Pneumatic Conveying System with Mathcad Prime 2.0

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A qusetion was raised if the capacity of an existing pneumatic conveying system, conveying High Density Polyethylene (HDPE) particles, can be increased from 20000 lbm/hr to 30000lbm/hr ?. The system data are given and summary results of calculations are given at the end of the worksheet. The system, simply composed of the following pipe components:

Blower(1375 SCFM) and feeding+horizontal pipe (100ft)+bend+vertical pipe (50ft)+bend +horizontal pipe (325 ft)+cyclone to the atmosphere. All bends with R/D>6.
The points for calculations of the system are, starting from cyclone atmospheric point (a), given by:

Atmospheric point: a
Horizontal pipe points: b-c
bend points: c-d
Vertical pipe points: d-e
Bend points: e-f
Horizontal pipe points: f-g
Blower/Feeding points: g

Note: Units in this worksheet are used everywhere which simplifies the conversions.

"———————————————————System Data———————————————————"

$Capacity := 30000 \ \frac{lb}{hr}$ "Increment required for the plant capacity"

$Q\_Blower := 1375 \ cfm$ "Existing blower flow rate"

$\Delta p\_Blower := 0.3 \ psi$ "Pressure drop in the blower"

$T_g := 68 \ °F = 527.67 \ R$ "Gas or air (used) temp at standard conditions"

$dp := 4 \ mm$ "Particle diameter"

$k := 0.00015\ ft$ "Pipe roughness"

$\Delta p\_cyclone := 0.368\ in\_Hg = 0.1807\ psi$ "Pressure drop in the cyclone"

$D := 0.5\ ft$ "Pipe diameter ft"

$V_T := 30.6\ \frac{ft}{s}$ "Terminal velocity at the feeding point"

$p_a := 14.7\ psi$ "Exit pressure boundary conditions"

$gas\_constant := \frac{10.73\ psi \cdot ft^3}{lb \cdot mol \cdot R}$ "Gas used is air"

$\rho_p := 59\ \frac{lb}{ft^3}$ "Particle desnsity"

$M := 29\ \frac{lb}{lb \cdot mol}$ "Molecular weight of air"

$g := 32.2\ \frac{ft}{s^2}$ "Gravity acceleration"

$g_c := \frac{32.174\ ft \cdot lb}{lbf \cdot s^2}$ "Constant"

$\mu_g := 0.0000114\ \frac{lb}{ft \cdot s}$ "Gas (air) viscosity at 68 Degree F"

$L_{bc} := 325\ ft$ "Horizontal pipe length"

$L_{de} := 50\ ft$ "Vertical rise pipe length"

$L_{fg} := 100\ ft$ "Inlet horizontal pipe length"

"———————————————————Calculations———————————————————"

"Ponit a"

$$\rho_{gSTP} := \frac{p_a \cdot M}{gas\_constant \cdot T_g} = 0.0753\ \frac{lb}{ft^3}$$ "Gas density from ideal gas law"

"Point b"

$$P_b := p_a + \Delta p\_cyclone = 14.8807\ psi$$

$$A := \frac{\pi \cdot D^2}{4} = 0.1963\ ft^2$$ "pipe area"

$$V_{gb} := \frac{Q\_Blower \cdot p_a}{A \cdot P_b} = 115.296\ \frac{ft}{s}$$ "Gas velocity at point b"

$$\rho_{gb} := \rho_{gSTP} \cdot \frac{P_b}{p_a} = 0.0762\ \frac{lb}{ft^3}$$ "Gas (air)density at point b"

$$Re := \frac{\rho_{gb} \cdot V_{gb} \cdot D}{\mu_g} = 3.8543 \cdot 10^5$$ "Reynolds number"

"Gas friction factor using Churchill's equation"

$$b := \left(\frac{37530}{Re}\right)^{16} = 6.5316 \cdot 10^{-17}$$

$$a := \left(2.457 \cdot \ln\left(\frac{1}{\left(\frac{7}{Re}\right)^{0.9} + 0.27 \cdot \frac{k}{D}}\right)\right)^{16} = 2.7804 \cdot 10^{21}$$

$$f := 2 \cdot \left(\left(\frac{8}{Re}\right)^{12} + \frac{1}{(a+b)^{\frac{3}{2}}}\right)^{\frac{1}{12}} = 0.0042$$

$$\mu := \frac{Capacity}{\rho_{gSTP} \cdot Q\_Blower} = 4.8296$$ "Solids mass– to– air ratio"

$$Fr := \frac{{V_{gb}}^2}{} = 825.6626$$

$$g \cdot D$$

$$Frp := \frac{V_T^{\,2}}{g \cdot dp} = 2.2159 \cdot 10^3$$

$dp = 0.0131\ ft$ "dp in ft now"

$$\lambda_z := 0.082 \cdot \mu^{-0.3} \cdot Fr^{-0.86} \cdot Frp^{0.25} \cdot \left(\frac{D}{dp}\right)^{0.1} = 0.0016$$ "dp must be in ft"

$$\Delta P_{bc} := \left(4 \cdot f + \lambda_z \cdot \mu\right) \cdot \frac{L_{bc}}{D} \cdot \frac{\rho_{gb} \cdot V_{gb}^{\,2}}{2 \cdot g_c} \cdot \frac{ft^2}{144\ in^2} = 1.7239\ psi$$

"Point c"

$P_c := P_b + \Delta P_{bc} = 16.6047\ psi$ "Pressure at point c"

$V_{gc} := \frac{Q\_Blower}{A} \cdot \frac{p_a}{P_c} = 103.3258\ \frac{ft}{s}$ "Gas velocity at point c"

$\rho_{gc} := \rho_{gSTP} \cdot \frac{P_c}{p_a} = 0.085\ \frac{lb}{ft^3}$ "Gas density at point c"

"Pipe bend cd"

$B := 0.5$ "For R/D>6"

$\Delta P_{bend} := B \cdot (1 + \mu) \cdot \frac{\rho_{gc} \cdot V_{gc}^{\,2}}{2 \cdot g_c} = 0.2856\ psi$ "Pressure drop in bend cd"

"point d"

$P_d := P_c + \Delta P_{bend} = 16.8903\ psi$ "Pressure at point d"

$V_{gd} := \frac{Q\_Blower \cdot p_a}{A \cdot P_d} = 101.5785\ \frac{ft}{s}$ "Gas velocity at point d"

$\rho_{gd} := \rho_{gSTP} \cdot \frac{P_d}{p_a} = 0.0865\ \frac{lb}{ft^3}$ "Gas density at point d"

"vertical pipe de"

$$Fr := \frac{V_{gd}^{\,2}}{} = 640.881$$

$g \cdot D$

$$\lambda_z := 0.082 \cdot \mu^{-0.3} \cdot Fr^{-0.86} \cdot Frp^{0.25} \cdot \left(\frac{D}{dp}\right)^{0.1} = 0.0019$$

$y := 1 - 0.123 \cdot 0.0131^{0.3} \cdot 59^{0.5} = 0.7427$ "dp=0.0132ft, ρ−particles=59lbm/ft^3"

$$V_p := V_{gd} \cdot y = 75.4379 \ \frac{ft}{s}$$

$$\varepsilon := 1 - \frac{Capacity}{A \cdot \rho_p \cdot V_p} = 0.9905$$

$$\rho_0 := \varepsilon \cdot \rho_{gd} + (1 - \varepsilon) \cdot \rho_p = 0.6483 \ \frac{lb}{ft^3}$$

"Vertical pressure drop de"

$$\Delta P_{vert} := (4 \cdot f + \lambda_z \cdot \mu) \cdot \frac{L_{de}}{D} \cdot \frac{\rho_{gd} \cdot V_{gd}^{\ 2}}{2 \cdot g_c} + \rho_0 \cdot L_{de} \cdot \frac{g}{g_c} = 0.4767 \ psi$$

"point e"

$P_e := \Delta P_{vert} + P_d = 17.367 \ psi$ "Pressure at point e"

$V_{ge} := \dfrac{Q\_Blower \cdot p_a}{A \cdot P_e} = 98.7904 \ \dfrac{ft}{s}$ "Gas velocity at point e"

$\rho_{ge} := \rho_{gSTP} \cdot \dfrac{P_e}{p_a} = 0.089 \ \dfrac{lb}{ft^3}$ "Gas density at point e"

"pipe bend ef"

$\Delta P_{bend} := B \cdot (1 + \mu) \cdot \dfrac{\rho_{ge} \cdot V_{ge}^{\ 2}}{2 \cdot g_c} = 0.2731 \ psi$ "Pressure drop in bend ef"

"point f"

$P_f := P_e + \Delta P_{bend} = 17.6401 \ psi$ "Pressure at point f"

$$V_{gf} := \frac{Q\_Blower \cdot p_a}{A \cdot P_f} = 97.261 \ \frac{ft}{s}$$ "gas velocity at point f"

$$\rho_{gf} := \rho_{gSTP} \cdot \frac{P_f}{p_a} = 0.0904 \ \frac{lb}{ft^3}$$ "gas density at point f"

"horizontal pipe fg"

$$Fr := \frac{{V_{gf}}^2}{g \cdot D} = 587.5594$$

$$\lambda_z := 0.082 \cdot \mu^{-0.3} \cdot Fr^{-0.86} \cdot Frp^{0.25} \cdot \left(\frac{D}{dp}\right)^{0.1} = 0.0021$$

"horizontal pressure drop fg"

$$\Delta P_{fg} := \left(4 \cdot f + \lambda_z \cdot \mu\right) \cdot \frac{L_{fg}}{D} \cdot \frac{\rho_{gf} \cdot {V_{gf}}^2}{2 \cdot g_c} = 0.4949 \ psi$$

$$P_g := P_f + \Delta P_{fg} = 18.1349 \ psi$$

$$V_{gg} := \frac{Q\_Blower \cdot p_a}{A \cdot P_g} = 94.6069 \ \frac{ft}{s}$$

$$\rho_{gg} := \rho_{gSTP} \cdot \frac{P_g}{p_a} = 0.0929 \ \frac{lb}{ft^3}$$

"The additional acceleration"

$$\Delta P_{accel} := \frac{\rho_{gg} \cdot {V_{gg}}^2}{2 \cdot g_c} \cdot \left(1 + 2 \cdot \mu \cdot y\right) = 0.7333 \ psi$$

$$P_{gtotal} := P_g + \Delta P_{accel} = 18.8683 \ psi$$

"Pressure drop across the blower"

$$P_{in} := p_a - 0.3 \ psi = 14.4 \ psi$$ "Inlet pressure to blower"

$$\Delta P_{blower} := P_{gtotal} - P_{in} = 4.4683 \ psi$$

"Saltation Velocity using Rizk Correlation"

$dp = 4\ mm$ *"HDPE particle dia. mm"*

$\delta := 1.44 \cdot dp + 1.96\ mm = 7.72\ mm$

$\psi := 1.1 \cdot dp + 2.5\ mm = 6.9\ mm$

$$V_{Saltation} := \left(\frac{Capacity \cdot \frac{hr}{3600\ s} \cdot 10^{7.72}}{A \cdot \rho_{gSTP}} \left(\sqrt{g \cdot D}\right)^{6.9} \right)^{\frac{1}{(6.9+1)}} = 71.1999\ \frac{ft}{s}$$

"————————————————————Summary of the Results————————————————————"

"Pressure at the pipe point"

$P_b = 14.8807\ psi$

$P_c = 16.6047\ psi$

$P_d = 16.8903\ psi$

$P_e = 17.367\ psi$

$P_f = 17.6401\ psi$

$P_g = 18.1349\ psi$

"Pressure drop at the pipe points"

$\Delta P_{bc} = 1.7239\ psi$

$\Delta P_{vert} = 0.4767\ psi$

$\Delta P_{fg} = 0.4949\ psi$

$\Delta P_{blower} = 4.4683\ psi$

"Gas (air) velocity at the various points in the pipe"

$V_{gb} = 115.296\ \frac{ft}{s}$ $V_{gd} = 101.5785\ \frac{ft}{s}$ $V_{gf} = 97.261\ \frac{ft}{s}$

$V_{gc} = 103.3258\ \frac{ft}{s}$ $V_{ge} = 98.7904\ \frac{ft}{s}$ $V_{gg} = 94.6069\ \frac{ft}{s}$

"Saltation Velocity"

$$V_{Saltation} = 71.1999 \frac{ft}{s}$$

"Conclusion"

The smallest velocity in the pipe line occurs at point g=94.6 ft/s, hence the velocity everywhere in the pipe line exceeds the Saltation velocity. we assume that the blower is capable of the 4.45 psi pressure increase, the velocity provided by the blower flow rate of 1375 SCFM exceeds the saltation velocity everywhere in the pipe line, therefore, the blower and the pipe line system is capable of conveying 30000lbm/hr of solids.

"References"

1- Theory and design of dilute phase pneumatic conveying system.
A.T. Agarwal USA Vol. 17 No.1 Jan/Feb-2006.
2- Pneumatic conveying design guide 2nd ed., David Mills 2004- Elsevier.
3- Handbook of fluidization and fluid particle systems edited by Wen-Ching Yang 2003.
4- On the prediction of pickup and saltation velocities in pneumatic conveying,
L.M Amarante Mesquita, Brazlian journal of chemical engineering, Vol.31 March 2014.
5- A tutorial on pipe flow equations, Donald W. Schroeder,Jr. Aug.2001/Stoner Associates Inc.
6- Design example dilute phase pneumatic conveying. Author/publisher are not known.