T-BRIDGED OSCILLATOR

The oscillator is composed of an operational amplifier in non-inverting configuration and a T-bridge consisting of four lumped devices, two resistors and other two reactive. After having written the results of the analysis, they are used to find the trigger condition of the oscillations by applying the Barkhausen criterion. It follows an example completed with graphics. Afterwards are traced the Bode plots of the transfer function and its representation on the Nichols plane. Finally, it is made the sensitivity analysis.

Reference:C:\new folder\Nichols Chart.xmcd(R)
Reference:C:\new folder\global data.xmcd(R)
Reference:C:\new folder\Dirac Pulse - formulas.xmcd(R)

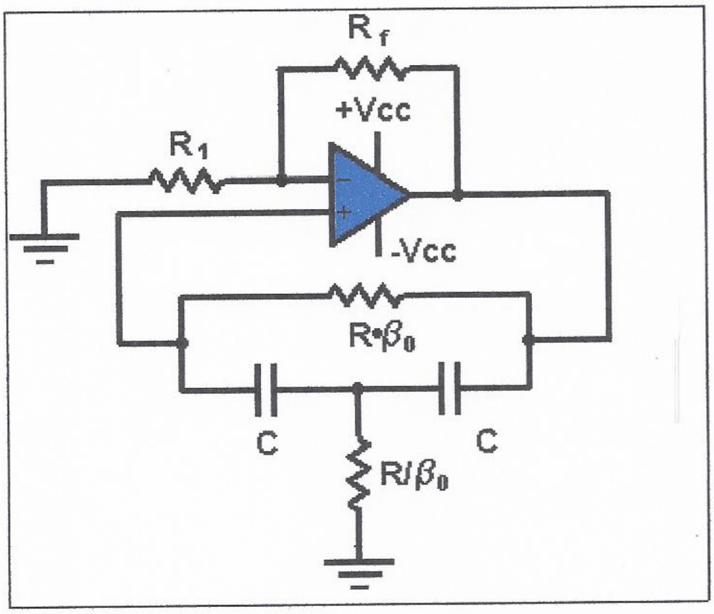


Fig.:1

Oscillator analysis:

Transfer function of the T-bridge:

Place $\omega_0 = \frac{1}{R \cdot C}$

$$H(s) = \frac{s^2 + s \cdot \frac{2 \cdot \omega_0}{\beta_0} + \omega_0^2}{s^2 + s \cdot \omega_0 \cdot \left(\frac{2 + \beta_0^2}{\beta_0}\right) + \omega_0^2} \qquad (1)$$

Open loop gain transfer function:

$$GH(s) = \left(1 + \frac{Rf}{R_1}\right) \cdot \frac{s^2 + s \cdot \frac{2 \cdot \omega_0}{\beta_0} + \omega_0^2}{s^2 + s \cdot \omega_0 \cdot \left(\frac{2 + \beta_0^2}{\beta_0}\right) + \omega_0^2} \tag{2}$$

Same function but in the domain of s=jω:

$$GH(j \cdot \omega) = \left(1 + \frac{Rf}{R_1}\right) \cdot \frac{-\omega^2 + j \cdot \omega \cdot \omega_0 \cdot \left(\frac{2}{\beta_0}\right) + \omega_0^2}{-\omega^2 + j \cdot \omega \cdot \omega_0 \cdot \left(\frac{2 + \beta_0^2}{\beta_0}\right) + \omega_0^2} \tag{3}$$

Magnitude of the open loop gain:

$$\left|GH(j \cdot \omega)\right| = \left(1 + \frac{Rf}{R_1}\right) \cdot \frac{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left[\omega \cdot \omega_0 \cdot \left(\frac{2}{\beta_0}\right)\right]^2}}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left[\omega \cdot \omega_0 \cdot \left(\frac{2 + \beta_0^2}{\beta_0}\right)\right]^2}} \tag{4}$$

Magnitude of the open loop gain for ω infinitive

$R_1 := R_1$ $Rf := Rf$ $R := R$ $C := C$ $\beta_0 := \beta_0$

$$\lim_{\omega \to \infty} \left[\left(1 + \frac{Rf}{R_1}\right) \cdot \frac{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left[\omega \cdot \omega_0 \cdot \left(\frac{2}{\beta_0}\right)\right]^2}}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left[\omega \cdot \omega_0 \cdot \left(\frac{2 + \beta_0^2}{\beta_0}\right)\right]^2}}\right] \rightarrow \frac{Rf + R_1}{R_1} \tag{5}$$

Phase of the open loop gain:

$$\phi(\omega) = \operatorname{atan}\left[\frac{\omega \cdot \omega_0 \cdot \left(\frac{2}{\beta_0}\right)}{\omega_0^2 - \omega^2}\right] - \operatorname{atan}\left[\frac{\omega \cdot \omega_0 \cdot \left(\frac{2 + \beta_0^2}{\beta_0}\right)}{\omega_0^2 - \omega^2}\right] \tag{6}$$

Barkhausen condition on the phase:

$$\operatorname{atan}\left[\frac{\omega \cdot \omega_0 \cdot \left(\frac{2}{\beta_0}\right)}{\omega_0^2 - \omega^2}\right] - \operatorname{atan}\left[\frac{\omega \cdot \omega_0 \cdot \left(\frac{2 + \beta_0^2}{\beta_0}\right)}{\omega_0^2 - \omega^2}\right] = 0 \tag{7}$$

$$\phi(\omega) = \operatorname{atan}\left[\frac{\omega\cdot\omega_0\cdot\left(\frac{2}{\beta_0}\right)}{\omega_0^2-\omega^2}\right] - \operatorname{atan}\left[\frac{\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right)}{\omega_0^2-\omega^2}\right] \tag{8}$$

The phase is null for:

$$\boxed{\omega = \omega_0 = \frac{1}{R\cdot C}} \tag{9}$$

<u>Oscillation frequency:</u>

$$f_0 = \frac{\omega_0}{2\cdot\pi} \tag{10}$$

Search of phase's max and min:

$$\frac{\partial}{\partial\omega}\phi(\omega) = 0$$

Open loop gain phase first derivative:

$$\frac{\partial}{\partial\omega}\phi(\omega) = \frac{-C\cdot R\cdot\beta_0^3\cdot\left(C^2\cdot R^2\cdot\omega^2+1\right)\cdot\left[C^2\cdot R^2\cdot\omega^2\cdot\left[\beta_0^2\cdot\left(C^2\cdot R^2\cdot\omega^2-4\right)-4\right]+\beta_0^2\right]}{\left[C^2\cdot R^2\cdot\omega^2\cdot\left[\beta_0^2\cdot\left(C^2\cdot R^2\cdot\omega^2+\beta_0^2+2\right)+4\right]+\beta_0^2\cdot C^2\cdot R^2\cdot\omega^2\cdot\left[\beta_0^2\cdot\left(C^2\cdot R^2\cdot\omega^2-2\right)+4\right]+\beta_0^2\right]} \tag{11}$$

Angular frequecies for which the phase derivative is null:

$$\omega_{00} = \frac{1}{R\cdot C}\cdot\begin{bmatrix} j \\ -j \\ \dfrac{\sqrt{2\cdot\beta_0^2+\sqrt{3\cdot\beta_0^4+8\cdot\beta_0^2+4}+2}}{\beta_0} \\ \dfrac{\sqrt{2\cdot\beta_0^2-\sqrt{3\cdot\beta_0^4+8\cdot\beta_0^2+4}+2}}{\beta_0} \\ \dfrac{-\sqrt{2\cdot\beta_0^2+\sqrt{3\cdot\beta_0^4+8\cdot\beta_0^2+4}+2}}{\beta_0} \\ \dfrac{-\sqrt{\left(2\cdot\beta_0^2-\sqrt{3\cdot\beta_0^4+8\cdot\beta_0^2+4}+2\right)}}{\beta_0} \end{bmatrix} \tag{12}$$

$$\omega_{\mu 1} = \frac{1}{2}\cdot\sqrt{2}\cdot\frac{\sqrt{2+3\cdot\beta_0^2}+\sqrt{2+\beta_0^2}}{\beta_0\cdot R\cdot C} \qquad \omega_{\mu 2} = \frac{1}{2}\cdot\sqrt{2}\cdot\frac{\sqrt{2+3\cdot\beta_0^2}-\sqrt{2+\beta_0^2}}{\beta_0\cdot R\cdot C} \tag{13}$$

<u>Now apply the Barkhausen criterion which states that |GH|=1 when</u> $\omega=\omega_o$**:**

$$\lim_{\omega \to \omega_0} |GH(j\cdot\omega)| = \lim_{\omega \to \omega_0} \left[\left(1 + \frac{Rf}{R_1}\right) \cdot \frac{\sqrt{\left[\frac{1}{(R\cdot C)^2} - \omega^2\right]^2 + \left[\frac{\omega}{R\cdot C}\cdot\left(\frac{2}{\beta_0}\right)\right]^2}}{\sqrt{\left[\frac{1}{(R\cdot C)^2} - \omega^2\right]^2 + \left[\frac{\omega}{R\cdot C}\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right)\right]^2}} \right] = 1$$

results: $$2\cdot\frac{\left(1+\frac{Rf}{R_1}\right)}{\left(2+\beta_0^2\right)} = 1 \qquad (14)$$

namely: $$1 + \frac{Rf}{R_1} = \frac{2+\beta_0^2}{2} \qquad (15)$$

from which, solving the equation for Rf, you get the following condition for the oscillation triggering:

$$\boxed{Rf = \frac{1}{2}\cdot\beta_0^2\cdot R_1} \qquad (16)$$

POLES of GH: $$s^2 + \frac{s}{R\cdot C}\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right) + \frac{1}{(R\cdot C)^2} = 0 \qquad (17)$$

$$p = \begin{bmatrix} \left(2+\beta_0^2 - \sqrt{4+\beta_0^4}\right) \\ \left(2+\beta_0^2 + \sqrt{4+\beta_0^4}\right) \end{bmatrix} \cdot \frac{-1}{2\cdot\beta_0\cdot R\cdot C} \qquad (18)$$

ZEROS of GH: $$s^2 + \frac{s}{R\cdot C}\cdot\left(\frac{2}{\beta_0}\right) + \frac{1}{(R\cdot C)^2} = 0 \qquad (19)$$

$$z = \begin{pmatrix} -1 + j\cdot\sqrt{-1+\beta_0^2} \\ -1 - \sqrt{1-\beta_0^2} \end{pmatrix} \cdot \frac{1}{R\cdot C\cdot\beta_0} \qquad (20)$$

Stability factor calculation $S_f = \omega_0\cdot\frac{\partial}{\partial\omega}\phi(\omega)$.

Open loop gain phase first derivative:

$$\frac{\partial}{\partial\omega}\phi(\omega) = \frac{\left(-\omega^6\cdot\omega_0 - \omega_0^7\right)\cdot\beta_0^5 + \omega^2\cdot\omega_0^5\cdot\beta_0^3\cdot\left(3\cdot\beta_0^2+4\right) + \omega^4\cdot\omega_0^3\cdot\beta_0^3\cdot\left(3\cdot\beta_0^2+4\right)}{\beta_0^4\cdot\left(\omega^8+\omega_0^8\right) + 2\cdot\omega^4\cdot\omega_0^4\cdot\left(-\beta_0^6+\beta_0^4+8\right) + \omega^2\cdot\omega_0^6\cdot\beta_0^2\cdot\left(\beta_0^4+8\right) + \omega^6\cdot\omega_0^2\cdot\beta_0^2\cdot\left(\beta_0^4+8\right)} \qquad (11')$$

Substituting in the previous result the pulsation ω_0 to ω, one gets:

$$\lim_{\omega \to \omega_0} \frac{\left(-\omega^6 \cdot \omega_0 - \omega_0^7\right) \cdot \beta_0^5 + \omega^2 \cdot \omega_0^5 \cdot \beta_0^3 \cdot \left(3 \cdot \beta_0^2 + 4\right) + \omega^4 \cdot \omega_0^3 \cdot \beta_0^3 \cdot \left(3 \cdot \beta_0^2 + 4\right)}{\beta_0^4 \cdot \left(\omega^8 + \omega_0^8\right) + 2 \cdot \omega^4 \cdot \omega_0^4 \cdot \left(-\beta_0^6 + \beta_0^4 + 8\right) + \omega^2 \cdot \omega_0^6 \cdot \beta_0^2 \cdot \left(\beta_0^4 + 8\right) + \omega^6 \cdot \omega_0^2 \cdot \beta_0^2 \cdot \left(\beta_0^4 + 8\right)} \text{ simplify } \to \frac{\beta_0}{10 \cdot \pi \cdot (}$$

$$\frac{\partial}{\partial \omega} \phi(\omega) = \frac{1}{\omega_0} \cdot \frac{\beta_0^3}{2 + \beta_0^2} \qquad (21)$$

multiplying both members for ω_0, results that the stability factor depends only from β_0, namely:

$$S_f = \left(\frac{\partial}{\partial \omega} \phi(\omega)\right) \cdot \omega_0 = \frac{\beta_0^3}{2 + \beta_0^2} \qquad (22)$$

$$\boxed{S_f = \frac{\beta_0^3}{2 + \beta_0^2}} \qquad (23)$$

Oscillator design using equations (9), (15), (22):

DATA: $R_1 := 1.5 \cdot k\Omega$

$C := 2.2 \cdot nF$

Desired oscillation frequency $f_0 := 100 \cdot kHz$

$\omega_0 := f_0 \cdot 2 \cdot \pi$ $\qquad \omega_0 = 6.283 \times 10^5 \cdot \frac{rad}{sec}$

(9) $R := \frac{1}{\omega_0 \cdot C}$ $\qquad R = 0.723 \cdot k\Omega$

Desired frequency stability factor: $Sf := 25$

Taking into account the definition of the stability factor (22): $Sf = \frac{\beta_0^{\,3}}{2 + \beta_0^{\,2}}$

one can write an equation in β_0: $\beta_0^{\,3} - Sf \cdot \beta_0^{\,2} - 2 \cdot Sf = 0$

Solving for β_0 (real root only) you get :

$$\beta_0 := \left[\frac{1}{27} \cdot Sf \cdot \left[27 + Sf^2 + 3 \cdot \sqrt{\left(27 + 2 \cdot Sf^2\right) \cdot 3}\right]\right]^{\frac{1}{3}} + \left[\frac{1}{27} \cdot Sf \cdot \left[27 + Sf^2 - 3 \cdot \sqrt{\left(27 + 2 \cdot Sf^2\right) \cdot 3}\right]\right]^{\frac{1}{3}} + \frac{1}{3} \cdot Sf \quad (24)$$

Once defined β_0 ,one can dimension the resistors:

$\beta_0 = 25.079$ $\qquad Rf := \frac{1}{2} \cdot \beta_0^{\,2} \cdot R_1$ (15)

$R = 723.432 \cdot \Omega$ $\qquad Rf = 471.736 \cdot k\Omega$

$\frac{Rf + R_1}{Rf} = 1.003$ $\qquad R \cdot \beta_0 = 18.143 \cdot k\Omega$

$Rf = 471.736 \cdot k\Omega$ $\qquad \frac{R}{\beta_0} = 28.846 \cdot \Omega$

$R_1 = 1.5 \cdot k\Omega$

$1 + \frac{Rf}{R_1} = 315.491$

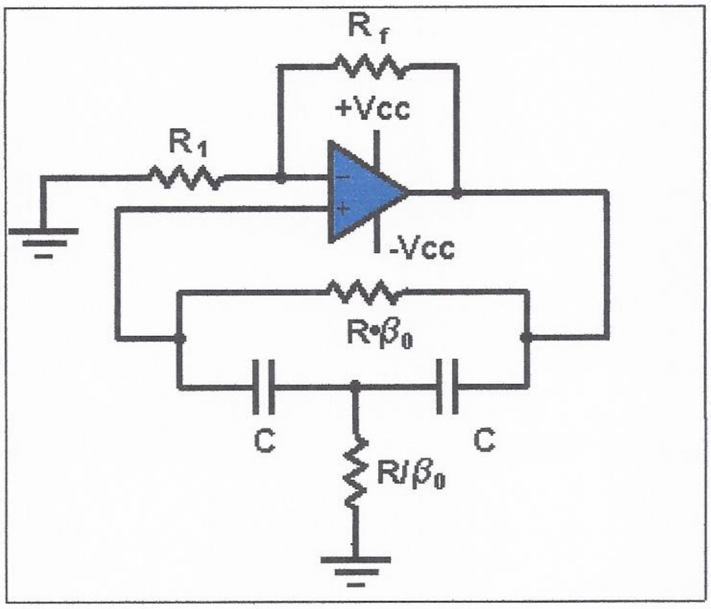


Fig.:2

Oscillations will start very slowly. For a quick start you should slightly vary a parameter.

Verify:

$$1-\left(1+\frac{Rf}{R_1}\right)\cdot\frac{\sqrt{\left(\omega_0{}^2-\omega_0{}^2\right)^2+\left[\omega_0{}^2\cdot\left(\frac{2}{\beta_0}\right)\right]^2}}{\sqrt{\left(\omega_0{}^2-\omega_0{}^2\right)^2+\left[\omega_0{}^2\cdot\left(\frac{2+\beta_0{}^2}{\beta_0}\right)\right]^2}}=0$$

POLES of GH:

$$p:=\begin{bmatrix}\left(2+\beta_0{}^2-\sqrt{4+\beta_0{}^4}\right)\\ \left(2+\beta_0{}^2+\sqrt{4+\beta_0{}^4}\right)\end{bmatrix}\cdot\frac{-\omega_0}{2\cdot\beta_0} \qquad (19')$$

ZEROS of GH:

$$z:=\begin{pmatrix}-1+j\cdot\sqrt{-1+\beta_0{}^2}\\ -1-\sqrt{1-\beta_0{}^2}\end{pmatrix}\cdot\frac{\omega_0}{\beta_0} \qquad (20')$$

$$p=\begin{pmatrix}-0.025\\ -15.783\end{pmatrix}\cdot\frac{\text{Mrads}}{\text{sec}} \qquad z=\begin{pmatrix}-0.025+0.628j\\ -0.025-0.628j\end{pmatrix}\cdot\frac{\text{Mrads}}{\text{sec}}$$

Open loop Phase function

$$\phi(\omega):=\text{atan}\left[\frac{\omega\cdot\omega_0\cdot\left(\frac{2}{\beta_0}\right)}{\omega_0{}^2-\omega^2}\right]-\text{atan}\left[\frac{\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0{}^2}{\beta_0}\right)}{\omega_0{}^2-\omega^2}\right] \qquad (8')$$

Angular frequecies for which the phase derivative is null:

$$\omega_{00}:=\omega_0\cdot\begin{bmatrix}j\\ -j\\ \frac{\sqrt{2\cdot\beta_0{}^2+\sqrt{3\cdot\beta_0{}^4+8\cdot\beta_0{}^2+4}+2}}{\beta_0}\\ \frac{\sqrt{2\cdot\beta_0{}^2-\sqrt{3\cdot\beta_0{}^4+8\cdot\beta_0{}^2+4}+2}}{\beta_0}\\ \frac{-\sqrt{2\cdot\beta_0{}^2+\sqrt{3\cdot\beta_0{}^4+8\cdot\beta_0{}^2+4}+2}}{\beta_0}\\ \frac{-\sqrt{\left(2\cdot\beta_0{}^2-\sqrt{3\cdot\beta_0{}^4+8\cdot\beta_0{}^2+4}+2\right)}}{\beta_0}\end{bmatrix}$$

$$\omega_{00}{}^T=(0.628j\quad -0.628j\quad 1.215\quad 0.325\quad -1.215\quad -0.325)\cdot\frac{\text{Mrads}}{\text{sec}}$$

Phase max and min acceptable real frequencies:

$$\omega_{\mu 1} := \frac{\omega_0}{2}\cdot\sqrt{2}\cdot\frac{\sqrt{2+3\cdot\beta_0^{\,2}}+\sqrt{2+\beta_0^{\,2}}}{\beta_0} \qquad \omega_{\mu 2} := \frac{\omega_0}{2}\cdot\sqrt{2}\cdot\frac{\sqrt{2+3\cdot\beta_0^{\,2}}-\sqrt{2+\beta_0^{\,2}}}{\beta_0} \qquad (13')$$

$$\omega_{\mu 1} = 1.215\cdot\frac{\text{Mrads}}{\text{sec}} \qquad \omega_{\mu 2} = 0.325\cdot\frac{\text{Mrads}}{\text{sec}}$$

Geometrical mean frequency: $\omega_0 = \sqrt{\omega_{\mu 1}\cdot\omega_{\mu 2}}$ (14')

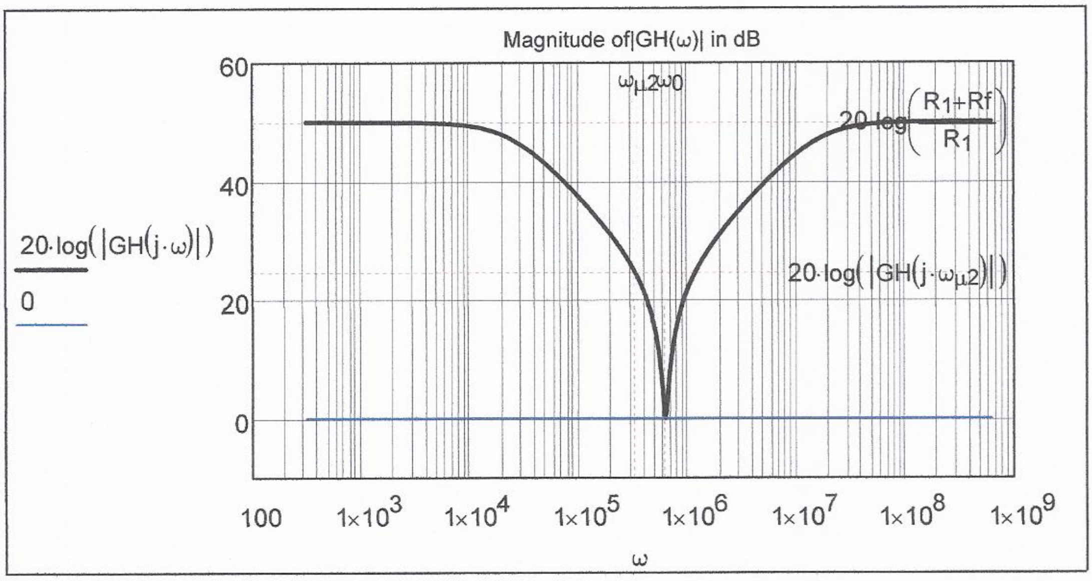
$$\sqrt{\omega_{\mu 1}\cdot\omega_{\mu 2}} = \frac{1}{R\cdot C} \qquad (15')$$

$$\omega_0 = 0.628\cdot\frac{\text{Mrads}}{\text{sec}} \qquad \sqrt{\omega_{\mu 1}\cdot\omega_{\mu 2}} = 0.628\cdot\frac{\text{Mrads}}{\text{sec}} \qquad \frac{1}{R\cdot C} = 0.628\cdot\frac{\text{Mrads}}{\text{sec}}$$

BODE plots of the open loop gain:

$$GH(s) := \left(1+\frac{Rf}{R_1}\right)\cdot\frac{s^2+s\cdot\frac{2\cdot\omega_0}{\beta_0}+\omega_0^{\,2}}{s^2+s\cdot\omega_0\cdot\left(\frac{2+\beta_0^{\,2}}{\beta_0}\right)+\omega_0^{\,2}} \qquad (2')$$

$$\omega_0 = 628.319\cdot\frac{\text{krads}}{\text{sec}} \qquad \omega := \frac{\omega_0}{2000}, \frac{\omega_0}{2000}+\frac{\omega_0}{200} .. \omega_0\cdot 1000 \qquad 20\cdot\log\left(\left|GH\left(j\cdot\omega_{00_2}\right)\right|\right) = 24.99$$



$$|p_0| = 2.501\times10^4\cdot\frac{\text{rad}}{\text{sec}} \qquad |p_1| = 1.578\times10^7\cdot\frac{\text{rad}}{\text{sec}}$$

$$|z_0| = 6.283\times10^5\cdot\frac{\text{rad}}{\text{sec}} \qquad |z_1| = 6.283\times10^5\cdot\frac{\text{rad}}{\text{sec}}$$

$$\omega_0 = 6.283\times10^5\cdot\frac{\text{rad}}{\text{sec}} \qquad \beta_0 = 25.079$$

Fig.:3

$$\omega_{00}^{\,T} = (0.628j \quad -0.628j \quad 1.215 \quad 0.325 \quad -1.215 \quad -0.325)\cdot\frac{\text{Mrads}}{\text{sec}}$$

$\phi(\omega_{\mu 1}) = 1.458\cdot\text{rad}$ $\quad \arg(GH(j\cdot\omega_{\mu 2})) = -1.458$ $\quad \phi(\omega_{\mu 2}) = -1.458\cdot\text{rad}$ $\quad Sf = 25$

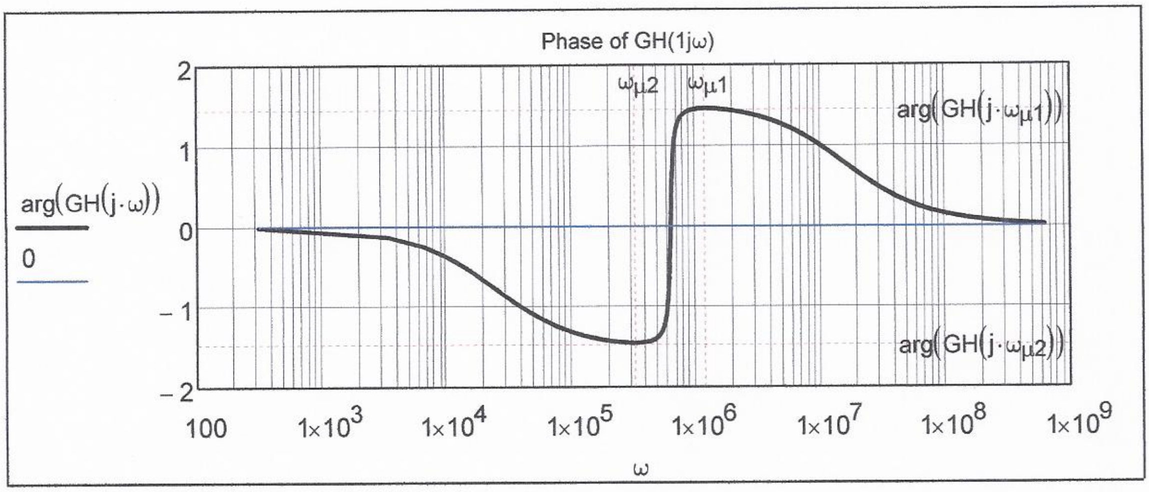

Phase of GH(1jω)
ωμ2
ωμ1
arg(GH(j·ωμ1))
arg(GH(j·ωμ2))
arg(GH(j·ω))
0
2
1
0
−1
−2
100
1×10^3
1×10^4
1×10^5
1×10^6
1×10^7
1×10^8
1×10^9
ω

Fig.:4

NYQUIST plot of the open loop gain function:

$$\omega := -\omega_0, -\omega_0 + \frac{\omega_0}{500} .. \omega_0 \qquad \beta_0 = 25.079$$

$$\omega_{00}^{T} = (0.628j \quad -0.628j \quad 1.215 \quad 0.325 \quad -1.215 \quad -0.325) \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$X1 := GH\left(j \cdot \omega_{00_3}\right) \qquad X2 := GH\left(j \cdot \omega_{00_2}\right)$$

$$X3 := GH\left(-j \cdot \omega_{\mu 2}\right) \qquad X4 := GH\left(j \cdot \omega_{\mu 1}\right)$$

For ω from -∞ to +∞ counterclockwise rotation

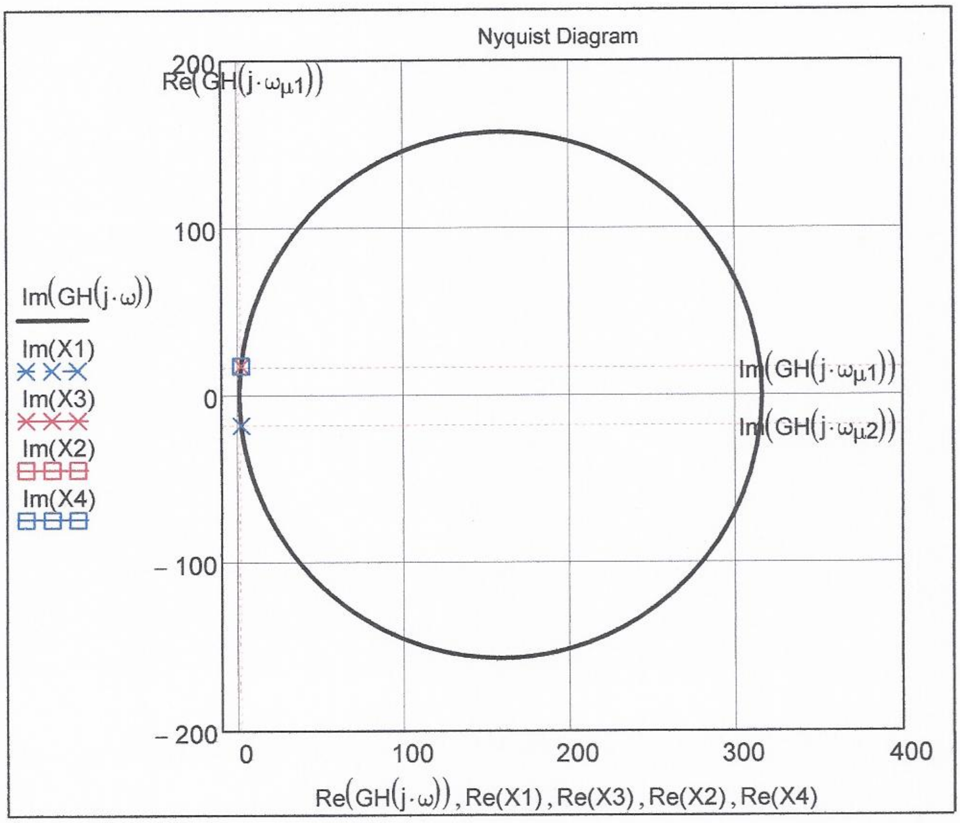


Fig.:5

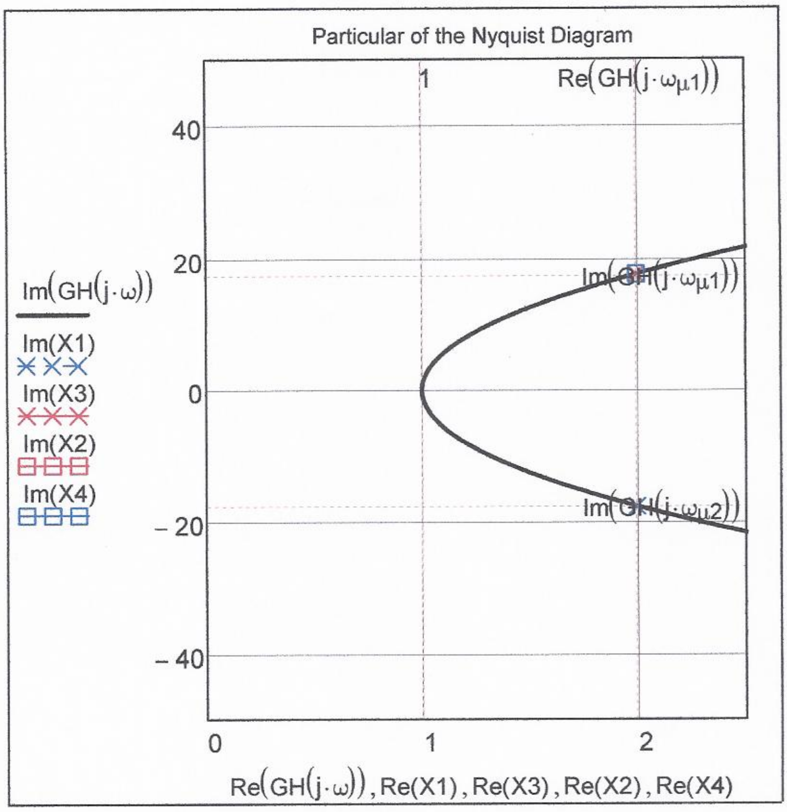


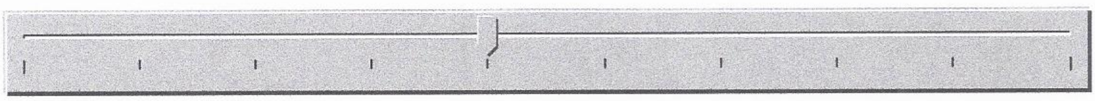
Fig.:6

In the following Nichols chart, the magnitude in dB of the open loop gain vs. the phase in radians both with ω as a parameter, is in **light green** depicted. The **light red** line refers to the **0dB open loop gain.**

$$\phi(\omega) = \arg(G(j\cdot\omega)\cdot H(j\cdot\omega)) \qquad \alpha_{nch}(\omega) = \arg(G(j\cdot\omega)\cdot H(j\cdot\omega)) \tag{25}$$

Scroll The Slider to Zoom In or Out The Nichols Chart

$mnp_{nch} :=$



$m_{nch} := 1.0$ $\qquad n_{nch} := 1$ $\qquad p_{nch} := mnp_{nch}\cdot 10$ $\qquad p_{nch} = 50$

$$\alpha_{nch} := -\pi\cdot m_{nch}\cdot 1.1\,, -\pi\cdot m_{nch}\cdot 1.1 + \frac{2\cdot\pi\cdot m_{nch}}{10^4} \,..\, m_{nch}\cdot\pi$$

The values of the *gain margin* and the *phase margin* can be deduced observing the graph. Therefore have a measure of the degree of stability of the system with feedback.

$\frac{\omega_0}{10^9} = 6.283\times 10^{-4}\cdot\frac{rad}{sec}$ $\qquad n := 2$ $\qquad m := 10$

$$\omega := -m^n\cdot\omega_0\,, -m^n\cdot\omega_0 + \frac{m^n\cdot\omega_0 + m^n\cdot\omega_0}{4.5\cdot 10^4} \,..\, m^n\cdot\omega_0$$

$$\arg\left(GH\left(j\cdot\omega_{00_3}\right)\right) = -1.458 \qquad 20\cdot\log\left(\frac{R_1 + Rf}{R_1}\right) = 49.98 \qquad (26)$$

$$\lim_{\omega\to\infty} |GH(j\cdot\omega)| = 20\cdot\log\left(\frac{R_1 + Rf}{R_1}\right) \qquad 20\cdot\log\left(\frac{R_1 + Rf}{R_1}\right) = 49.98 \qquad 20\log\left[\left|GH\left[j\cdot(1\cdot\omega_0)\right]\right|\right] = -9.643\times 10^{-16}$$

Phase max $\arg\left(GH\left(j\cdot\omega_{\mu 2}\right)\right) = -1.458$ Phase min $\arg\left(GH\left(j\cdot\omega_{\mu 1}\right)\right) = 1.458$

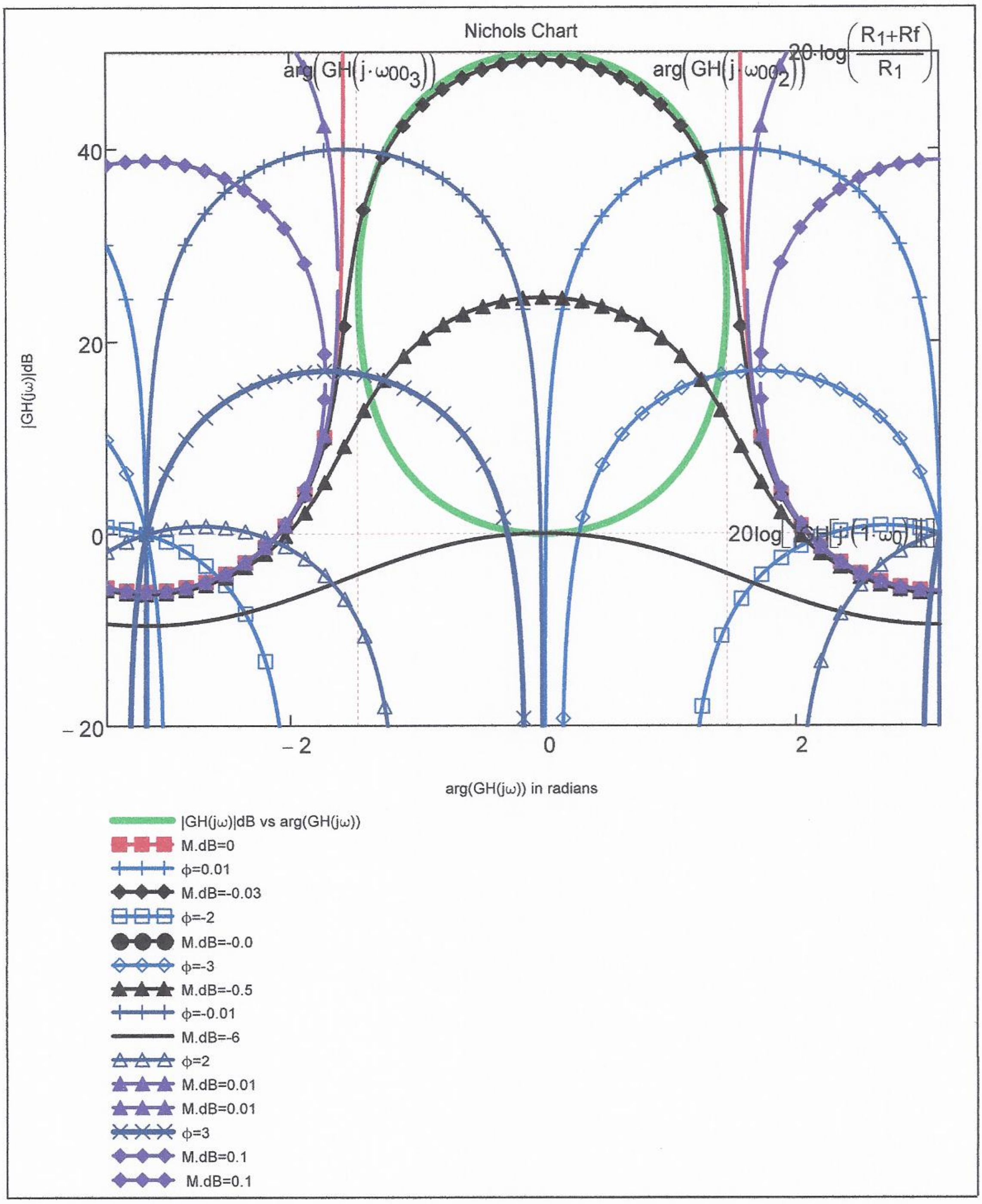


Fig.:7

Transfer function: $$W(s) = \frac{G}{1 + GH(s)} \quad (27)$$

namely: $$W(s) = \frac{\left(1 + \frac{Rf}{R_1}\right)\cdot\left[s^2 + s\cdot\omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right) + \omega_0^2\right]}{s^2 + s\cdot\omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right) + \omega_0^2 + \left(1 + \frac{Rf}{R_1}\right)\cdot\left(s^2 + s\cdot\frac{2\cdot\omega_0}{\beta_0} + \omega_0^2\right)} \quad (28)$$

Now try to simplify:

$$W(s) = \frac{\left(1 + \frac{Rf}{R_1}\right)\cdot\left[s^2 + s\cdot\omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right) + \omega_0^2\right]}{s^2 + s\cdot\omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right) + \omega_0^2 + \left(1 + \frac{Rf}{R_1}\right)\cdot\left(s^2 + s\cdot\frac{2\cdot\omega_0}{\beta_0} + \omega_0^2\right)} = \frac{R_1 + Rf}{2\cdot R_1 + Rf}\cdot\frac{s^2 + s\cdot\omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right) + \omega_0^2}{s^2 + \omega_0\cdot\frac{R_1\cdot\beta_0^2 + 2\cdot Rf + 4\cdot R_1}{\beta_0\cdot(Rf + 2\cdot R_1)}\cdot s + \omega_0^2}$$

$Rf := Rf$ $\quad R_1 := R_1$ $\quad R := R$ $\quad \beta_0 := \beta_0$ $\quad C := C$ $\quad \omega := \omega$ $\quad \omega_0 := \omega_0$

it results $$W(s) := \frac{R_1 + Rf}{2\cdot R_1 + Rf}\cdot\frac{s^2 + s\cdot\omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right) + \omega_0^2}{s^2 + \omega_0\cdot\frac{R_1\cdot\beta_0^2 + 2\cdot Rf + 4\cdot R_1}{\beta_0\cdot(Rf + 2\cdot R_1)}\cdot s + \omega_0^2} \quad (29)$$

Place: $$A_w := \frac{R_1 + Rf}{2\cdot R_1 + Rf} \qquad a_1 := \omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right) \qquad b_1 := \frac{R_1\cdot\beta_0^2 + 2\cdot Rf + 4\cdot R_1}{\beta_0\cdot(Rf + 2\cdot R_1)} \quad (30)$$

$A_w = 0.997$ $\qquad a_1 = 15.808\cdot\frac{Mrads}{sec}$ $\qquad b_1 = 0.159$

$a_1 := a_1$ $\qquad b_1 := b_1$ $\qquad \omega_0 := \omega_0$ $\qquad A_w := A_w$

$$A_w\cdot\frac{s^2 + s\cdot a_1 + \omega_0^2}{s^2 + \omega_0\cdot b_1\cdot s + \omega_0^2} \;\Bigg|\; \begin{array}{l} \text{invlaplace} \\ \text{simplify} \\ \text{collect}, e^{-\frac{b_1\cdot t\cdot\omega_0}{2}} \\ \text{factor} \end{array} \rightarrow$$

"Inverse Laplace transform (Dirac pulse response):

$$w(t) := \left[e^{-\frac{b_1\cdot t\cdot\omega_0}{2}}\cdot\left[\cosh\left(\frac{t\cdot\omega_0\cdot\sqrt{b_1^2 - 4}}{2}\right)\cdot(a_1 - b_1\cdot\omega_0) - \frac{b_1\cdot\sinh\left(\frac{t\cdot\omega_0\cdot\sqrt{b_1^2-4}}{2}\right)\cdot\sqrt{b_1^2 - 4}\cdot(a_1 - b_1\cdot\omega_0)}{(b_1^2 - 4)}\right] + \Delta_\varepsilon(t, \varepsilon_{gd})\right]\cdot A_w$$

(31)

$$T_0 := \frac{2\cdot\pi}{\omega_0} \qquad t := \frac{T_0}{10}, \frac{T_0}{10} + \frac{10\cdot T_0 - \frac{T_0}{10}}{1000} .. 10\cdot T_0$$

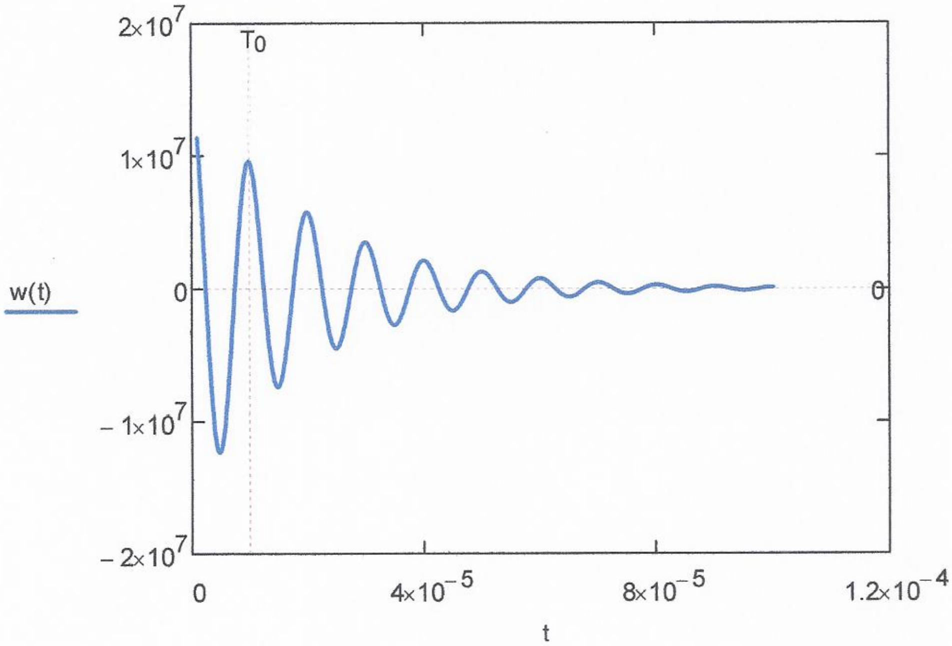


Fig.:8

T.F. in the ω domain:

$$W(j\cdot\omega) = \frac{R1 + Rf}{2\cdot R1 + Rf}\cdot\frac{\omega_0^2 - \omega^2 + j\cdot\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right)}{\omega_0^2 - \omega^2 + j\cdot\omega\cdot\omega_0\cdot\frac{R_1\cdot\beta_0^2 + 2\cdot Rf + 4\cdot R_1}{\beta_0\cdot(Rf + 2\cdot R_1)}} \tag{32}$$

Phase of the t.f.:

$$\phi_w(\omega) := \operatorname{atan}\left[\omega\cdot\omega_0\cdot\frac{(2+\beta_0^2)}{\beta_0\cdot(\omega_0^2 - \omega^2)}\right] - \operatorname{atan}\left[\frac{\omega\cdot\omega_0\cdot(R_1\cdot\beta_0^2 + 2\cdot Rf + 4\cdot R_1)}{(\omega_0^2-\omega^2)\cdot[\beta_0\cdot(Rf + 2\cdot R_1)]}\right] \tag{33}$$

Magnitude of the t. f.:

$$|W(j\cdot\omega)| = \frac{R1+Rf}{2\cdot R1 + Rf}\cdot\frac{\sqrt{(\omega_0^2-\omega^2)^2 + \left[\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right)\right]^2}}{\sqrt{(\omega_0^2-\omega^2)^2 + \left[\omega\cdot\omega_0\cdot\frac{(\beta_0^2+4)\cdot R_1 + 2\cdot Rf}{R_1\cdot\beta_0}\right]^2}} \tag{34}$$

$$\lim_{\omega \to \infty}\left[\frac{R1 + Rf}{2\cdot R1 + Rf}\cdot\frac{\sqrt{1+\left[\frac{\omega\cdot\omega_0}{\omega_0^{\,2}-\omega^2}\cdot\left(\frac{2+\beta_0^{\,2}}{\beta_0}\right)\right]^2}}{\sqrt{1+\left[\frac{\omega\cdot\omega_0}{\omega_0^{\,2}-\omega^2}\cdot\frac{\left(\beta_0^{\,2}+4\right)\cdot R_1+2\cdot Rf}{R_1\cdot\beta_0}\right]^2}}\right] = \frac{Rf + R_1}{Rf + 2\cdot R_1} \tag{35}$$

$$inf := 20\cdot\log\left(\frac{Rf + R_1}{Rf + 2\cdot R_1}\right) \tag{36}$$

$$\lim_{\omega \to \omega_0}\left[\frac{R1 + Rf}{2\cdot R1 + Rf}\cdot\frac{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^{\,2}}{\beta_0}\right)\right]^2}}{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\frac{\left(\beta_0^{\,2}+4\right)\cdot R_1+2\cdot Rf}{R_1\cdot\beta_0}\right]^2}}\right] = \frac{\left(R_1 + Rf\right)\cdot\left(2+\beta_0^{\,2}\right)}{\left(\beta_0^{\,2}+4\right)\cdot R_1 + 2\cdot Rf} \tag{37}$$

$$maxW := 20\cdot\log\left[\frac{\left(R_1 + Rf\right)\cdot\left(2+\beta_0^{\,2}\right)}{\left(\beta_0^{\,2}+4\right)\cdot R_1 + 2\cdot Rf}\right] \qquad maxW = 43.959 \tag{38}$$

Bode Plots

$$\omega := \frac{\omega_0}{2000}, \frac{\omega_0}{2000} + \frac{\omega_0}{200} .. \omega_0\cdot 1000$$

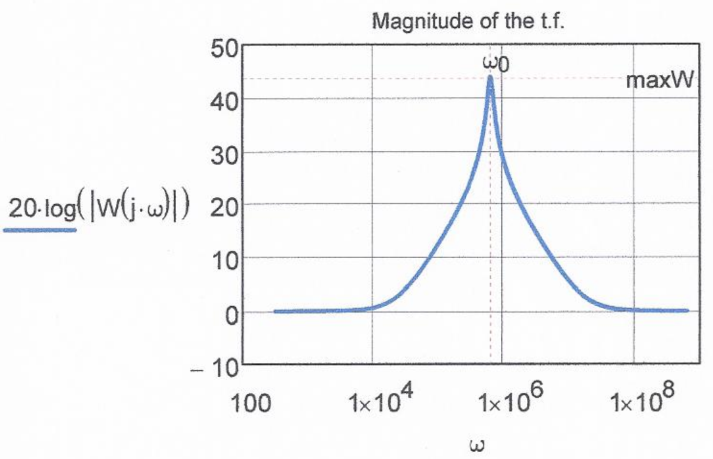


Fig.:9

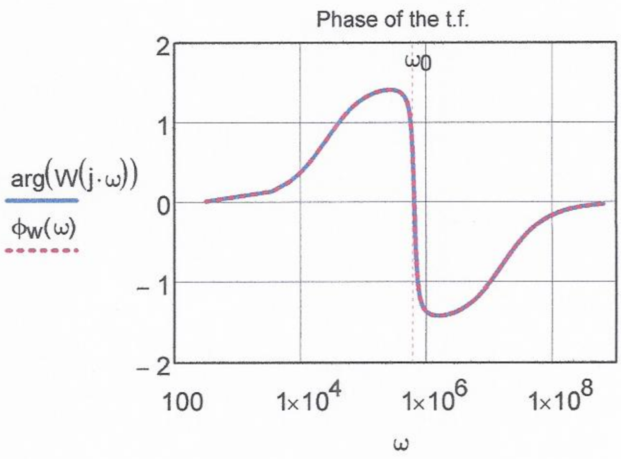


Fig.:10

$$\omega := -m^n\cdot\omega_0, -m^n\cdot\omega_0 + \frac{m^n\cdot\omega_0 + m^n\cdot\omega_0}{4.5\cdot 10^4} .. m^n\cdot\omega_0$$

Nichols chart used as a new reference system

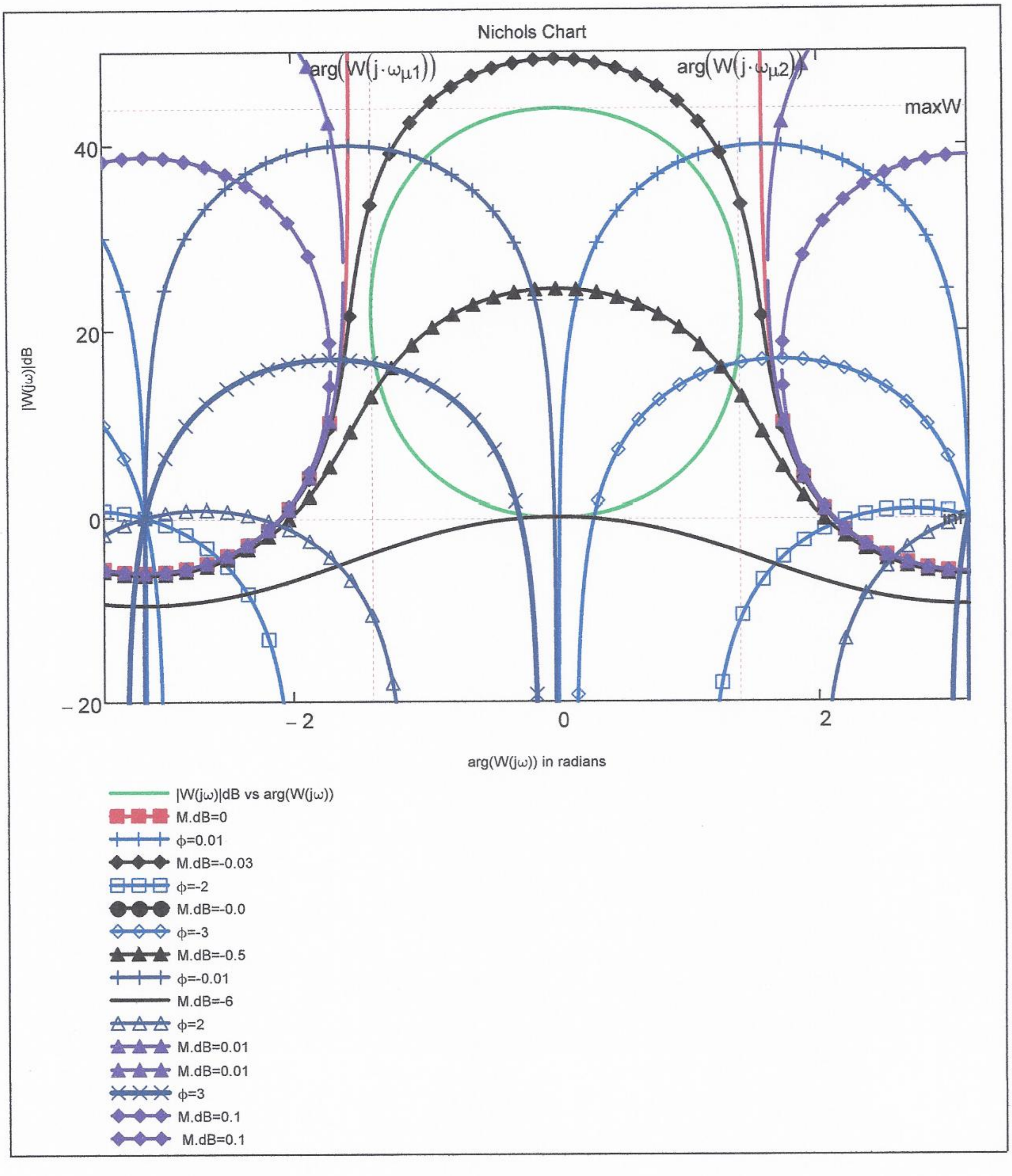

Nichols Chart
arg(W(j·ωμ1))
arg(W(j·ωμ2))
maxW
40
20
0
− 20
− 2
0
2
|W(jω)|dB
arg(W(jω)) in radians
|W(jω)|dB vs arg(W(jω))
M.dB=0
ϕ=0.01
M.dB=-0.03
ϕ=-2
M.dB=-0.0
ϕ=-3
M.dB=-0.5
ϕ=-0.01
M.dB=-6
ϕ=2
M.dB=0.01
M.dB=0.01
ϕ=3
M.dB=0.1
M.dB=0.1

Fig.:11

Sensitivity

$$F(\omega) = |GH(j\cdot\omega)| = \left(1 + \frac{Rf}{R_1}\right)\cdot\frac{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left[\omega\cdot\omega_0\cdot\left(\frac{2}{\beta_0}\right)\right]^2}}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left[\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^2}{\beta_0}\right)\right]^2}} \qquad (4')$$

$$A_w = \frac{R_1 + Rf}{2\cdot R_1 + Rf}$$

$$\boxed{\omega_0 = \frac{1}{R\cdot C}} \qquad (9')$$

$$\boxed{Rf = \frac{1}{2}\cdot\beta_0^2\cdot R_1} \qquad (16')$$

$$\boxed{S_f = \frac{\beta_0^3}{2+\beta_0^2}} \qquad (23)$$

Sensitivity calculation of the circuit performance A_w

$$\varepsilon = \frac{\Delta R_1}{R_1} = \frac{\Delta Rf}{Rf} = \frac{\Delta C}{C} = 1\% \qquad \varepsilon := 1\%$$

$$\frac{\partial}{\partial R_1}A_w = \frac{\partial}{\partial R_1}\frac{R_1 + Rf}{2\cdot R_1 + Rf} \qquad \frac{\partial}{\partial R_1}\frac{R_1 + Rf}{2\cdot R_1 + Rf} \rightarrow \frac{1}{Rf + 2\cdot R_1} - \frac{2\cdot(Rf + R_1)}{(Rf + 2\cdot R_1)^2}$$

$$\frac{R_1}{\frac{R_1+Rf}{2\cdot R_1+Rf}}\cdot\frac{\partial}{\partial R_1}A_w = \frac{R_1}{\frac{R_1+Rf}{2\cdot R_1+Rf}}\cdot\left[\frac{1}{Rf + 2\cdot R_1} - \frac{2\cdot(Rf + R_1)}{(Rf + 2\cdot R_1)^2}\right]$$

$$\frac{R_1}{\frac{R_1+Rf}{2\cdot R_1+Rf}}\cdot\left[\frac{1}{Rf + 2\cdot R_1} - \frac{2\cdot(Rf + R_1)}{(Rf + 2\cdot R_1)^2}\right] = -\frac{Rf\cdot R_1}{(Rf + 2\cdot R_1)\cdot(Rf + R_1)}$$

$$\boxed{S_{R.1} = -\frac{Rf\cdot R_1}{(Rf + 2\cdot R_1)\cdot(Rf + R_1)}} \qquad (39)$$

$$\frac{R_1}{\frac{R_1+Rf}{2\cdot R_1+Rf}}\cdot\frac{\partial}{\partial Rf}\frac{R_1 + Rf}{2\cdot R_1 + Rf} \text{ simplify } \rightarrow \frac{R_1^2}{(Rf + 2\cdot R_1)\cdot(Rf + R_1)}$$

$$S_{Rf} = \frac{Rf}{A_w}\cdot\frac{\partial}{\partial Rf}A_w = \frac{R_1^2}{(Rf + 2\cdot R_1)\cdot(Rf + R_1)} \qquad (40)$$

$$S_{Aw} = \frac{\Delta A_w}{A_w} = \left|S_{R.1}\right| \cdot \frac{\Delta R_1}{R_1} + \left|S_{Rf}\right| \cdot \frac{\Delta Rf}{Rf} = \frac{Rf \cdot R_1}{(Rf + 2 \cdot R_1) \cdot (Rf + R_1)} \cdot \frac{\Delta R_1}{R_1} + \frac{R_1^{\ 2}}{(Rf + 2 \cdot R_1) \cdot (Rf + R_1)} \cdot \frac{\Delta Rf}{Rf}$$

$$S_{Aw} := \frac{R_1 \cdot (Rf + R_1)}{(Rf + 2 \cdot R_1) \cdot (Rf + R_1)} \cdot \varepsilon \qquad (41)$$

$$S_{Aw} = 3.16 \times 10^{-3} \cdot \% \qquad (42)$$

Sensitivity calculation of the circuit performance Rf

$$\boxed{Rf = \frac{1}{2} \cdot \beta_0^{\ 2} \cdot R_1} \qquad (16')$$

$$\frac{\partial}{\partial R_1} Rf = \frac{\partial}{\partial R_1}\left(\frac{1}{2} \cdot \beta_0^{\ 2} \cdot R_1\right) \qquad \frac{\partial}{\partial R_1}\left(\frac{1}{2} \cdot \beta_0^{\ 2} \cdot R_1\right) \rightarrow \frac{\beta_0^{\ 2}}{2} \qquad (43)$$

$$S_{Rf} = \frac{R_1}{\left(\frac{1}{2} \cdot \beta_0^{\ 2} \cdot R_1\right)} \cdot \frac{\beta_0^{\ 2}}{2} \cdot \frac{\Delta R_1}{R_1} = \frac{\Delta R_1}{R_1} \qquad (44)$$

$$S_{Rf} := \varepsilon \qquad S_{Rf} = 1 \cdot \% \qquad (45)$$

Sensitivity calculation of the circuit performance ω_0

$$\boxed{\omega_0 = \frac{1}{R \cdot C}} \qquad (9')$$

Angular frequency sensitivity with circuit parameter R:

$$R := R \qquad C := C \qquad \frac{\partial}{\partial R} \frac{1}{R \cdot C} \rightarrow -\frac{1}{C \cdot R^2}$$

$$\boxed{\frac{\partial}{\partial R} \omega_0 = \frac{-1}{R^2 \cdot C}} \qquad (46)$$

$$S_{\omega.0.R} = \frac{R}{\omega_0} \cdot \frac{\partial}{\partial R} \omega_0 = \frac{R}{\frac{1}{R \cdot C}} \cdot \frac{-1}{R^2 \cdot C} = -1$$

$$\frac{\partial}{\partial C} \frac{1}{R \cdot C} \rightarrow -\frac{1}{C^2 \cdot R}$$

$$\boxed{\frac{\partial}{\partial C} \omega_0 = \frac{-1}{R \cdot C^2}}$$

$$S_{\omega.0.C} = \frac{C}{\omega_0}\cdot\frac{\partial}{\partial C}\omega_0 = \frac{C}{\frac{1}{R\cdot C}}\cdot\frac{-1}{R\cdot C^2} = -1$$

$$S_{\omega.0} = \left|S_{\omega.0.R}\right|\cdot\frac{\Delta R}{R} + \left|S_{\omega.0.C}\right|\cdot\frac{\Delta C}{C} = \frac{\Delta R}{R} + \frac{\Delta C}{C}$$

$$S_{\omega.0} := 2\cdot\varepsilon \qquad (47)$$

$$S_{\omega.0} = 2\cdot\%$$

Sensitivity calculation of the open loop gain

Magnitude of the open loop gain:

$$F(\omega) := \left(1+\frac{Rf}{R_1}\right)\cdot\frac{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2}{\beta_0}\right)\right]^2}}{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^{\,2}}{\beta_0}\right)\right]^2}} \qquad (4')$$

$$S_{RfF} = \frac{Rf}{F}\cdot\frac{\partial}{\partial Rf}F = \frac{Rf}{\left(1+\frac{Rf}{R_1}\right)\cdot\frac{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2}{\beta_0}\right)\right]^2}}{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^{\,2}}{\beta_0}\right)\right]^2}}}\cdot\left[\frac{1}{R_1}\cdot\frac{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2}{\beta_0}\right)\right]^2}}{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^{\,2}}{\beta_0}\right)\right]^2}}\right] = \frac{Rf}{R_1\cdot\left(1+\frac{Rf}{R_1}\right)}$$

$$S_{RfF} := \frac{Rf}{R_1\cdot\left(1+\frac{Rf}{R_1}\right)} \qquad (48)$$

$$S_{\omega.0} = \frac{\omega_0}{F}\cdot\frac{\partial}{\partial\omega_0}F = \frac{\omega_0}{\left(1+\frac{Rf}{R_1}\right)\cdot\frac{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2}{\beta_0}\right)\right]^2}}{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^{\,2}}{\beta_0}\right)\right]^2}}}\cdot\frac{\partial}{\partial\omega_0}\left[\left(1+\frac{Rf}{R_1}\right)\cdot\frac{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2}{\beta_0}\right)\right]^2}}{\sqrt{\left(\omega_0^{\,2}-\omega^2\right)^2+\left[\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^{\,2}}{\beta_0}\right)\right]^2}}\right]$$

$Rf := Rf \quad R_1 := R_1 \quad \omega_0 := \omega_0 \quad \beta_0 := \beta_0 \quad \omega := \omega$

$$\frac{\partial}{\partial \omega_0}\left[\left(1+\frac{Rf}{R_1}\right)\cdot\frac{\sqrt{\left(\omega_0^{2}-\omega^{2}\right)^{2}+\left[\omega\cdot\omega_0\cdot\left(\frac{2}{\beta_0}\right)\right]^{2}}}{\sqrt{\left(\omega_0^{2}-\omega^{2}\right)^{2}+\left[\omega\cdot\omega_0\cdot\left(\frac{2+\beta_0^{2}}{\beta_0}\right)\right]^{2}}}\right] \text{ simplify } \rightarrow$$

$$\frac{\partial}{\partial \omega_0}F = -\frac{\omega^{2}\cdot\omega_0\cdot\beta_0^{6}\cdot\left(\omega^{4}-\omega_0^{4}\right)\cdot\sqrt{\frac{\omega^{2}\cdot\left[\beta_0^{2}\cdot\left(\omega^{2}-2\cdot\omega_0^{2}\right)+4\cdot\omega_0^{2}\right]+\omega_0^{4}\cdot\beta_0^{2}}{\beta_0^{2}}}\cdot\sqrt{\frac{\beta_0^{2}\cdot\omega^{4}+\omega_0^{2}\cdot\left(\beta_0^{4}+2\cdot\beta_0^{2}+4\right)\cdot\omega^{2}+\omega_0^{4}\cdot\beta_0}{\beta_0^{2}}}}{R_1\cdot\left[\beta_0^{2}\cdot\omega^{4}+-2\cdot\omega_0^{2}\cdot\left(\beta_0^{2}-2\right)\cdot\omega^{2}+\omega_0^{4}\cdot\beta_0^{2}\right]\cdot\left[\beta_0^{2}\cdot\omega^{4}+\omega_0^{2}\cdot\left[\beta_0^{2}\cdot\left(\beta_0^{2}+2\right)+4\right]\cdot\omega^{2}+\omega_0^{4}\cdot\beta_0^{2}\right.}$$

$$S_{\omega.0F}(\omega) := \frac{-\omega_0^{2}\cdot\beta_0^{4}\cdot\left(\beta_0^{2}+4\right)\cdot\omega^{6}+\omega_0^{6}\cdot\beta_0^{4}\cdot\left(\beta_0^{2}+4\right)\cdot\omega^{2}}{\beta_0^{4}\cdot\omega^{8}+\omega_0^{2}\cdot\beta_0^{2}\cdot\left(\beta_0^{4}+8\right)\cdot\omega^{6}+-2\cdot\omega_0^{4}\cdot\left[\beta_0^{4}\cdot\left(\beta_0^{2}-1\right)-8\right]\cdot\omega^{4}+\omega_0^{6}\cdot\beta_0^{2}\cdot\left(\beta_0^{4}+8\right)\cdot\omega^{2}+\omega_0^{8}\cdot\beta_0^{4}} \quad (49)$$

$$\omega := \omega_0 \qquad S_F := \left|S_{RfF}\right|\cdot S_{Rf}+\left|S_{\omega.0F}(\omega)\right|\cdot S_{\omega.0} \qquad S_F = 0.997\cdot\% \quad (50)$$

SUMMARY:

$Sf = 25$

$\beta_0 = 25.079$

$Rf = 471.736\cdot k\Omega$ $\quad S_{Rf} = 1\cdot\%$

$R_1 = 1.5\cdot k\Omega$ $\quad \varepsilon = 1\cdot\%$

$R = 723.432\cdot\Omega$

$\frac{R}{\beta_0} = 28.846\cdot\Omega$

$R\cdot\beta_0 = 18.143\cdot k\Omega$

$\frac{Rf}{R_1} = 314.491$

$C = 2.2\cdot nF$ $\quad \varepsilon = 1\cdot\%$

$\omega_0 = 628.319\cdot\frac{krads}{sec}$ $\quad S_{\omega.0} = 2\cdot\%$

$\frac{\omega_0}{2\cdot\pi} = 100\cdot kHz$

$Rf_{real} := 51.0k\Omega$

$1+\frac{Rf}{R_1} = 315.491$

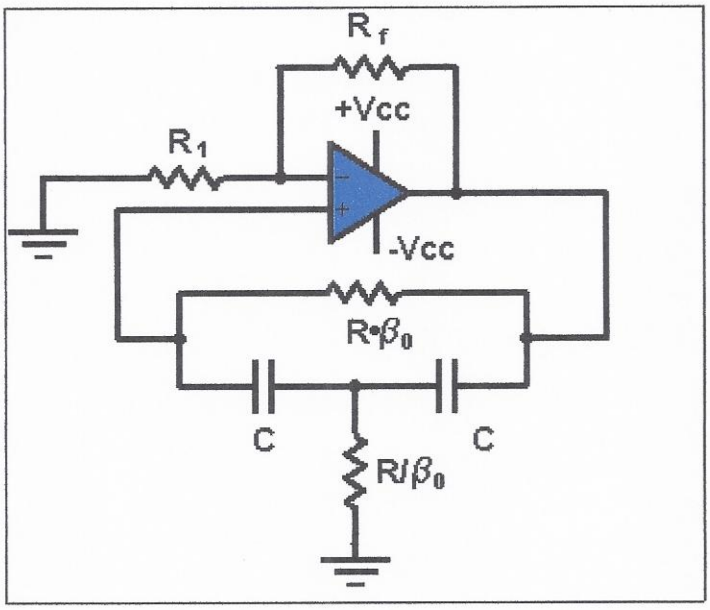


Fig.:12