

### Steel Properties (For steel reinforcement, ASTM A615M Grade 420 MPa)

$$f_y := 420 \text{ MPa} = 60.916 \text{ ksi}$$

$$E_s := 2 \cdot 10^5 \text{ MPa}$$

### Concrete Properties

$$\gamma_{\text{concrete}} := 25 \frac{\text{kN}}{\text{m}^3} = (2.549 \cdot 10^3) \frac{\text{kgf}}{\text{m}^3}$$

$$f'_c := 32 \cdot \text{MPa}$$

$$\lambda := 1.0$$

$$E_c := 4700 \cdot \sqrt{f'_c \cdot \text{MPa}} = (2.659 \cdot 10^7) \text{ kPa} \quad n := \frac{E_s}{E_c} = 7.522$$

$$f_r := 0.62 \cdot \lambda \cdot \sqrt{f'_c \cdot \text{MPa}} = 3.507 \text{ MPa}$$

: modulus of rupture of concrete (ACI 318-08, eq. 9-10)

### Geometric Properties

$$b := 1000 \text{ mm}$$

$$h := 900 \text{ mm}$$

$$c_c := 75 \text{ mm}$$

$$w_{\text{limit}} := 0.1 \text{ mm}$$

$$\text{Dia} := [10 \text{ mm} \quad 12 \text{ mm} \quad 16 \text{ mm} \quad 20 \text{ mm} \quad 25 \text{ mm} \quad 32 \text{ mm}]^T$$

$$\text{Area} := [0.7854 \text{ cm}^2 \quad 1.131 \text{ cm}^2 \quad 2.011 \text{ cm}^2 \quad 3.142 \text{ cm}^2 \quad 4.909 \text{ cm}^2 \quad 8.042 \text{ cm}^2]^T$$

$$\text{Dn} := 6 \quad \text{: Rebar Selection D10=1, D12=2, D16=3, D20=4, D25=5, D32=6}$$

$$s := 150 \text{ mm} \quad d_s := \text{Dia}_{\text{Dn}} = 32 \text{ mm} \quad A_s := \text{Area}_{\text{Dn}} \cdot \frac{b}{s} = (5.361 \cdot 10^3) \text{ mm}^2$$

$$I_g := \frac{b \cdot h^3}{12} = 0.061 \text{ m}^4$$

$$y_t := \frac{h}{2} = 0.45 \text{ m}$$

$$d := h - c_c = 0.825 \text{ m}$$

$$d_c := c_c + \frac{d_s}{2} = 91 \text{ mm}$$

$$d := 809 \text{ mm}$$

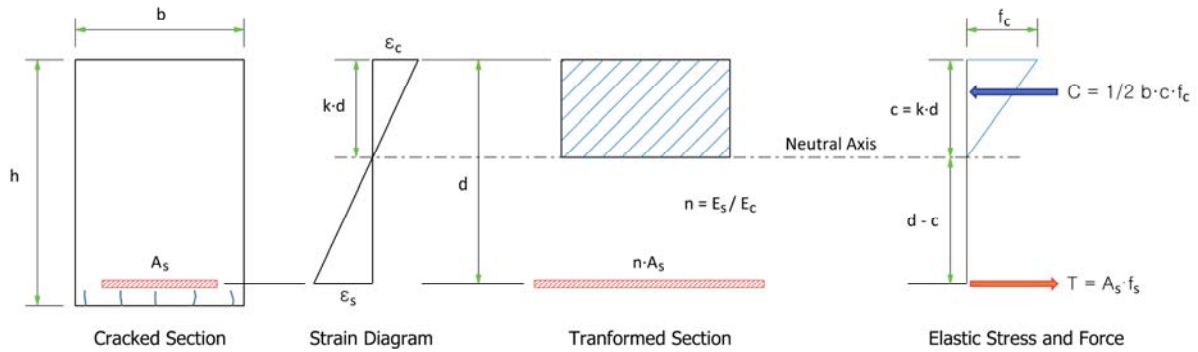
### Cracking Moment and Moment at Service Loads

$$M_{\text{serv}} := 250 \text{ kN} \cdot \text{m}$$

$$M_{\text{cr}} := \frac{f_r \cdot I_g}{y_t} = 473.479 \text{ kN} \cdot \text{m}$$

: cracking moment (ACI 318-08, eq. 9-9)

## Calculation of Tensile Stress in Reinforcement at Service Loads



$$B := \frac{b}{n \cdot A_s} = 0.025 \text{ mm}^{-1} \quad c := \frac{\sqrt{2 \cdot B \cdot d + 1} - 1}{B} = 218.283 \text{ mm} \quad \text{: neutral axis depth}$$

$$I_{cr} := \frac{b \cdot c^3}{3} + n \cdot A_s \cdot (d - c)^2 = (1.754 \cdot 10^{10}) \text{ mm}^4 \quad \text{: moment of inertia of cracked section transformed to concrete}$$

$$f_s := \frac{M_{srv}}{A_s \cdot \left(d - \frac{c}{3}\right)} = 63.336 \text{ MPa} \quad f_s := \frac{M_{srv}}{I_{cr}} n \cdot (d - c) = 63.336 \text{ MPa} \quad \text{: steel stress}$$

$$f_c := \frac{M_{srv}}{I_{cr}} c = 3.111 \text{ MPa} \quad \text{: concrete stress}$$

- verificaton -

$$T := A_s \cdot f_s = 339.564 \text{ kN} \quad C := 0.5 \cdot b \cdot c \cdot f_c = 339.564 \text{ kN} \quad \text{: internal couple } T = C$$

$$M := T \cdot \left(d - \frac{c}{3}\right) = 250 \text{ kN} \cdot \text{m} \quad M_{srv} = 250 \text{ kN} \cdot \text{m} \quad \text{: internal couple mement } M = M_{srv}$$

- if there is a compression axial force, the tensile stress minus compress can be applied to calca lte the crack width.

$$P := 0 \text{ kN} \quad A_c := b \cdot h - A_s$$

$$\sigma_s := \frac{P \cdot E_s}{E_s \cdot A_s + E_c \cdot A_c} = 0 \text{ MPa} \quad \sigma_s := \frac{P}{A_s + \frac{1}{n} \cdot A_c} = 0 \text{ MPa}$$

$$f_s := f_s - \sigma_s = (6.334 \cdot 10^4) \text{ kPa} \quad \text{: steel stress considering compression}$$

## 1. Calculation of Crack Width (ACI)

### Step 1. Calculation of Spacing and Crack Width (ACI 318-08)

$$s_{\max} := 380 \cdot \frac{280}{f_s} \cdot \text{MPa} \cdot \text{mm} - 2.5 \cdot c_c = (1.492 \cdot 10^3) \text{ mm} \quad : \text{maxium spacing (ACI 318-08, eq. 10-4)}$$

$$\text{Judge} := \text{if } (s_{\max} > s, \text{“OK”}, \text{“NG”}) = \text{“OK”}$$

### Step 2. Comparison of Cracking Moment and Moment at Service Loads

$$M_{\text{cr}} = 473.479 \text{ kN} \cdot \text{m}$$

$$M_{\text{srv}} = 250 \text{ kN} \cdot \text{m}$$

$$\text{Judge} := \text{if } (M_{\text{cr}} > M_{\text{srv}}, \text{“OK”}, \text{“NG”}) = \text{“OK”}$$

### Step 3. Calculation of Crack Width (ACI 224.1R)

$$\beta := \frac{h - c}{d - c} = 1.154 \quad d_c = 91 \text{ mm} \quad A := 2 d_c \cdot s = (2.73 \cdot 10^4) \text{ mm}^2$$

$$w := 2 \cdot \frac{f_s}{E_s} \cdot \beta \cdot \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} = 0.086 \text{ mm} \quad : \text{crack width (ACI 224.1R-07, eq. 1-1)}$$

$$w := \begin{cases} \text{if } c_c > 50 \text{ mm} \\ \quad \left\| \begin{array}{l} c_c \leftarrow 50 \text{ mm} \\ d_c \leftarrow c_c + 0.5 \cdot d_s \end{array} \right. \\ \quad \left\| \begin{array}{l} w \leftarrow 2 \cdot \frac{f_s}{E_s} \cdot \beta \cdot \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \\ w \end{array} \right. \end{cases} = 0.073 \text{ mm} \quad : \text{crack width calculated with reduced cover depth, 50 mm (ACI 350-06, Clause 10.6.4)}$$

$$\text{Judge} := \begin{cases} \text{if } M_{\text{srv}} \leq M_{\text{cr}} \\ \quad \left\| \begin{array}{l} \text{“Not cracked”} \\ \text{else if } w \leq w_{\text{limit}} \\ \quad \left\| \begin{array}{l} \text{“Lower than limit”} \\ \text{else} \\ \quad \left\| \begin{array}{l} \text{“NG”} \end{array} \right. \end{array} \right. \end{array} \right. \end{cases} = \text{“Not cracked”}$$