Ex.7 Convolution (Response of electric circuit, with rectangular input; 0.5Hz,1 [V])
R=1Ω,L=1 H (Series connection)

$R := 1$ $\quad L := 1$ $\quad Z(s) := R + s \cdot L$ $\quad h(t) := e^{-t}$ impulse response

$h(t) := e^{-t} \xrightarrow{laplace} \frac{1}{s+1}$ $\quad H(s) := \frac{1}{s+1}$ $\quad T := 2$ $\quad h(t) := e^{-t}$

$$e(t) = \Phi(t) - 2 \cdot \Phi\left(t - \frac{T}{2}\right) + 2 \cdot \Phi(t - T) - 2 \cdot \Phi\left(t - \frac{3 \cdot T}{2}\right) + 2 \cdot \Phi(t - 2 \cdot T) \ldots$$

$$e(t) := \Phi(t) - \sum_{n=1}^{m} \left(2 \cdot \Phi\left(t - \frac{(2 \cdot n - 1) \cdot T}{2}\right) - 2 \cdot \Phi(t - n \cdot T)\right) - \Phi(t - m \cdot T)$$

$m := 1$

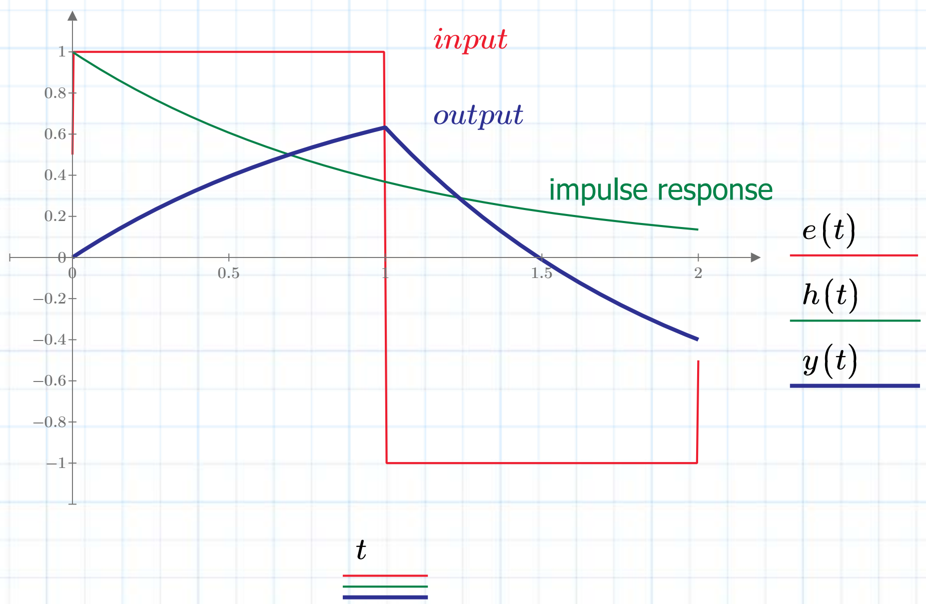
$$e(t) := \Phi(t) - \sum_{n=1}^{m} \left(2 \cdot \Phi\left(t - \frac{(2 \cdot n - 1) \cdot T}{2}\right) - 2 \cdot \Phi(t - n \cdot T)\right) - \Phi(t - m \cdot T) \xrightarrow{simplify} \Phi(t) - 2 \cdot \Phi(t - 1$$

$$e(t) := \Phi(t) - 2 \cdot \Phi\left(t - \frac{T}{2}\right) + \Phi(t - T)$$ *input*

$$E(s) := \frac{1}{s} - \frac{2 \cdot e^{-1 \cdot s}}{s} + \frac{e^{-2 \cdot s}}{s}$$

output

$$y(t) := E(s) \cdot H(s) \xrightarrow{invlaplace} \Phi(t-2) - 2 \cdot \Phi(t-1) - e^{-t} + 2 \cdot e^{1-t} \cdot \Phi(t-1) - e^{2-t} \cdot \Phi(t-2) + 1$$



[redacted]

$m := 2$

$$e(t) := \Phi(t) - \sum_{n=1}^{m} \left(2 \cdot \Phi\left(t - \frac{(2 \cdot n - 1) \cdot T}{2}\right) - 2 \cdot \Phi(t - n \cdot T)\right) - \Phi(t - m \cdot T) \xrightarrow{simplify} \Phi(t) - 2 \cdot \Phi(t - 1$$

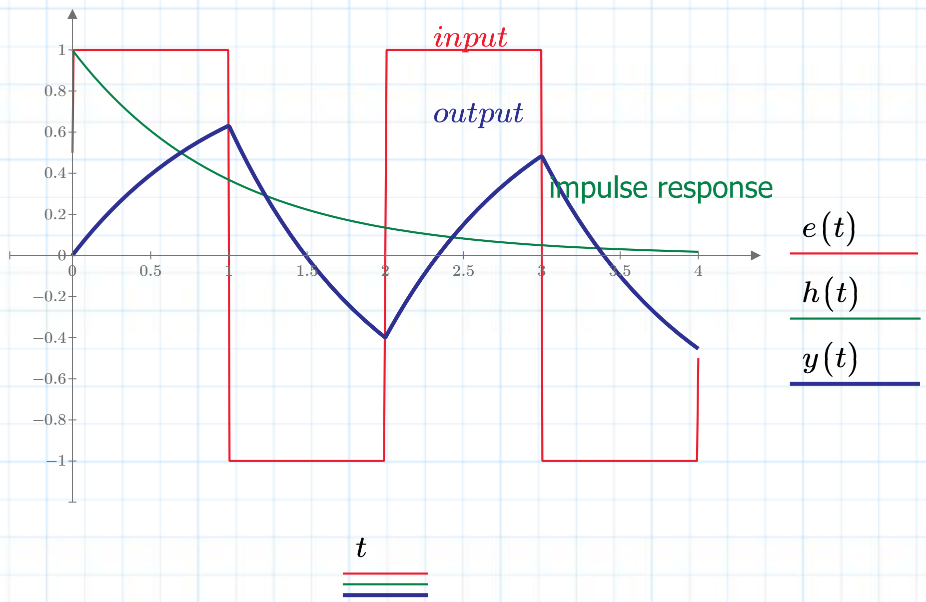
$$e(t) := \Phi(t) - 2 \cdot \Phi(t-1) + 2 \cdot \Phi(t-2) - 2 \cdot \Phi(t-3) + \Phi(t-4)$$ *input*

$$E(s) := \frac{1}{s} - \frac{2 \cdot e^{-1 \cdot s}}{s} + \frac{2 \cdot e^{-2 \cdot s}}{s} - \frac{2 \cdot e^{-3 \cdot s}}{s} + \frac{1 \cdot e^{-4 \cdot s}}{s}$$

output

$$y(t) := E(s) \cdot H(s) \xrightarrow{invlaplace} 2 \cdot \Phi(t-2) - 2 \cdot \Phi(t-1) - e^{-t} - 2 \cdot \Phi(t-3) + \Phi(t-4) + 2 \cdot e^{1-t} \cdot \Phi(t-1) - 2 \cdot e^{2-t} \cdot \Phi(t-2) + 2$$

$$y(t) := 2 \cdot \Phi(t-2) - 2 \cdot \Phi(t-1) - e^{-t} - 2 \cdot \Phi(t-3) + \Phi(t-4) + 2 \cdot e^{1-t} \cdot \Phi(t-1) - 2 \cdot e^{2-t} \cdot \Phi(t-2) + 2 \cdot e^{3-t} \cdot \Phi(t-3) - e^{4-t} \cdot \Phi(t-4) + 1$$



[redacted]

$m := 3$

$e(t) := \Phi(t) - \sum_{n=1}^{m} \left(2 \cdot \Phi\left(t - \frac{(2 \cdot n - 1) \cdot T}{2} \right) - 2 \cdot \Phi(t - n \cdot T) \right) - \Phi(t - m \cdot T) \xrightarrow{simplify} \Phi(t) - 2 \cdot \Phi(t-1) + 2 \cdot$

$e(t) := \Phi(t) - 2 \cdot \Phi(t-1) + 2 \cdot \Phi(t-2) - 2 \cdot \Phi(t-3) + 2 \cdot \Phi(t-4) - 2 \cdot \Phi(t-5) + \Phi(t-6)$

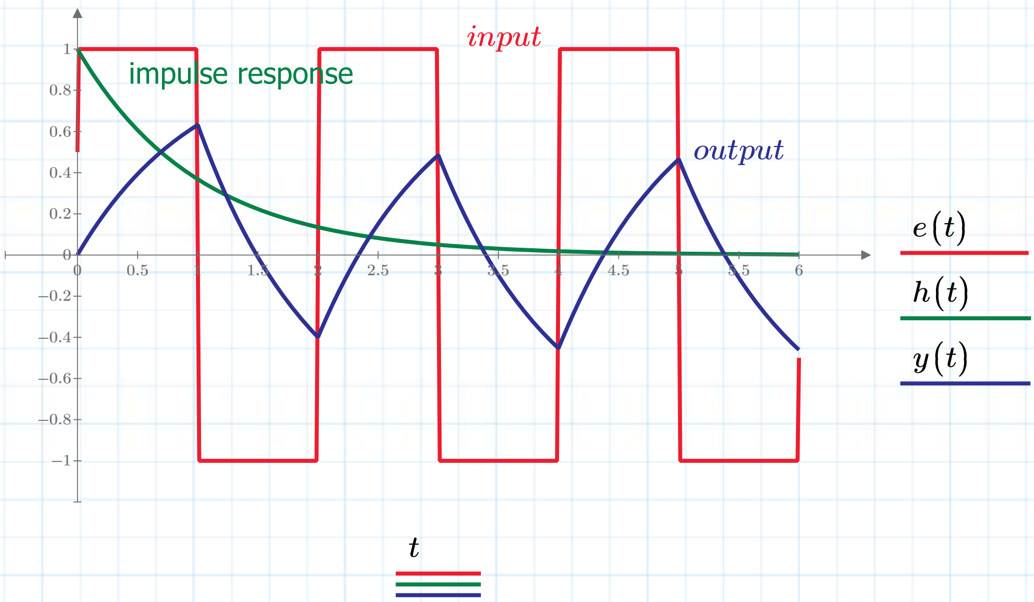
$E(s) := \frac{\left(e^{-s} - 1\right)^2 \cdot \left(e^{-2 \cdot s} + e^{-4 \cdot s} + 1\right)}{s}$

input

output

$y(t) := E(s) \cdot H(s) \xrightarrow{invlaplace} \Phi(t-1) \cdot \left(2 \cdot e^{1-t} - 2\right) - e^{-t} - \Phi(t-2) \cdot \left(2 \cdot e^{2-t} - 2\right) + \Phi(t-3) \cdot \left(2 \cdot e^{3-t} - 2\right) - \Phi(t-4) \cdot \left(2 \cdot e^{4-}\right.$

$y(t) := \Phi(t-1) \cdot \left(2 \cdot e^{1-t} - 2\right) - e^{-t} - \Phi(t-2) \cdot \left(2 \cdot e^{2-t} - 2\right) + \Phi(t-3) \cdot \left(2 \cdot e^{3-t} - 2\right) \hookleftarrow$
$- \Phi(t-4) \cdot \left(2 \cdot e^{4-t} - 2\right) + \Phi(t-5) \cdot \left(2 \cdot e^{5-t} - 2\right) - \Phi(t-6) \cdot \left(e^{6-t} - 1\right) + 1$



$y(5) = 0.465$ $\quad$ $y(6) = -0.461$

$(1 + A) \cdot e^{-1} - 1 = -A \xrightarrow{solve, A} -\frac{e^{-1} - 1}{e^{-1} + 1} = 0.462$

$1.462 \cdot e^{-1} - 1 = -0.462$

$e^{-t} = 0.001 \xrightarrow{solve, t} 6.9077552789821370521$

[redacted]