This worksheet is intended to demonstrate sampling of signals using Prime (Mathcad).

This topic like the other worksheet on Fourier can have difficult solutions, here the objective is to use a software for application in signal processing, come to appreciate software for signal processing.

The use of software here is for learning the subject matter and software. A learning experience, gaining software skills, and solving some basic to intermediate examples/problems. End result should be where we are able to take on difficult or complex problems.

Introduction to Sampling

The process of sampling involves the recording of the amplitude of a signal over a specific amount of time. The time interval at which the information is recorded is called the **sampling interval**. As stated by the **sampling theorem**, due to Nyquist, an accurate reconstruction of a signal requires that the sampling frequency be at least twice the frequency of the signal we wish to sample. To sample an analog signal, theoretically, we must multiply it by a sampler. What is a sampler? Ideally, a sampler can be a set of delta functions. For example, to sample an analog signal, we can multiply it by a shifted set of delta functions. Figure 4.1 shows the process of sampling using an ideal sampler.

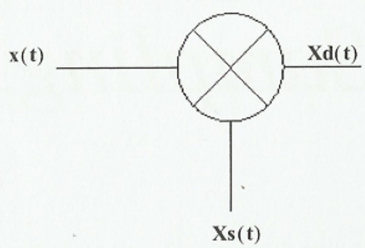


Figure 4.1: *Shows a typical sampler*

As shown in Figure 4.1 , we wish to sample an input signal $x(t)$, so we multiply the signal by a shifted set of a delta function, $x_s(t)$, and the resultant signal is $x_d(t) = x(t)x_s(t)$. Since we sample a signal at specific interval, it can be shown that $x_s(t) = \sum_{k=-\infty}^{\infty} (t - k \cdot T_s)$, where T_s is the sampling time. Using the equation, we can rearrange Figure 4-1 as shown in Figure 4.2.

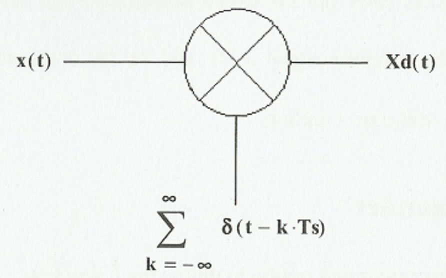


Figure 4.2: Shows the sampler with the impulse train

We use Figure 4.3 to show the process graphically . One of the signals shown is a continuous time analog signal, and the sampler is a discrete time signal. The signal on the right is the resultant signal, which is the product of the two inputs.

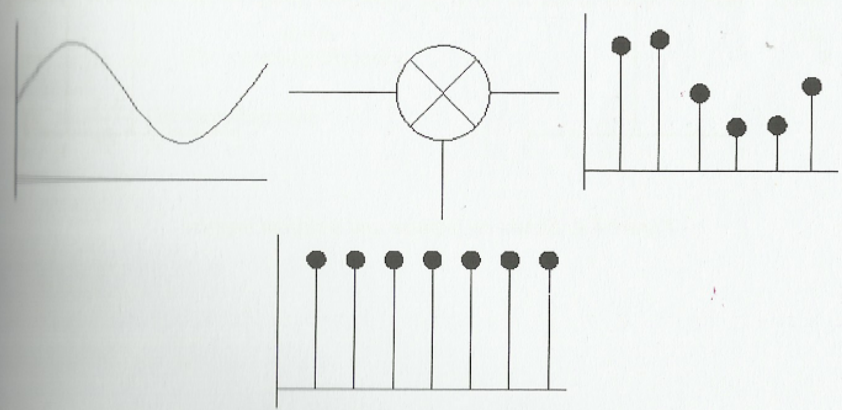


Figure 4.3: Shows the result of a signal after sampling

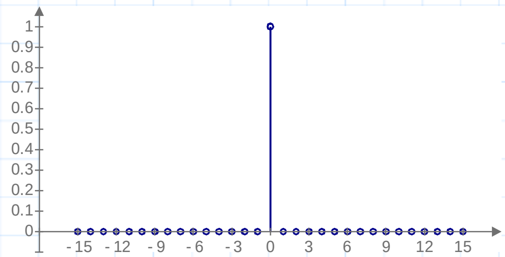
We can use MathCAD to carry out the whole process; we can define the analog signal $x(t)$, we can create the sampling signal $x_s(t)$, and we can do the multiplication process.

Example 4.1 - Sampling Understanding

$t := -15, -14 .. 15$ defining a range for t

$\delta(t) := if(t = 0, 1, 0)$

Delta function below as per if statement definition, in the stem plot

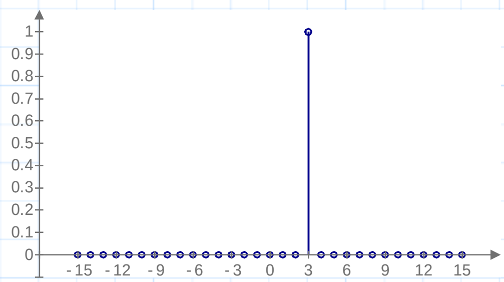


$\delta(t)$

t

The shifted plot when t=3, which means t=0 had been shifted to t=3. This exercise was done in the Fourier Series worksheet

$\delta(t-3)$



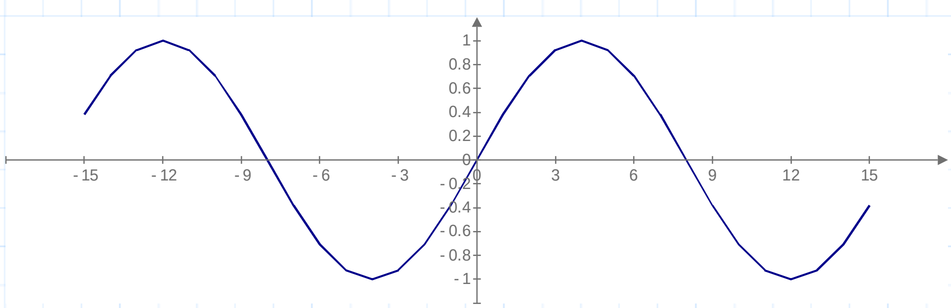
$\delta(t-3)$

t

Lets create a sinusoidal wave:

$\omega_0 := \frac{\pi}{8}$

$x(t) := \sin(\omega_0 \cdot t)$



$x(t)$

t

The sinusoidal signal is the signal we want to put through the sampling process.

$\omega_s := 16 \cdot \omega_0$ the sampling frequency; w = 2 pi f, so its 16 x 2 pi f

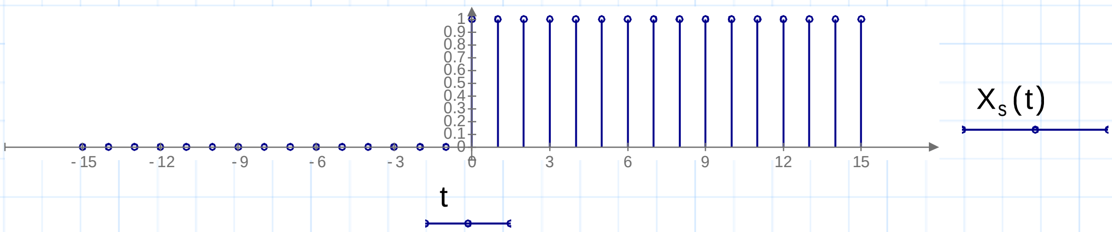
$T_s := \frac{2 \cdot \pi}{\omega_s}$ the sampling period multiplied by 2 pi or sampling time

Objective is to take 16 samples starting from 0.
Let N equal the number of samples.

$N := 16$

$X_s(t) := \sum_{k=0}^{N-1} \delta(t - k \cdot T_s)$ here is it NOT k that takes the value of 0 to 15 (N-1), so k Ts is the sampling interval at 16 locations starting from k=0

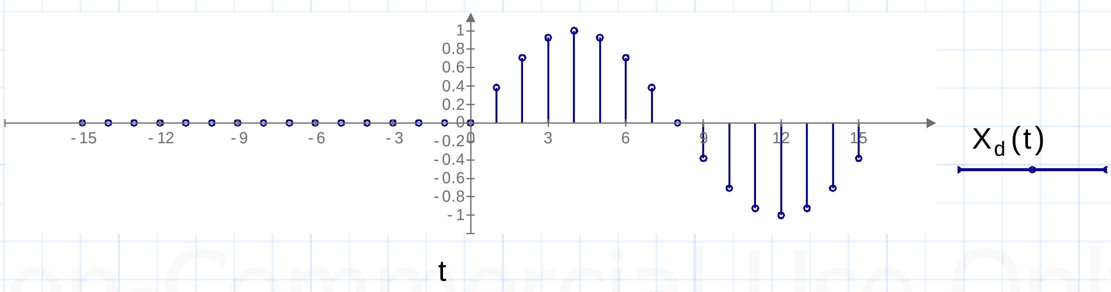
$T_s = 1$ calculated value of Ts



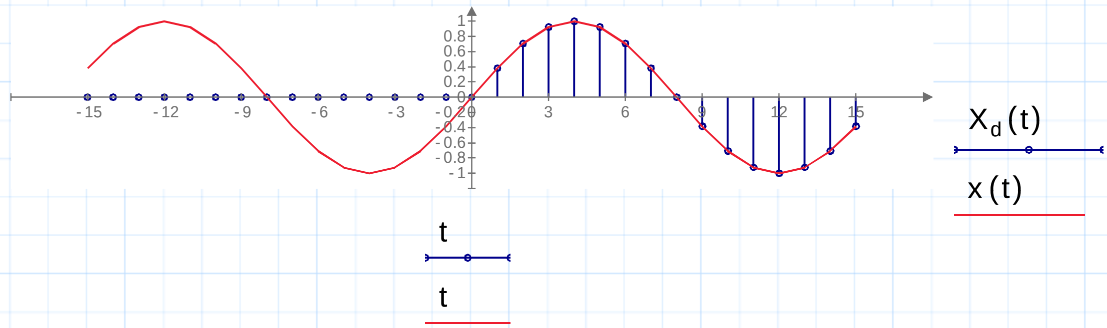
Here we had shifted the delta function 15 times to the right of t=0
Lets identify the plot above as a tool, and call it the sampler
Xs(t) is the sampler

Getting closer to the learning objective of this example.
Lets multiply the sampler (Xs(t) to the signal (ie equation x(t)) to be sampled.

$X_d(t) := x(t) \cdot X_s(t)$



Now lets place the signal to be sampled and the sampled signal on the same plot



The signal and the sampled signal are matching each other.

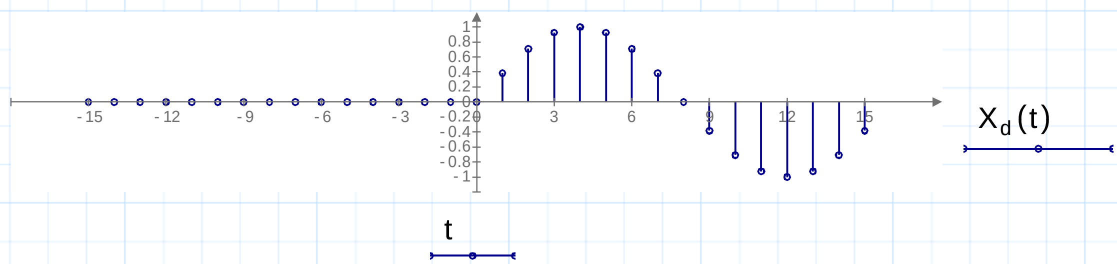
$$X_s(t) := \sum_{k=0}^{N-1} \delta\left(t - k \cdot T_s\right)$$

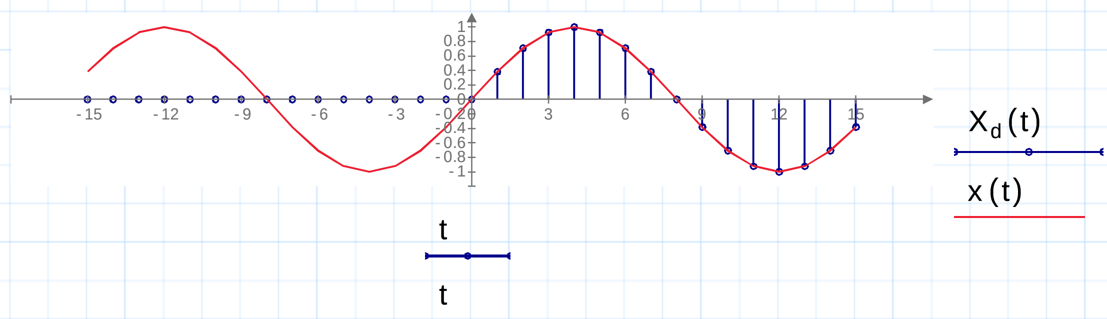
Now lets make kTs =nTs,
then we define n=0 to N-1,
and then plot the signals (n gives a more discrete representation to the sampler)

$$n := 0 .. N - 1$$

$$X_{s\_n}(t) := \sum_{n=0}^{N-1} \delta\left(t - n \cdot T_s\right)$$

$$\text{clear}\left(X_d\right)$$
$$X_d(t) := x(t) \cdot X_{s\_n}(t)$$





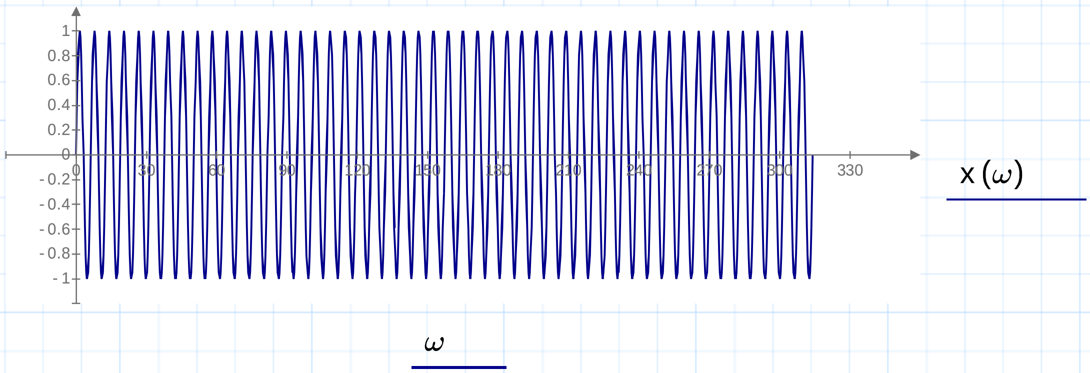
The signal and the sampled signal are matching each other again as expected.

Example 4.2 - Spelling Confusion Outt

w = 2 pi f, radian frequency

$\omega := 0, 0.1 \cdot \pi .. 50 \cdot 2 \cdot \pi$ from 0 to 50 x 2pi is 50 angular cycles per second

$x(\omega) := \sin(\omega)$ signal



What is the problem above? If there is one.

Would not the signal we want to sample be represented in time rather than radian frequency? We want to sample a signal which we do not know its frequency but have some representation of it with respect to time.

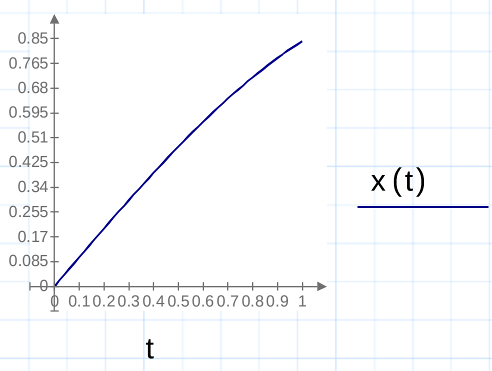
So the signal we want to sample should be with respect to time on the horizontal axis. Here we are saying the radian frequency 2 pi f, f = 50 represents a power signal for 1 second, using a sine wave. So it should be corrected for 1 second on the horizontal axis?

clear (x)

$f := 50 \quad T := \frac{1}{f} = 0.02$

$t := 0, T .. 1$

$x(t) := \sin(t)$

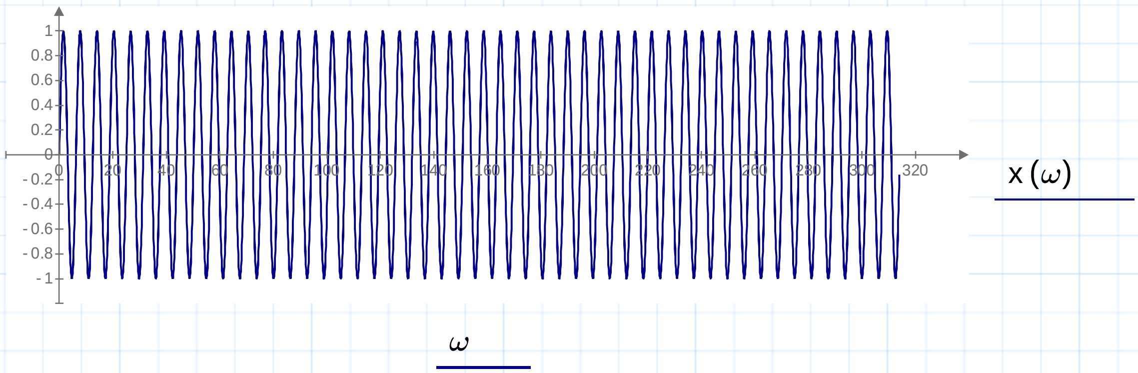


This is the plot from 0 to 1 at T interval, the sin(t) function does not return a sine wave within 1 second. Since the sine function returns its value in radians, perhaps we have to use values similar to w on the horizontal axis, w = 2pif. X axis from 0 to 15 or 0 to 2pif

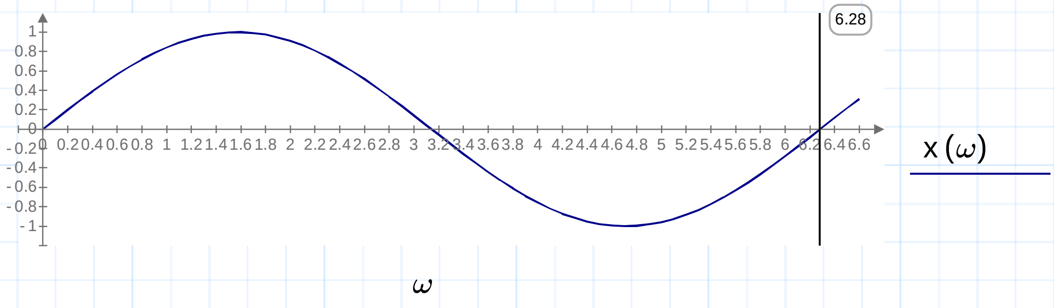
$w := 2 \cdot \pi \cdot f = 314.159265$

Does this represent a power singal at 50hz?
No, it is the range to 314 that will show the sine wave.

$\omega := 0, 0.1 .. 314$
$x(\omega) := \sin(\omega)$

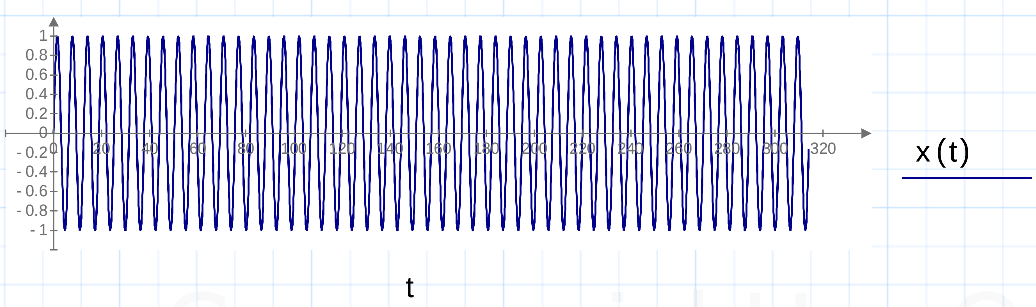


$\omega := 0, 0.1 .. 6.6$ Zoomed in plot to show 1 period



The period of the signal above is 6.28 which is 2pi (2x 3.14=6.28)
The signal is defined by the radian value. We can change w to t so long as the range values on the x axis are the same. It would give the same wave on the plot. So we simply substitute t for w, and set the plot in the time domain.

$t := 0, 0.1 .. 314$
$x(t) := \sin(t)$



The plot above has the same period of 6.28

We want to create a signal with a period, and then we want to sample it.
So the signal need's a shape, and we use the sine function to create its shape.

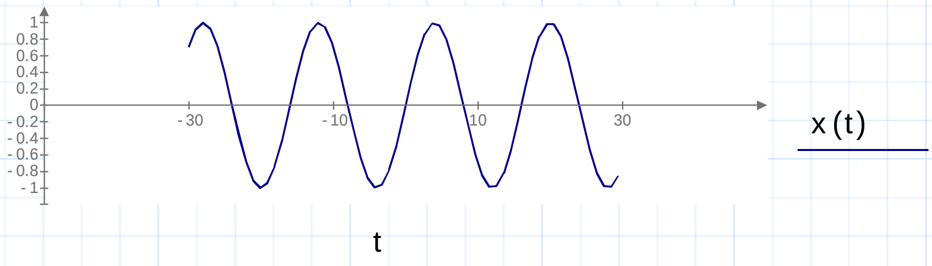
Now the signal we want to sample is shown below.

$t := -30, -29.01 .. 30$

$\omega_0 := \frac{\pi}{8}$ the signals angular frequency - radians

$\omega_0 = 0.392699$ multiplying 0.3926 to 't' each time

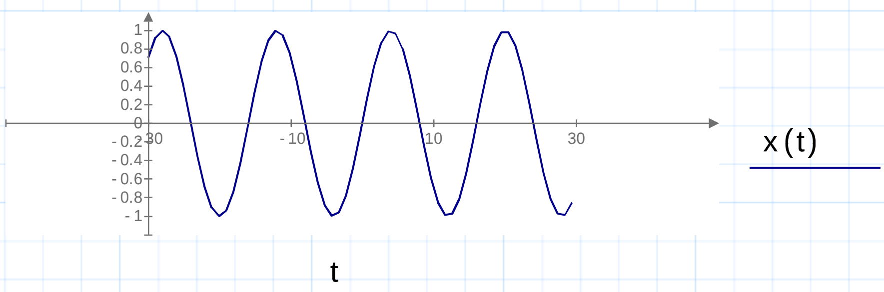
$x(t) := \sin(\omega_0 \cdot t)$



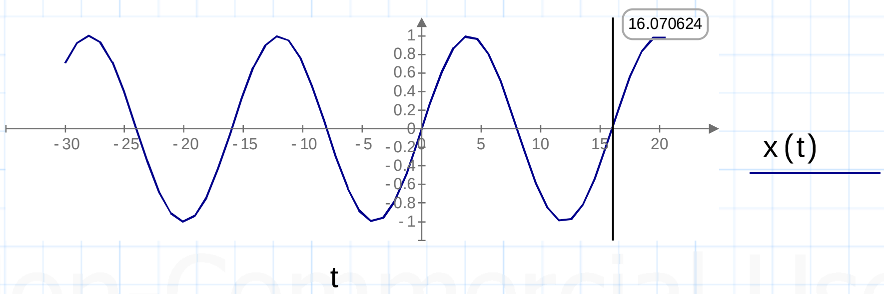
Now remove w (omega) and plug in the multiplier 'm' instead, so we do not associate it with w (omega) of signal theory. we just want to form a shape for the signal in the time domain.

$m := \omega_0 = 0.392699$

$x(t) := \sin(m \cdot t)$



Now plot for one period, as close as possible; same for sin(wo t) plot



The sampling frequency has to be greater than twice the signal frequency

The period is approximately 16.07 in the plot above

$T := 16.07$

$f := \frac{1}{T} = 0.0622$

$f_s := 2 \cdot f = 0.124456$ this is the sampling frequency 2 x f

$T := \frac{1}{f_s} = 8.035$

$T := 8$ set to integer

Now returning to the text book example the authors suggest a sampling frequency of 8 times w0. Where w0 = pi/8. ws = 8 x (pi/8) = pi

We showed above how we arrived to 8 from the inverse of T

$\omega_0 := \frac{\pi}{8}$ $\omega_s := 8 \cdot \omega_0 = 3.1416$

Now we set the sampling period Ts or sampling frequency Ts

Remember function is a sine wave its period is from 0 to 2pi in radian (angular frequency)
ws = 2 pi f, f = 1/Ts, ws = 2 pi /Ts, so Ts = 2 pi/ws

$T_s := \frac{2 \cdot \pi}{\omega_s} = 2$

$T_s = 2$ sampling interval

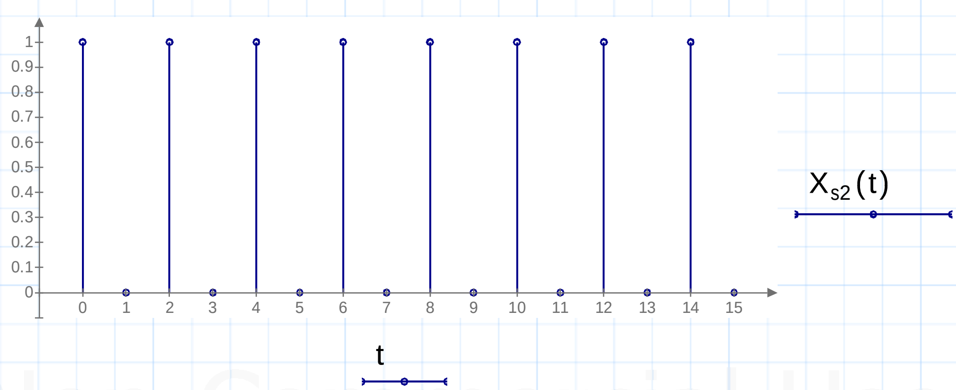
Next form the sampler:

$N := 16$ take 16 samples starting from 0

$t := 0, 1 .. N - 1$

$\delta(t) := \text{if}(t = 0, 1, 0)$ delta impulse function

$X_{s2}(t) := \sum_{n=0}^{N-1} \delta\left(t - n \cdot T_s\right)$ the sampler for this example Xs2



$$X_{s2}(t) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

The shifting property: in this example the period is 2, then shifting it by multiples of 2 would place the unit impulse function back to its position of t=0, resulting in a 1. If the shift is not in multiples of the sampling period Ts (2) then the result is 0.

n=3, t= 3 and n =4, t=4

d(t - nTs) = d(3 - 3x2) = d(3 - 6) = d(-3) = 0
d(t - nTs) = d(4 - 4x2) = d(4 - 8) = d(-4) = 1

If the function of d(t) resulted in =ve 6 it is the same for -ve 6, here the result would be 1

The individual results of Xs2 shown to the left

We formed the sampler next multiply the signal to be sampled to the sampler, to get the sampled signal in discrete intervals.

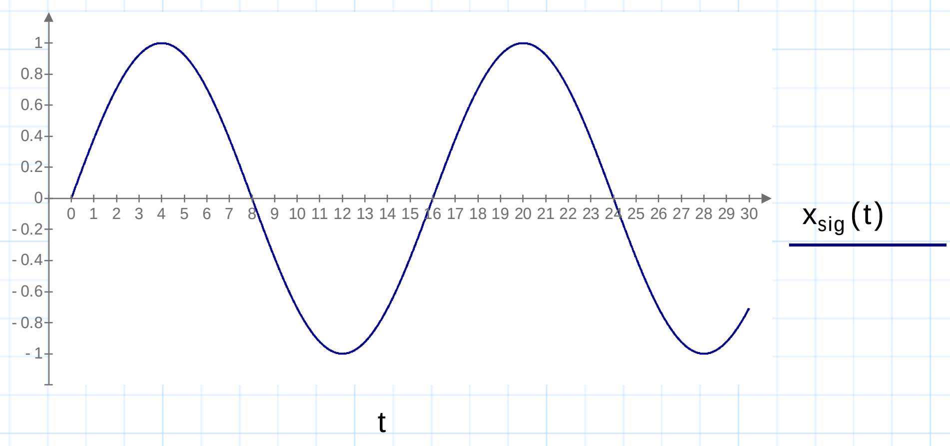
The form of the sampled signal in discrete form is used for further processing in the electronic circuit for whatvever application it is used for.

Signal to be sampled x_sig(t) below:

$$\omega_0 := \frac{\pi}{8}$$

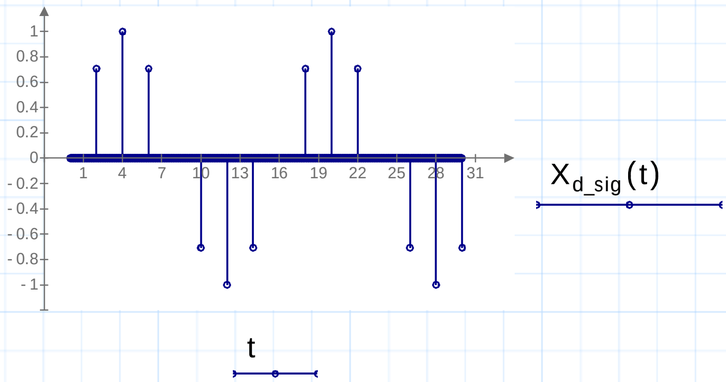
$t := 0, 0.01 .. 30$

$x_{sig}(t) := \sin(\omega_0 \cdot t)$

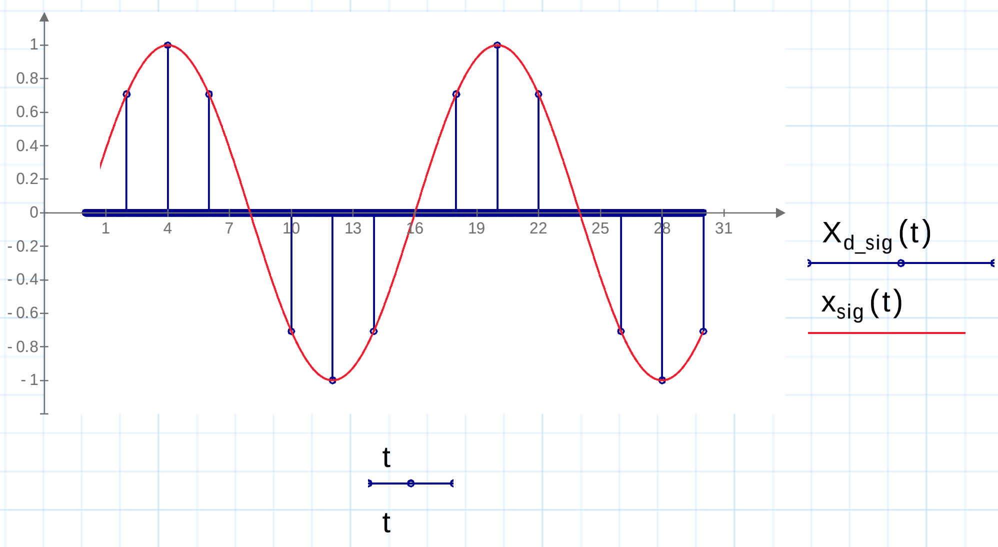


$X_{d\_sig}(t) := x_{sig}(t) \cdot X_{s2}(t)$

Note: the intervals and number of iterations for x_sig(t) and Xs2(t) must be the same.



Now with both the signal and sampled signal to show match.



Note:
The delta function here served to teach the underlying sampling theory.
In the applications for products it is not practical to use the delta function. The signal was in analog form, we create a sampler with intervals to lock-in to the signal, and generate a discrete (digital) sampled signal. In real applications an analog to digital converter associated to a clock is used. The approach is to create a window with length T to sample the continous signal, this is called the zero order hold or sample-and-hold circuit.

Example 4.2 (a)

Undersampling (Aliasing): Lets use the previous example to demonstrate this
Narrower the sampling interval T better the reconstruction which requires a higher frequency f

$T := \frac{1}{f_s} = 8.035$ from example 4.2

$T := 8$ then we set it to an integer in ex 4.2

Now set T to less than 2 times

$T := 4$ <--Set T = 1, 2, 4, and 6. Only 4 shows the peak values in step plot.
The delta function has to work as defined.

Where w0 = pi/8. ws = T x (pi/8) = pi
We showed above how we arrived to 8 from the inverse of T

$\omega_0 := \frac{\pi}{8}$ $\omega_s := T \cdot \omega_0 = 1.5708$

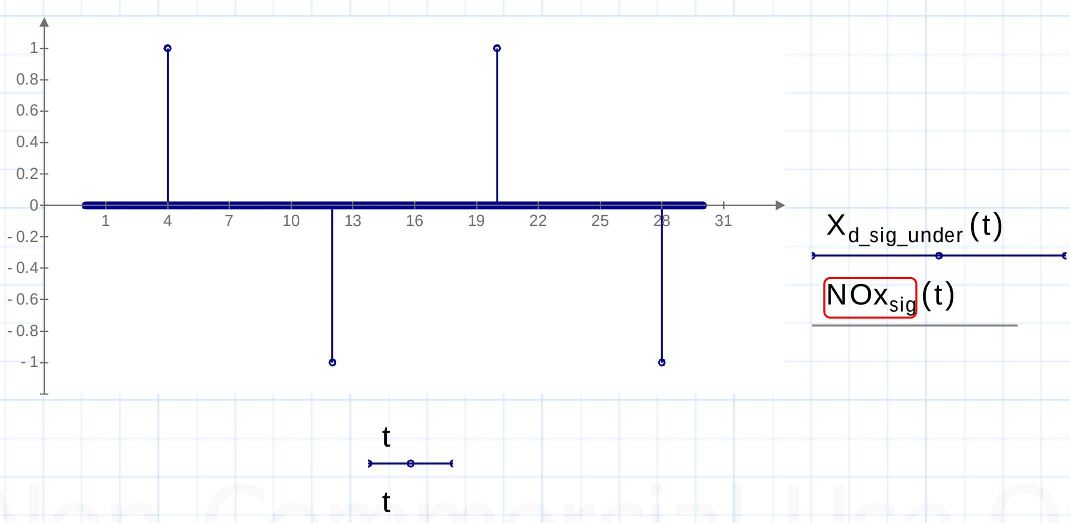
Now we set the sampling period Ts_under

Remember function is a sine wave its period is from 0 to 2pi in radian (angular frequency)
ws = 2 pi f, f = 1/Ts, ws = 2 pi /Ts, so Ts = 2 pi/ws

$T_{s\_under} := \frac{2 \cdot \pi}{\omega_s} = 4$

$X_{s2\_under}(t) := \sum_{n=0}^{N-1} \delta\left(t - n \cdot T_{s\_under}\right)$ the sampler for this example Xs2_under

$X_{d\_sig\_under}(t) := x_{sig}(t) \cdot X_{s2\_under}(t)$ The stem plot does not show the sampled wave accurately compared to T=8



Sampling in the Frequency Domain

4.4 Illustration of Sampling in the Frequency Domain

In the previous section, we illustrate the process of sampling in the time domain. To better understand the process of sampling, it is worthwhile to illustrate the process of sampling and aliasing in the frequency domain as well. Just to refresh our memory, in the time domain, we sample a signal $x(t)$ with the impulse train $x_s(t)$ and get $x_d(t)$ as shown in Figure 4.22.

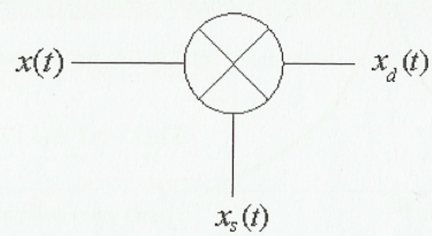


Figure 4.22: Shows an ideal sampler

The frequency domain representation of Figure 4.22 is shown in Figure 4.23. We take the Fourier transform of each signal and replace the multiplication sign by the convolution sign, since multiplication in time domain corresponds to convolution in frequency domain .

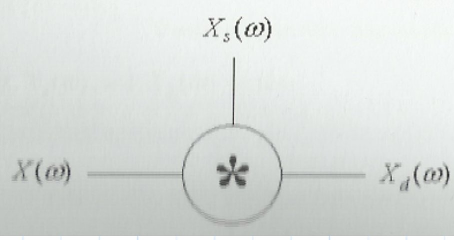


Figure 4.23: Shows the frequency domain representation

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BSE - Arkansas State U 1990. BSc - USAO Oklahoma 1986.

$$x(t)x_s(t) \Leftrightarrow X(\omega) * X_s(\omega)$$

Lets present the process graphically. Figure 4.24 shows the signal $x(t)$, while Figure 4.25 shows $x_s(t)$, where **T** represents the sampling time; Figure 4.26 shows the output $x_d(t)$.

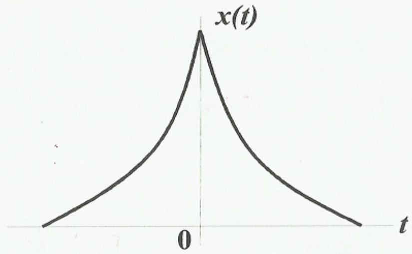

***Figure 4.24**: An analog signal to be sampled*

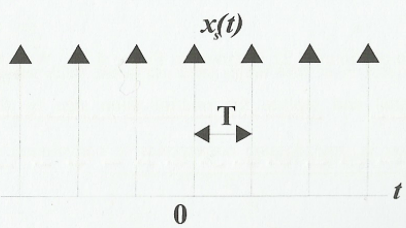

***Figure 4.25**: Shows the sampler with sampling time T*

$$x(t)x_s(t) \Leftrightarrow X(\omega) * X_s(\omega)$$

Lets present the process graphically. Figure 4.24 shows the signal $x(t)$, while Figure 4.25 shows $x_s(t)$, where **T** represents the sampling time; Figure 4.26 shows the output $x_d(t)$.

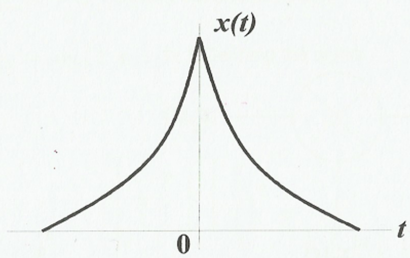

***Figure 4.24**: An analog signal to be sampled*

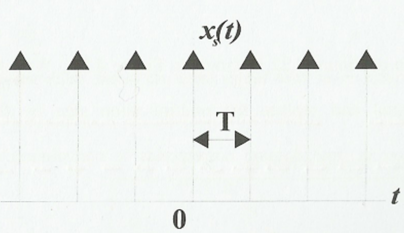

***Figure 4.25**: Shows the sampler with sampling time T*

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BSE - Arkansas State U 1990. BSc - USAO Oklahoma 1986.

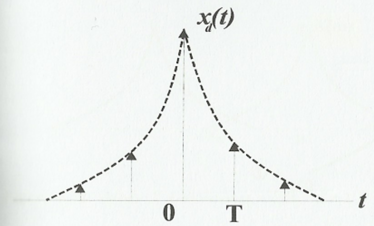


Figure 4.26: Shows the resulting signal after sampling

Below, we show the corresponding frequency domain of the above signal. To find $X_d(\omega)$, we convolve $X(\omega)$ and $X_s(\omega)$. From the convolution integral, we have

$$X_d(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\lambda)X_s(\omega-\lambda)d\lambda$$

Since $X_s(\omega)$ is an impulse train, then

$$X_s(\omega)=\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_s)$$

where ω_s is the sampling frequency; since we know that, when we convolve impulses, we simply shift the signal to the direction of the shifted impulse, the overall convolution of the signal is

$$X_d(\omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty}X(\omega-k\omega_s)$$

By sketching $X(\omega)$, $X_s(\omega)$, and $X_d(\omega)$ we have

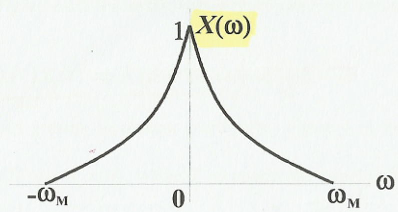


***Figure 4.27**: Shows the Fourier transform of $x(t)$*

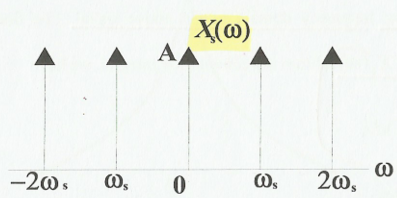


***Figure 4.28**: Shows the Fourier transform of $x_s(t)$ where $A=2\pi/T$*

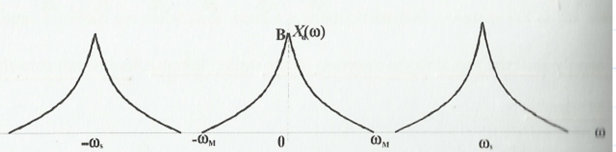


***Figure 4.29**: Shows $X_d(\omega)$ when $\omega_s>2\omega_m$, $B=1/T$*

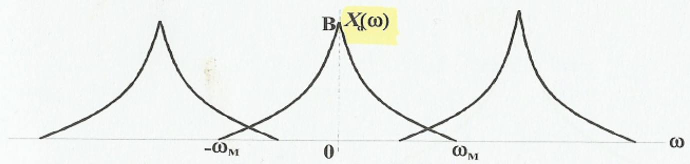


Figure 4.30: *Shows* $X_d(\omega)$ *when* $\omega_s < 2\omega_m$, $B = 1/T$

To recover the original signal , we can pass the signal through an ideal low-pass filter with amplitude of T. Figure 4.31 represents an ideal low-pass filter to which we apply $X_d(\omega)$. Figure 4.32 shows the filter's response. Finally, Figure 4.33 represents the reconstructed signal.

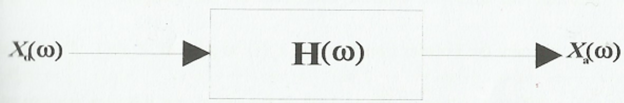


Figure 4.31: *Shows the block diagram of an ideal low-pass filter*

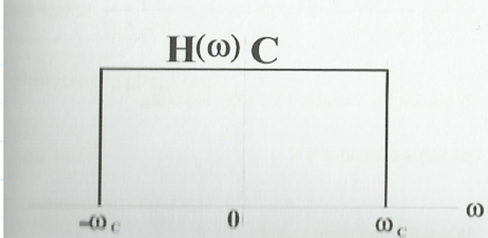


Figure 4.32: *Shows an ideal low-pass filter response where* $C = T$

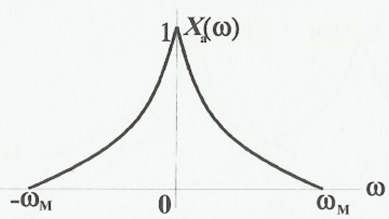


Figure 4.33: *Shows the original signal after reconstruction*

MathCAD Illustration of Sampling in Frequency Domain

We can use MathCAD to illustrate the sampling process in the frequency domain as follows. First, we define the signal $x(t)$ and then take the Fourier transform of $x(t)$ to get $X(\omega)$. Second, we take the Fourier transform of the sampler $x_s(t)$, to give use $X_s(\omega)$. Third, we take the Fourier transform of $x_d(t)$ to give us $X_d(\omega)$. Finally, we can plot $X_d(\omega)$ for different values of **Ws** to look at the effect, for instance we can plot $X_d(\omega)$ at $\omega_s > 2\omega_m$ and $\omega_s < 2\omega_m$.

Example - Procedure, Theory, and Problem.

$j := \sqrt{-1}$

$ORIGIN := -50$ set the origin (ORIGIN capital for Prime 2.0) to -50 for indexing

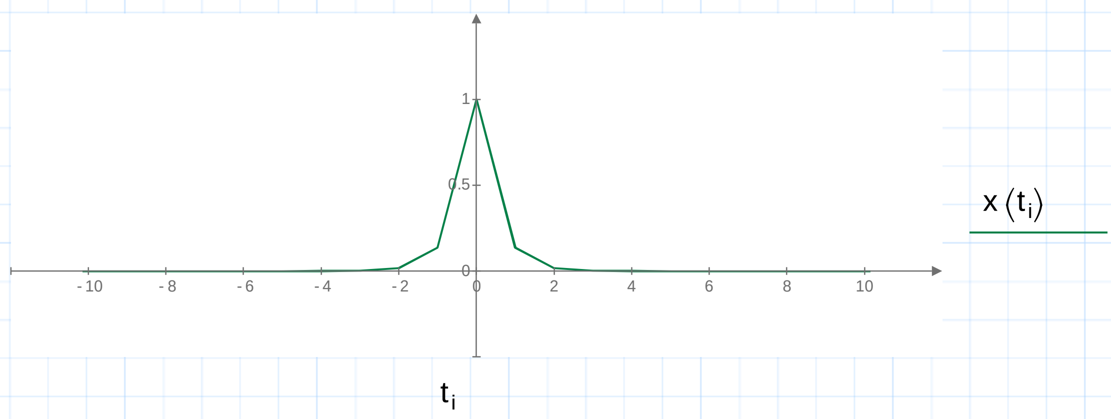
$i := -50..50$

$t_i := i$ setup the variable t for time indexing - note this technique/procedure

$a := 2$

$x(t_i) := e^{-|a \cdot t_i|}$

Plot x(t)



Next the Fourier transform:

$j := \sqrt{-1}$ clear (X)

$$X(\omega) := \int_{-3}^{3} x(t) \cdot e^{-j \cdot \omega \cdot t}\, dt$$

$$X1(\omega) := \int_{-3}^{0} x(t) \cdot e^{-j \cdot \omega \cdot t}\, dt \qquad X2(\omega) := \int_{0}^{3} x(t) \cdot e^{-j \cdot \omega \cdot t}\, dt$$

$$X1(\omega) := \int_{-3}^{0} x(t) \cdot e^{-j\cdot\omega\cdot t}\,dt \rightarrow \int_{-3}^{0} e^{-2\cdot|t|} \cdot e^{-(t\cdot\omega\cdot 1i)}\,dt$$

$$X2(\omega) := \int_{0}^{3} x(t) \cdot e^{-j\cdot\omega\cdot t}\,dt \rightarrow \begin{cases} \text{if } \omega \neq -2i \wedge \omega \neq 2i & \dfrac{\left(e^{-6-3i\cdot\omega}-1\right)\cdot(-2+\omega\cdot 1i)}{\omega^2+4} \\ \text{else if } \omega = -2i \vee \omega = 2i & \displaystyle\int_0^3 e^{-(2\cdot t)-t\cdot\omega\cdot 1i}\,dt \end{cases}$$

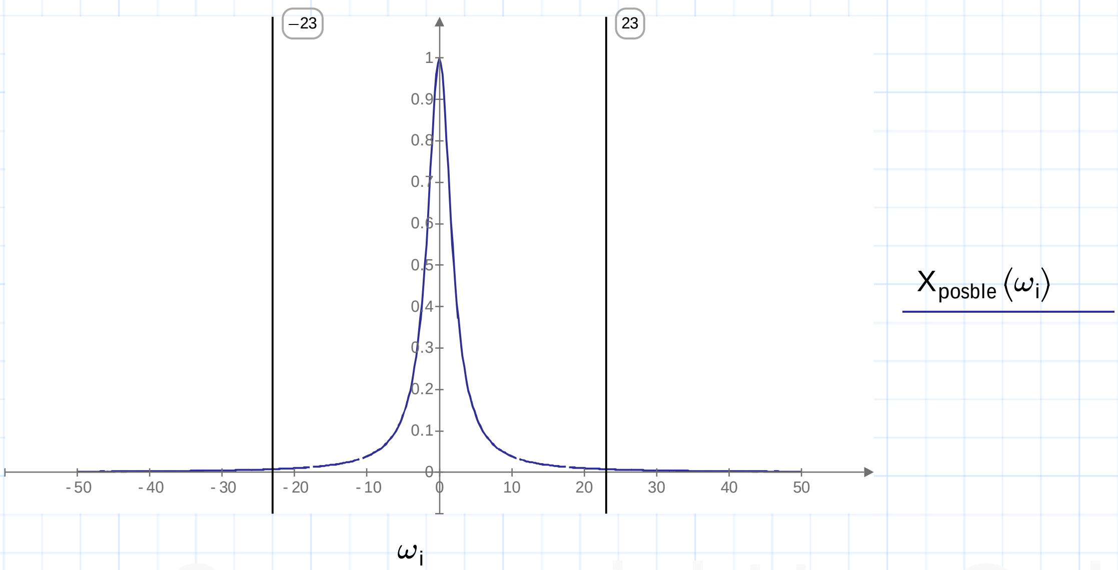
The above evaluation from Prime, the equation below was attempted. Complex integer evaluations are not easy to do manually and when a software is applied we need to have a manual calculation done to check its answer or have some expected result from past exercises.

$$X_{posble}(\omega_i) := \left(-\frac{\left(e^{-6}\cdot e^{3j\cdot\omega_i}-1\right)\cdot\left(2+\omega_i\cdot 1\cdot j\right)}{\omega_i^2+4}\right) + \left(\frac{\left(e^{-6-3i\cdot\omega_i}-1\right)\cdot\left(-2+\omega_i\cdot 1\,j\right)}{\omega_i^2+4}\right)$$

Set the value of w_i for i = -50 to 50

$$\omega_i := \left(\frac{\pi}{7}\right)\cdot(i)$$ <---- although it shows as an error it is correct, because its used as an index and not a scalar or matrix

Now plot the Fourier transform:



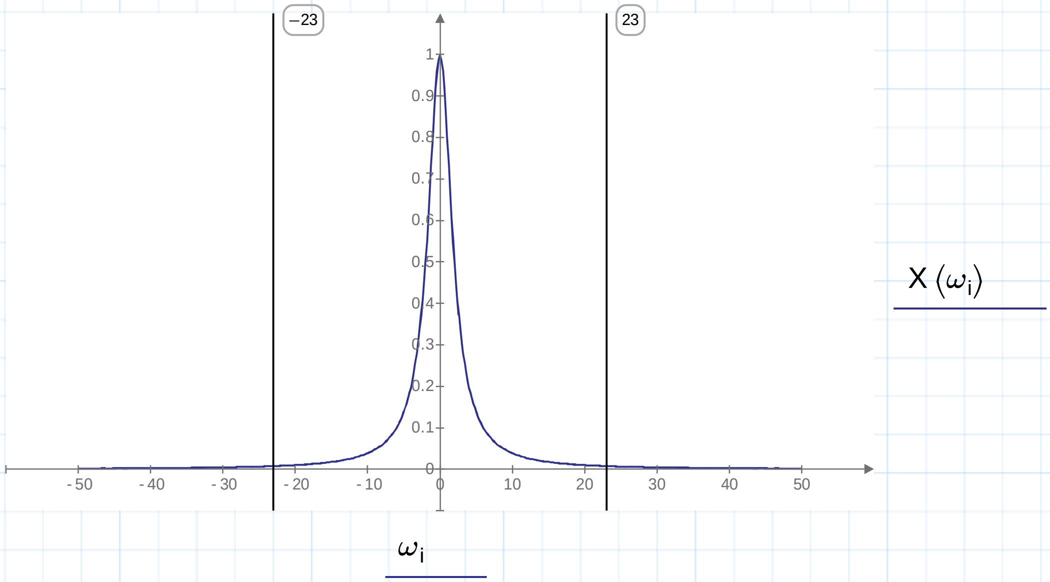
The authors of the textbook (tutorial) suggest the following equation - "For now, lets use the Fourier transform formula, as shown below.

$X1(\omega_i) := \left(\frac{-1}{2 + j \cdot \omega_i}\right) \cdot e^{(-6 - 3i \cdot \omega_i)}$ $X2(\omega_i) := \left(\frac{1}{-2 + j \cdot \omega_i}\right) \cdot e^{(-6 + 3i \cdot \omega_i)}$ $X3(\omega_i) := \left(\frac{1}{-2 + j \cdot \omega_i}\right)$

$X4(\omega_i) := \left(\frac{1}{2 + j \cdot \omega_i}\right)$

$X(\omega_i) := X1(\omega_i) + X2(\omega_i) - X3(\omega_i) + X4(\omega_i)$

The signal to be samples below shows the same shape and amplitude.
So we will proceed with the signal X(w_i).



Textbook set the value of w_m to 22.44 though indicating 23 is more accurate

$\omega_m := 22.44$ -23 to +23 is the bandwidth of the signal, before it touches the vertical axis = 0

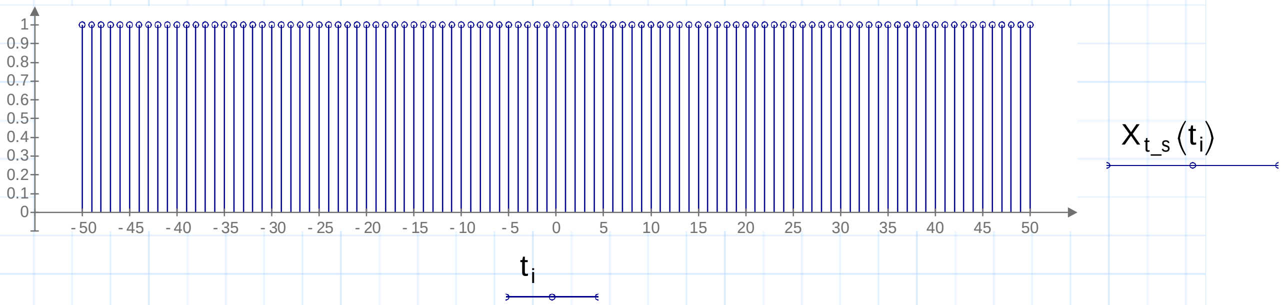
The signal is in the frequency domain after the fourier transform
now this signal can be multiplied to the sampler.

Next the sampler signal has to be formed:

$\delta(t) := \text{if}(t = 0, 1, 0)$ delta impulse function

$T_s := 1$ sampling interval

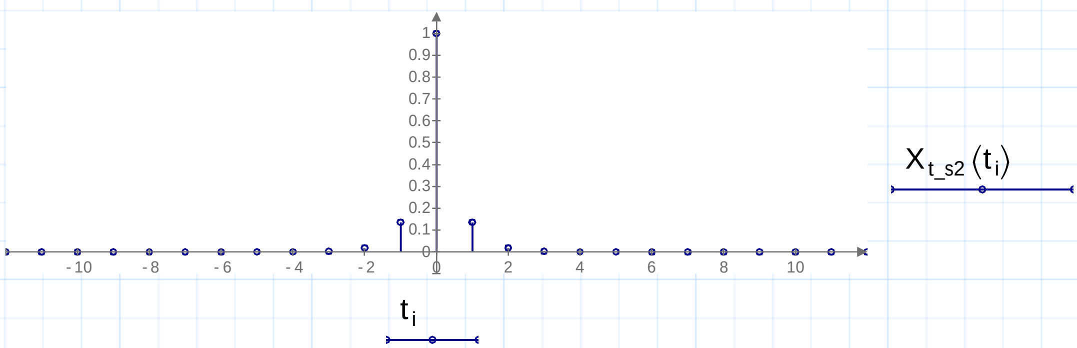
$X_{t\_s}(t_i) := \sum_{n=-50}^{50} \delta(t_i - n \cdot T_s)$ the sampler



Sampler above is in the time domain.

The sampler can be formed as follows:

$X_{t\_s2}(t_i) := \sum_{n=-50}^{50} x(n \cdot T_s) \cdot \delta(t_i - n \cdot T_s)$ understand this sampler with multiplier x(nTs)



The above sampler X_t_s2(ti) has the shape similar to the Fourier transform signal in frequency domain.

Now the Fourier transform of the first sampler (X_t_s(ti)) in time domain has to be found:
We use the first sampler (delta impulse train) which is in the frequency domain.

Sampler in frequency domain:

Fourier Transform Property:

Fourier transform (freq domain) of an impulse train (X_t_s(ti))
results with an identical impulse train but its amplitude is 2pi/T

$Amp := \frac{(2 \cdot \pi)}{T_s} = 6.28$

clear (ω_i)

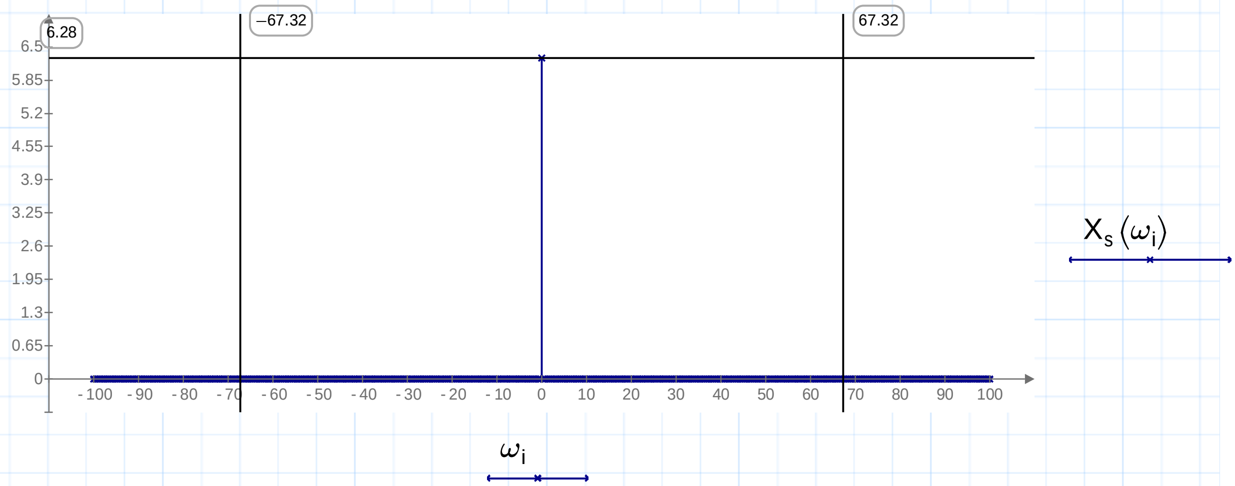
$\omega_i := i \cdot \omega_m$ redefine the indexing for the frequency domain

$\omega_s := 3 \cdot \omega_m$ set the value of the sampling frequency to 3 times the signal bandwidth - here the bandwidth is (0 - +wm) x 3, this is larger than -wm to wm or larger than 2wm

$T_s = 1$

$$X_s(\omega_i) := \left(\frac{2 \cdot \pi}{T_s}\right) \cdot \sum_{n=-50}^{50} \delta(\omega_i - n \cdot \omega_s)$$

$\omega_{s\_pos} := 3 \cdot \omega_m = 67.32$ $\omega_{s\_neg} := -3 \cdot \omega_m = -67.32$

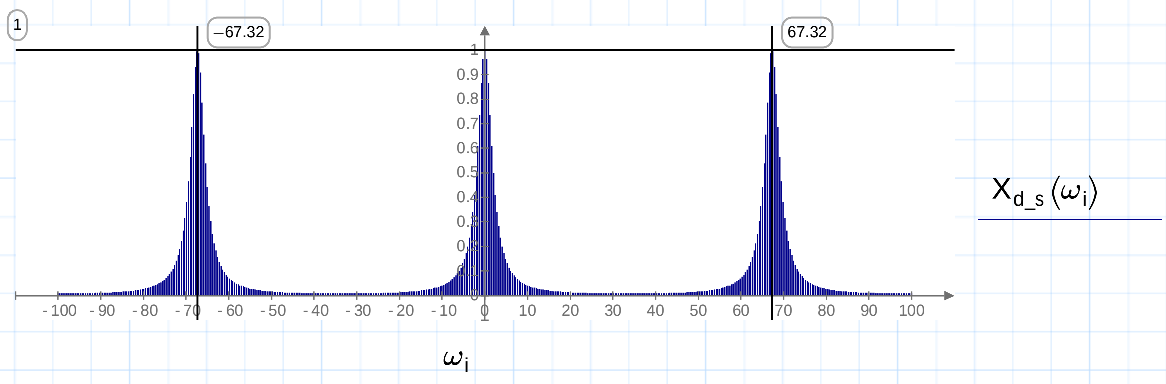


Fourier transform sampler in frequency domain above.
Above plot matches textbook plot - with sampling frequency and amplitude shown on plot

Finally convolve the signal in the frequency domain X(w_i) to the sampler in the frequency domain Xs(w_i).

Convolution with a delta function gives the function itself, shifted to the direction of the delta function.

$$X_{d\_s}(\omega_i) := \left(\frac{1}{T_s}\right) \cdot \sum_{n=-50}^{50} X(\omega_i - n \cdot \omega_s)$$

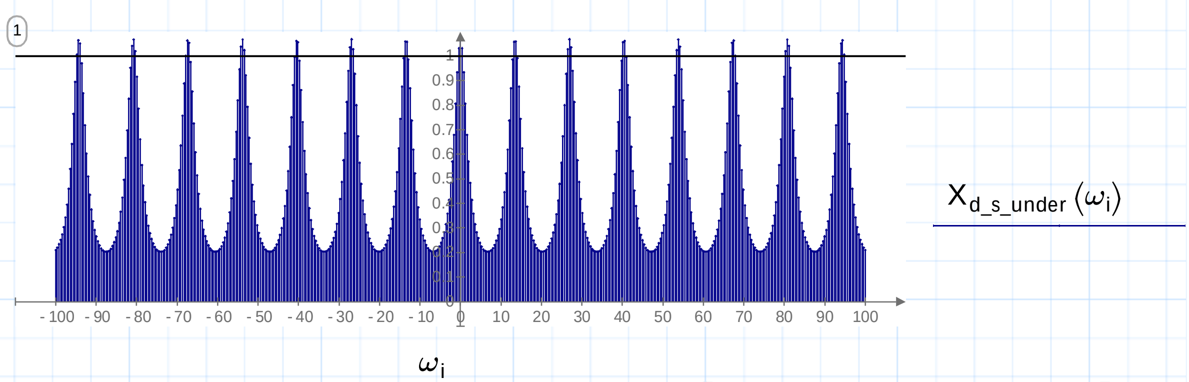


The plot above matches the textbook plot. Only difference is it is plotted in line trace plot and more triangular, however the plot data matches. w_s > 2w_m.

Lets see undersampling (Aliasing) in the frequency domain:
Case: w_s < 2 w_m

$\omega_s := \left(\frac{6}{10}\right) \cdot \omega_m$ here we set w_s much less then w_m

$$X_{d\_s\_under}(\omega_i) := \left(\frac{1}{T_s}\right) \cdot \sum_{n=-50}^{50} X(\omega_i - n \cdot \omega_s)$$

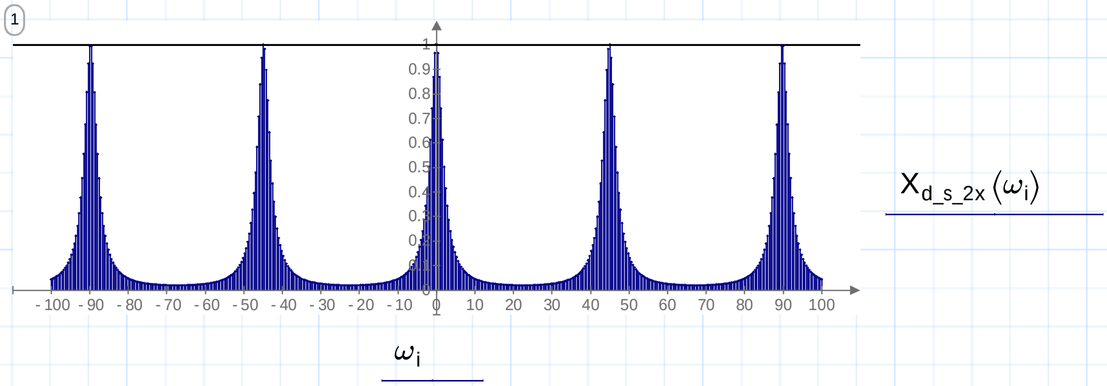


The signal above overlaps and does not replicate or reconstruct the original signal. Though under sampled, the output shows as if its more content in the plot as if correct! This is called signal overlap.

Case: w_s = 2 w_m

$\omega_s := 2 \cdot \omega_m$

$$X_{d\_s\_2x}(\omega_i) := \left(\frac{1}{T_s}\right) \cdot \sum_{n=-50}^{50} X(\omega_i - n \cdot \omega_s)$$



Improvement compared to the previous plot yet incorrect. Increasing the sampling frequency lowers the sampling period. With a lower sampling period the signal is better captured.

To better see the overlaps in the case when w_m < 2 wm, we plot each individual spike.

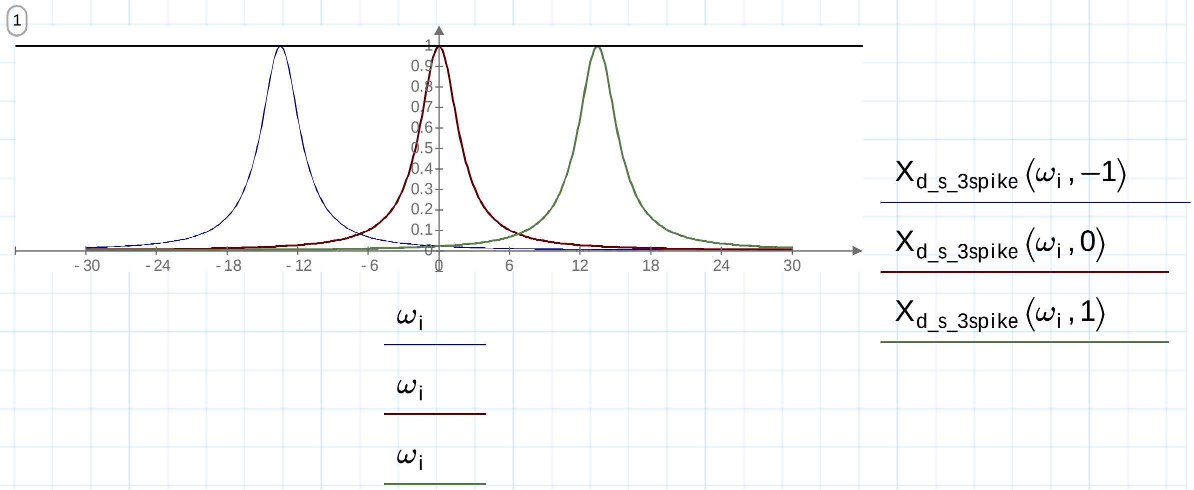
$n := -1, 0..1$ here we intend to plot 3 spikes

$\omega_s := \left(\frac{6}{10}\right) \cdot \omega_m$ make the sample frequency lower than original signal frequency

$\omega_i := i \cdot \left(\frac{\pi}{4}\right)$ reindexing w - this is important to zoom in the 3 spikes

$X_{d\_s\_3spike}(\omega_i, n) := \left(\frac{1}{T_s}\right) \cdot X(\omega_i - n \cdot \omega_s)$ Note: Summation term not included

See plot on next page



In the line trace plot above we can see the overlapping signals. Take note on how this was created in the lines above in the plot and calculation/coding lines.

Lets use Prime 2 to shade the areas the signals are overlapping.

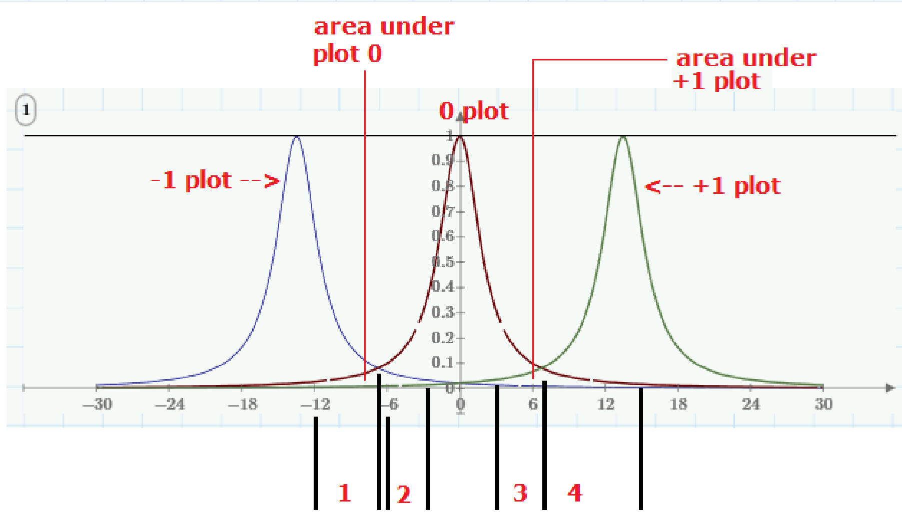
Go to the plot above and see roughly where the demarcations are for wi for each half of the area in the overlaps

$\text{Area1}(\omega_i) := \text{if}(-15 \le \omega_i \le -6.3, X_{d\_s\_3spike}(\omega_i, 0), 0)$ 1st area - defining the range

$\text{Area2}(\omega_i) := \text{if}(-6.3 \le \omega_i \le 0, X_{d\_s\_3spike}(\omega_i, -1), 0)$ 2nd area

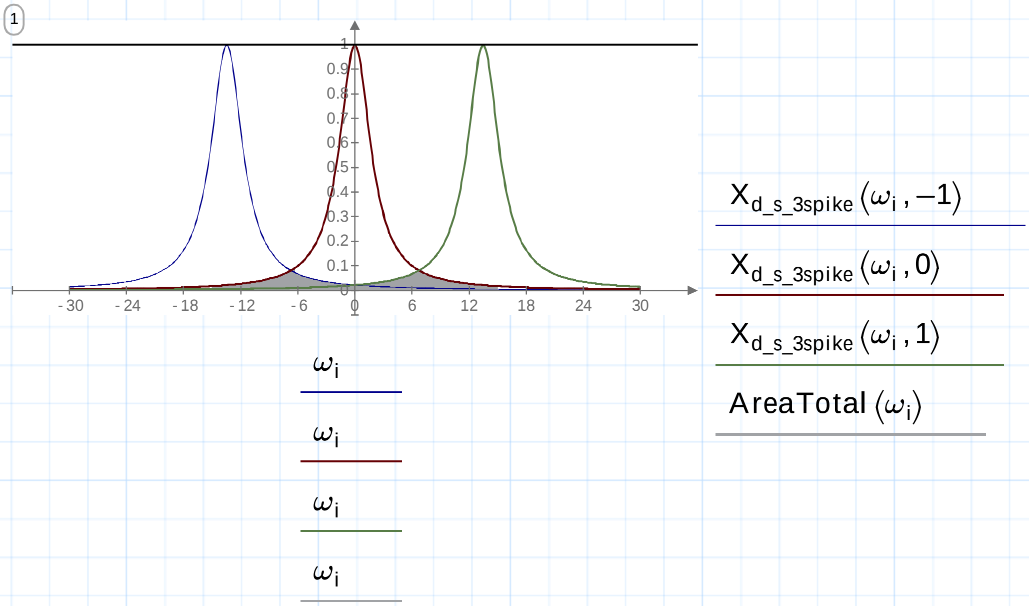
$\text{Area3}(\omega_i) := \text{if}(0 \le \omega_i \le 7, X_{d\_s\_3spike}(\omega_i, 1), 0)$ 3rd area

$\text{Area4}(\omega_i) := \text{if}(7 \le \omega_i \le 15, X_{d\_s\_3spike}(\omega_i, 0), 0)$ 4th area



<-- Areas shown in figure

$\text{AreaTotal}(\omega_i) := \text{Area1}(\omega_i) + \text{Area2}(\omega_i) + \text{Area3}(\omega_i) + \text{Area4}(\omega_i)$



Shaded overlapping areas shown in plot above.
A little tedious and requires careful observation on the values of w_i.

So far we had sampled the signal, got to shading the overlapped signal areas, its an achievement.
Is is the signal we had obtained thru convolution (for frequency domain) the original signal with respect to its shape and characteristics?
Maybe not but further reconstruction is possible by applying a filter to the process thus far, and this is performed in practical or real applications.

Next we apply an ideal low-pass filter to X_d_s(w_i) with an amplitude of T.

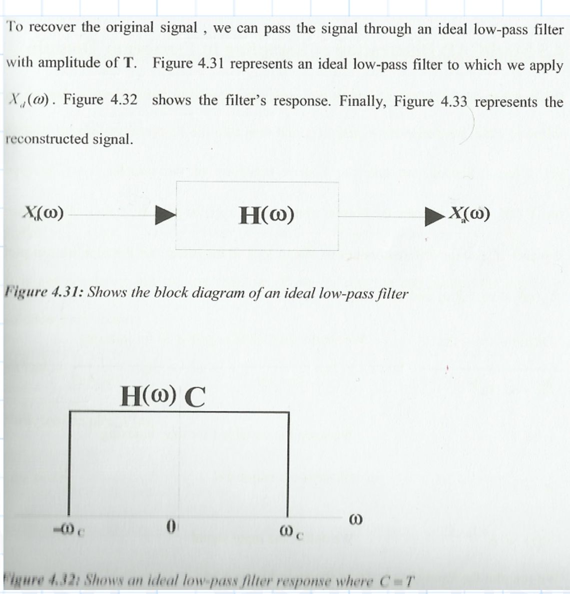

To recover the original signal , we can pass the signal through an ideal low-pass filter with amplitude of T. Figure 4.31 represents an ideal low-pass filter to which we apply $X_d(\omega)$. Figure 4.32 shows the filter's response. Finally, Figure 4.33 represents the reconstructed signal.

Figure 4.31: Shows the block diagram of an ideal low-pass filter

Figure 4.32: Shows an ideal low-pass filter response where C = T

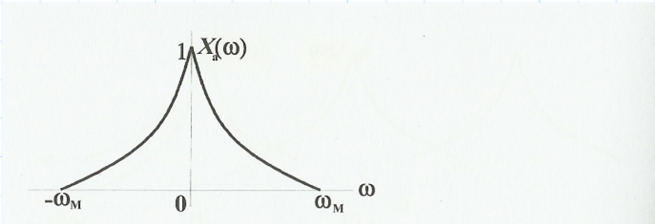


Figure 4.33: Shows the original signal after reconstruction

Now we define the filter function.

$w_i := i \cdot \omega_m$

$\omega_s := 3 \cdot \omega_m$ set the sampling 3 times the signal bandwidth

$X_{d\_s}(\omega_i) := \left(\frac{1}{T_s}\right) \cdot \sum_{n=-50}^{50} X(\omega_i - n \cdot \omega_s)$ the equation we used for the sampled signal

Set the cutoff frequency w_c:

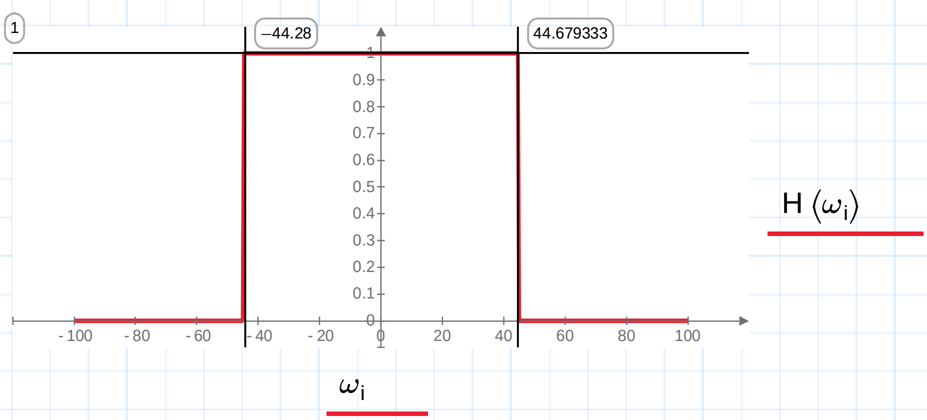
$\omega_s = 67.32$ $\omega_m = 22.44$

$\omega_c := \omega_s - \omega_m$ defining the cutoff frequency for the filter

$\omega_c = 44.88$

$H(\omega_i) := \text{if}(-\omega_c < \omega_i < \omega_c, T_s, 0)$ defining the low-pass filter

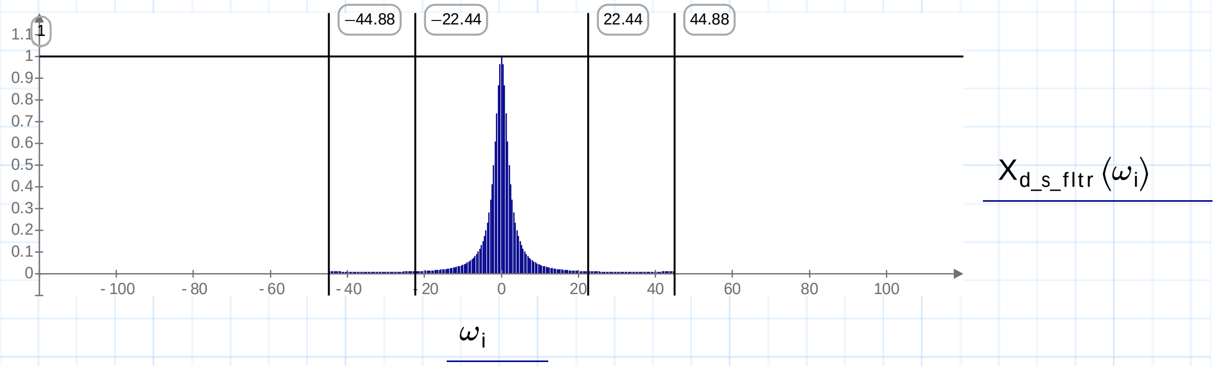
Plot the low-pass filter and add in the markers for cutoff frequency and amplitude of the low-pass filter signal



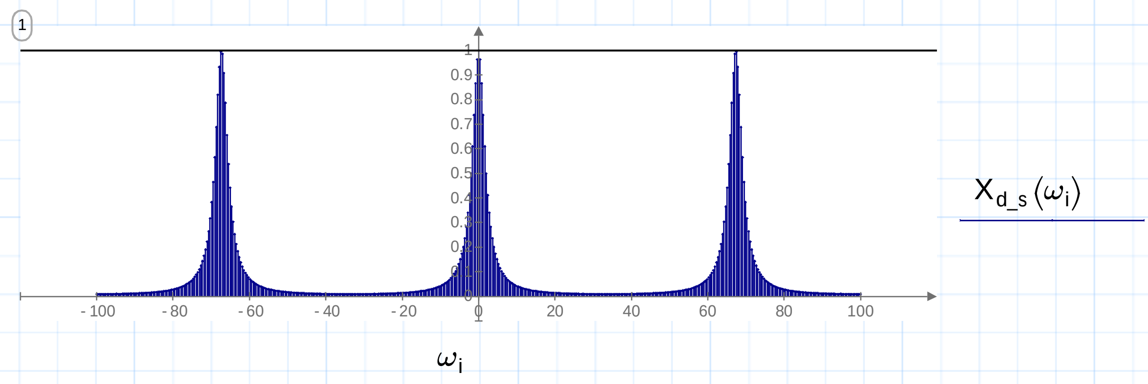
So frequencies within -w_c and wc will pass thru while the rest blocked. This will help reconstruct the sampled signal closer to the original.

Next we apply the low-pass filter to the sampled signal, by multipling both of them.

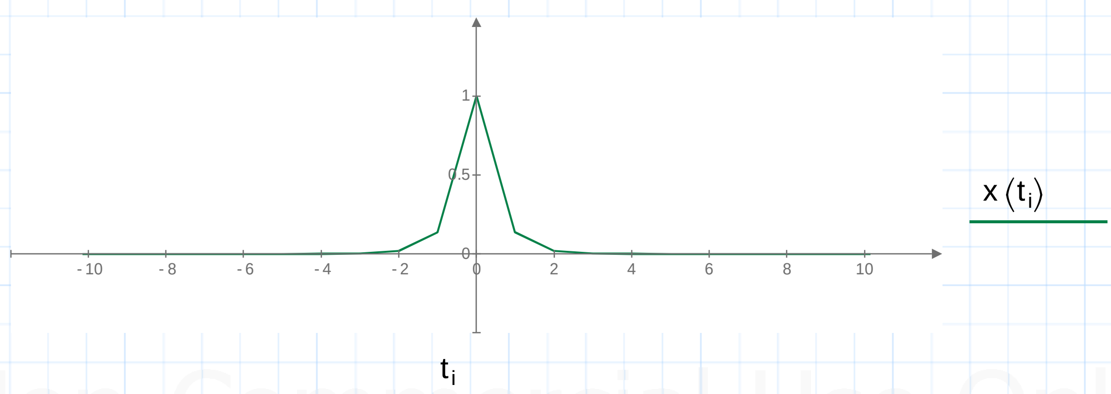
$X_{d\_s\_fltr}(\omega_i) := X_{d\_s}(\omega_i) \cdot H(\omega_i)$



Plot above is cleaner and closer to the original compared to the first sampled signal below which was overlapped.

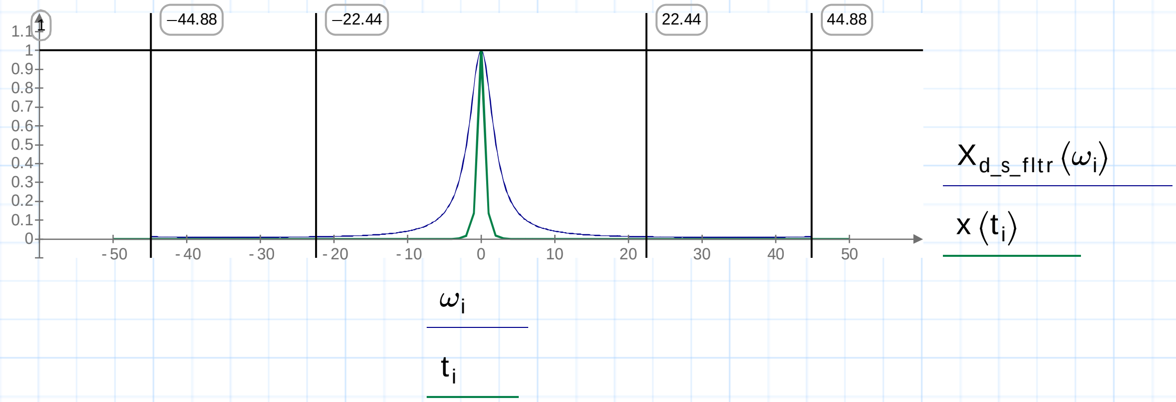


Below is the original signal in time domain which was to be processed - without any operation on it.



Plot below with the orignal without any operation in the time domain, and sampled-filtered signal in the frequency domain.

It shows a close shape while not exactly duplicate, with the same amplitude. The visible difference being in the signal widths but this is due to one is in the time domain and the other in the frequency domain.



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