot \int_{-L}^L f(x) \cdot \sin\left(\frac{n \cdot \pi \cdot x}{L}\right) dx \end{cases}$$

$$res := FC(f, Nt, L)$$

$$A := res^{(0)}$$

$$B := res^{(1)}$$

$$p(x) := A_0 + \sum_{n=1}^{Nt} \left(A_n \cdot \cos\left(\frac{n \cdot \pi \cdot x}{L}\right) + B_n \cdot \sin\left(\frac{n \cdot \pi \cdot x}{L}\right) \right)$$

$$a_0 := \frac{1}{2}$$

$$A = \begin{bmatrix} 0.5 \\ 1.567 \cdot 10^{-17} \\ 4.163 \cdot 10^{-17} \\ 2.054 \cdot 10^{-17} \\ 0 \\ -1.018 \cdot 10^{-16} \\ 5.551 \cdot 10^{-17} \\ 2.69 \cdot 10^{-17} \\ -2.498 \cdot 10^{-16} \\ -1.325 \cdot 10^{-17} \\ -1.852 \cdot 10^{-16} \\ 3.465 \cdot 10^{-16} \\ \vdots \end{bmatrix}$$

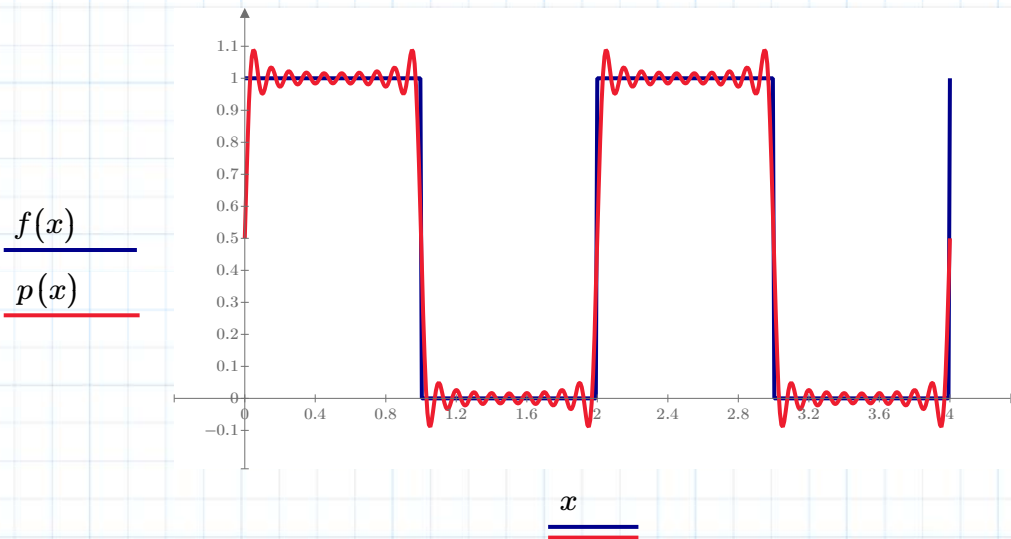
$$B = \begin{bmatrix} 0 \\ 0.637 \\ -7.272 \cdot 10^{-18} \\ 0.212 \\ -8.401 \cdot 10^{-17} \\ 0.127 \\ 4.008 \cdot 10^{-17} \\ 0.091 \\ -2.29 \cdot 10^{-16} \\ 0.071 \\ -6.658 \cdot 10^{-18} \\ 0.058 \\ \vdots \end{bmatrix}$$

$$b_1 := \frac{2}{\pi} = 0.637$$

$$b_3 := \frac{2}{3 \cdot \pi} = 0.212$$

$$b_5 := \frac{2}{5 \cdot \pi} = 0.127$$

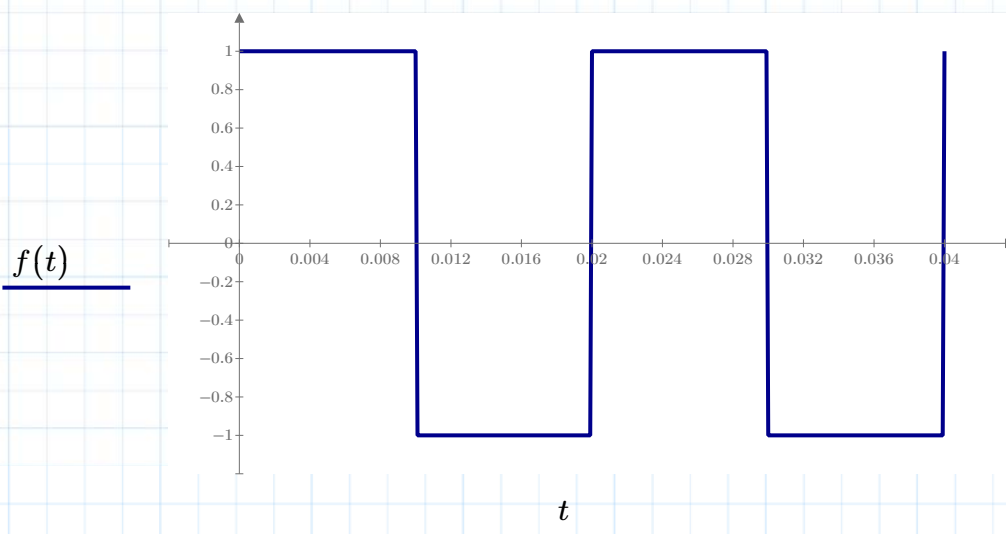
$$b_7 := \frac{2}{7 \cdot \pi} = 0.091$$



```
clear(f)      50 Hz Rectangular wave
```

```
f(t) := || if (0 ≤ t) ∧ (t ≤ 0.01)
          || 1
          || else if (-0.01 ≤ t) ∧ (t ≤ 0)
          || -1
          || else if (t > 0.01)
          || f(t - 0.02)
```

```
t := 0, 0.0001 .. 0.04
```



```
L := 0.01
```

```
Nt := 10
```

```
FC(f, N, L) := || Z(0) ← [ (1 / (2 · L)) · ∫-LL f(t) dt ]
                || for n ∈ 1 .. N
                || || Zn,0 ← (1 / L) · ∫-LL f(t) · cos(n · π · t / L) dt
                || || Zn,1 ← (1 / L) · ∫-LL f(t) · sin(n · π · t / L) dt
                || Z
```

```
res := FC(f, Nt, L)
```

```
A := res(0)
```

```
B := res(1)
```

```
p(t) := A0 + ∑n=1Nt ( An · cos(n · π · t / L) + Bn · sin(n · π · t / L) )
```

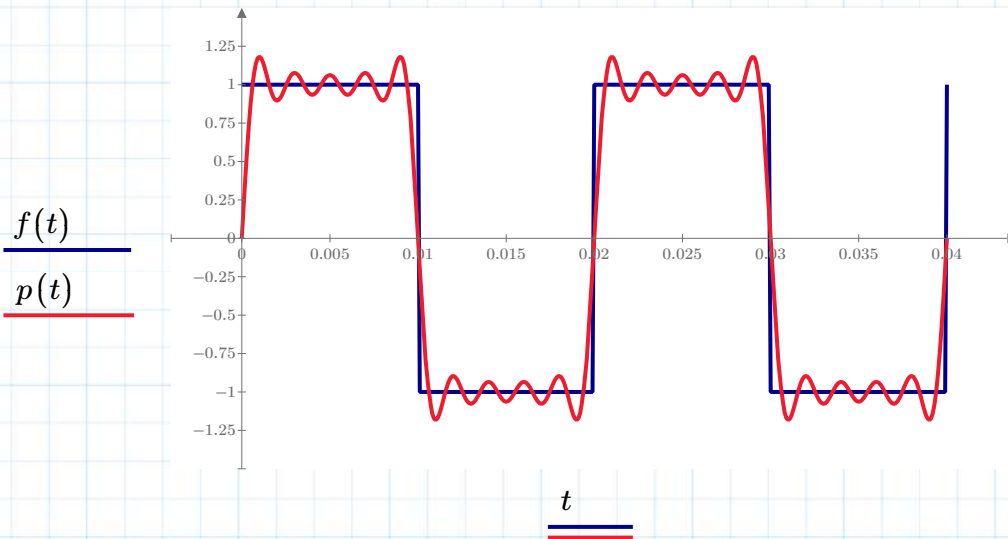
$$A = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1.273 \\ 4.647 \cdot 10^{-17} \\ 0.424 \\ -3.349 \cdot 10^{-17} \\ 0.255 \\ 3.064 \cdot 10^{-16} \\ 0.182 \\ 1.056 \cdot 10^{-15} \\ 0.141 \\ 1.615 \cdot 10^{-16} \end{bmatrix}$$

$$\frac{4}{\pi} = 1.273$$

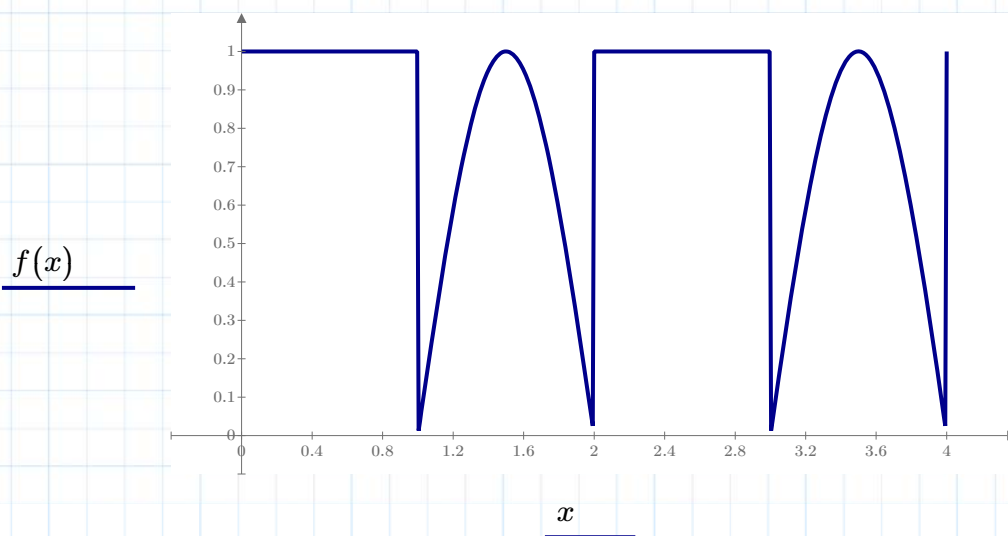
$$\frac{4}{3 \cdot \pi} = 0.424$$

$$\frac{4}{5 \cdot \pi} = 0.255$$



```
clear(f)
```

```
f(x) := || if (0 ≤ x) ∧ (x ≤ 1)
         || 1
         || else if (-1 ≤ x) ∧ (x ≤ 0)
         || -sin(x · π)
         || else if (x > 1)
         || f(x - 2)
```



$L := 1$ $Nt := 50$

```
FC(f, N, L) := || Z(0) ← [ ( 1 / (2 · L) · ∫-LL f(x) dx ) ]
                || for n ∈ 1 .. N
                || || Zn,0 ← 1/L · ∫-LL f(x) · cos( (n · π · x) / L ) dx
                || || Zn,1 ← 1/L · ∫-LL f(x) · sin( (n · π · x) / L ) dx
                || Z
```

```
res := FC(f, Nt, L)
```

```
A := res(0)
```

```
B := res(1)
```

```
p(x) := A0 + ∑n=1Nt ( An · cos( (n · π · x) / L ) + Bn · sin( (n · π · x) / L ) )
```

$$A = \begin{bmatrix} 0.818 \\ 1.419 \cdot 10^{-17} \\ -0.212 \\ 4.399 \cdot 10^{-18} \\ -0.042 \\ -1.783 \cdot 10^{-16} \\ -0.018 \\ 3.062 \cdot 10^{-17} \\ -0.01 \\ -7.394 \cdot 10^{-17} \\ -0.006 \\ 4.813 \cdot 10^{-16} \\ \vdots \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0.137 \\ -1.696 \cdot 10^{-17} \\ 0.212 \\ -3.507 \cdot 10^{-17} \\ 0.127 \\ -7.909 \cdot 10^{-18} \\ 0.091 \\ -1.231 \cdot 10^{-16} \\ 0.071 \\ -6.501 \cdot 10^{-18} \\ 0.058 \\ \vdots \end{bmatrix}$$

