CHAPTER 10 BUILDING THERMAL CONTROL

10.3 Thermal Control and Transient Response of a Heating System

The building, HVAC system and control transfer functions are combined through block diagram algebra to obtain the overall system s-transfer functions. These transfer functions are then studied in the frequency domain ($s = j \cdot \omega$) for stability and other analyses or are used for transient analysis of overall system response to setpoint and load variations using numerical inversion of Laplace transforms. This technique is described in **Section 10.2**.

PID (Proportional-Integral-Derivative) Controller

The transfer function for a PID (proportional-integral-derivative) controller is equal to the ratio of the controller output to the input (error) in the Laplace domain. Its output is proportional to the error, the integral of the error over time, and the rate of change of the error.

The transfer function of a PID controller is given by (Stephanopoulos 1984):

$$G_c(s) = K_p \cdot \left(1 + \frac{1}{\tau_i \cdot s} + \tau_D \cdot s\right)$$

where K_p is the proportional gain, τ_i is the integral time, and τ_D is the derivative time. Normally, a proportional or a proportional-integral controller is satisfactory for HVAC systems.

PID Controller Constants Based on the Ziegler-Nichols Method

PID control constants are selected based on various tuning methods, such as the Ziegler Nichols method. First, we determine the crossover frequency and the corresponding ultimate gain K_u, that is, the proportional gain for which the system is at the stability limit. The following values are recommended for P, PI, and PID controllers:

Proportional controller: $K_p = 0.5 \cdot K_u$

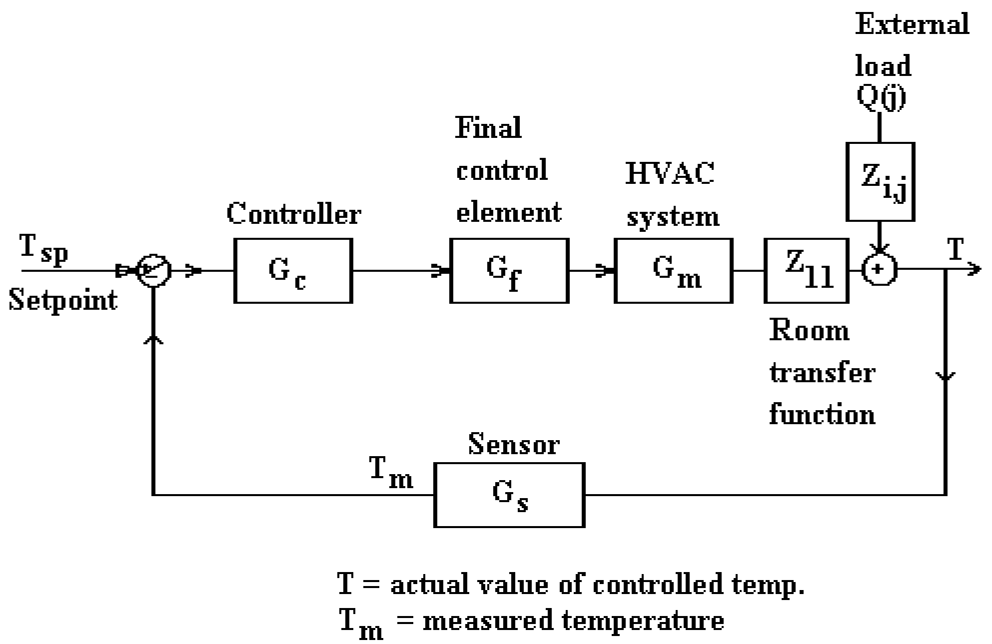
P-I controller: $K_p = \frac{K_u}{2.2}$ $\tau_i = \frac{P_u}{1.2}$

P_u is the ultimate period (corresponding to the crossover frequency)

PID controller: $K_p = \frac{K_u}{1.7}$ $\tau_i = \frac{P_u}{2}$ $\tau_D = \frac{P_u}{8}$

Convective Heating System

Consider a room with convective heating, simple feedback control and *K* inputs-loads *Q(j)* influencing room temperature *T(1)* (also denoted T_{ai}) through transfer functions Z_{ij} (see figure below). The resulting room temperature variation as a function of setpoint and load changes is then obtained through block diagram algebra as follows:



$$T_{ai} = T_{sp} \cdot \frac{G_c \cdot G_f \cdot G_m \cdot Z_{11}}{1 + G_c \cdot G_f \cdot G_m \cdot Z_{11} \cdot G_s} + \sum_j \left(Q(j) \cdot \frac{Z_{1,j}}{1 + G_c \cdot G_f \cdot G_m \cdot Z_{11} \cdot G_s} \right) \qquad (1)$$

$$j = 1, 2 .. K$$

where T_{sp} is the setpoint and *G* are the transfer functions indicated in the block diagram. All variables are a function of *s*.

Let

$$G_{sp}(s) = \frac{G_c \cdot G_f \cdot G_m \cdot Z_{11}}{1 + G_c \cdot G_f \cdot G_m \cdot Z_{11} \cdot G_s}$$ setpoint transfer function

$$G_{OL} = G_c \cdot G_f \cdot G_m \cdot Z_{11} \cdot G_s$$ open loop transfer function

Characteristic equation:

$$1+G_c \cdot G_f \cdot G_m \cdot Z_{11} \cdot G_s$$

The loop transfer function may be used for stability analysis using the Nyquist criterion or the Bode criterion. For example, the Bode criterion may be used to determine the period and gain at the stability limit, known as the ultimate period and ultimate gain K_u respectively, at which the phase angle of the loop transfer function (for proportional control) is equal to 180 deg. The ultimate gain is determined based on the fact that at the crossover frequency the magnitude of the open loop transfer function (amplitude ratio) is equal to one at the stability limit.

The selection of the actual gain is then based on various tuning techniques such as the Ziegler-Nichols method. Normally, tuning is performed on-site for HVAC systems because the system parameters are usually not accurately known; self-tuning adaptive algorithms may also be used. However, the analysis is useful for comparison of alternative control algorithms, the effects of sensors with different time constants and of system parameters such as coil time constant and building time constant.

Example: A room is heated by a 2kW fan convector. The air temperature is sensed by a fast response thermocouple, the output is compared with a set-point voltage and the amplified error signal is used to drive an *SCR* (thyristor) control on the fan heater element. The sensor is 3 m away from the heater and the air moves away from the heater at 0.1 m/s; assume that this will introduce a sensor delay. Using the data below, determine the ultimate gain (stability limit) and the Ziegler-Nichols settings for proportional-integral control. Then, determine the response of room temperature to one *degC* step change of the setpoint with the numerical inverse Laplace transform method.

Room data:

$U := 160 \ \frac{watt}{\Delta°C}$ total *U*-value of room

$Vol := 40 \ m^3$ room volume

$\rho c_{air} := 1200 \ \frac{joule}{m^3 \ \Delta°C}$

$C := Vol \cdot \rho c_{air}$ $C = 48000 \ \frac{joule}{\Delta°C}$

consider only thermal capacity of air

$\tau_r := \frac{C}{U}$ $\tau_r = 300 \ s$

room time constant for fast convective heating;
you may increase it by a factor of 2-3 to account
for the effect of lightweight room contents.

Sensor:

$\tau_s := 15\ s$

$t_d := 3 \dfrac{m}{0.1 \dfrac{m}{s}}$ time constant

$t_d = 30\ s$ time delay due to air travel from heater to thermostat

Heater:

$K_h := 2\ kW$

$\tau_h := 20\ s$ heater capacity and time constant

First establish the component transfer functions:

Room transfer function:

$Z_{11}(s) = \dfrac{Tai(s)}{q_{aux}(s)}$ $Z_{11}(s) := \dfrac{1}{U \cdot (\tau_r \cdot s + 1)}$

Heater transfer function:

$G_m = \dfrac{q_{aux}(s)}{degC}$ $G_m(s) := \dfrac{K_h}{\tau_h \cdot s + 1}$

Sensor transfer function:

$G_s(s) = \dfrac{T_{measured}}{T_{actual}}$ $G_s(s) := \dfrac{\exp(-t_d \cdot s)}{\tau_s \cdot s + 1}$

We will first consider proportional control. For simplicity and for generality of this example, the *SCR* constant K_{scr} is combined with the proportional control constant K_p to give the modified gain *K*:

$G_c \cdot G_f = K = K_{scr} \cdot K_p$ Let $K := 1$ initially

Open-Loop Transfer Function and Stability Analysis

$$G_{OL}(s) := K \cdot G_m(s) \cdot Z_{11}(s) \cdot G_s(s)$$

$$j := \sqrt{-1}$$

Frequency range:

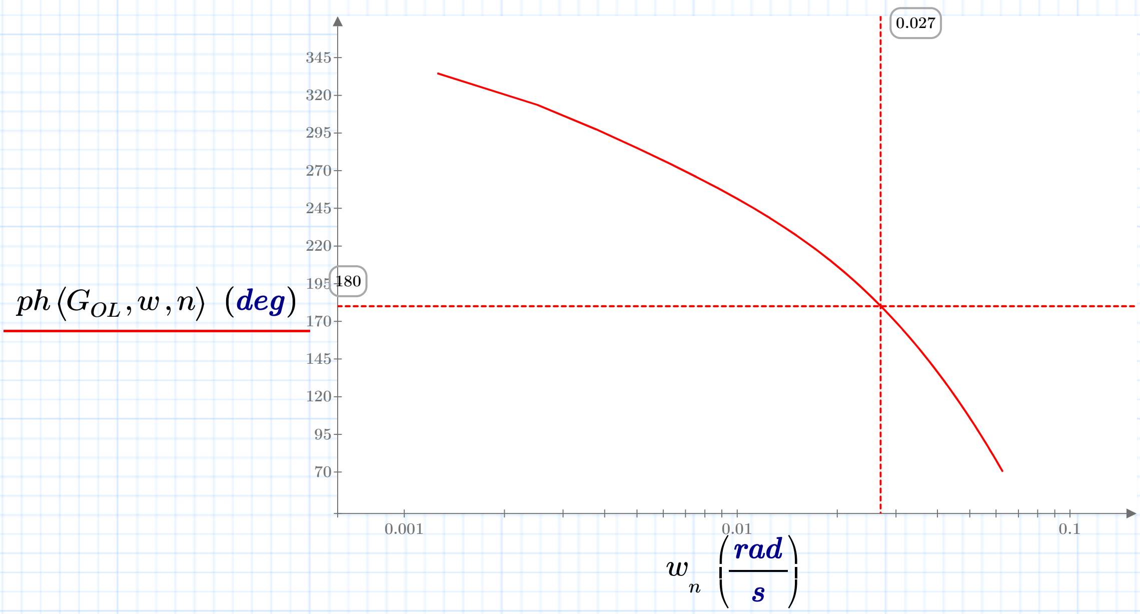
$$P := 5000\ s$$

Period

$$n := 1, 2 .. 50 \qquad w_n := \frac{2 \cdot \pi \cdot n}{P}$$

Phase angle:

$$ph(G_{OL}, w, n) := \arg\left(G_{OL}\left(j \cdot w_n\right)\right) + 2 \cdot \pi \cdot \left(\text{if}\left(\arg\left(G_{OL}\left(j \cdot w_n\right)\right) < 0, 1, 0\right)\right)$$



Approximate crossover frequency is equal to

$$w_{co} := 0.027\ \frac{rad}{s}$$

Calculation of ultimate gain (at stability limit):

$K_u := \frac{1}{\left|G_{OL}(j \cdot w_{co})\right|}$ $K_u = 0.801 \ \frac{1}{K}$ ultimate gain

$P_u := \frac{2 \cdot \pi}{w_{co}}$ $P_u = 232.711 \ s$ ultimate period

$K_p := \frac{K_u}{2.2}$ $K_p = 0.364 \ \frac{1}{K}$

$\tau_i := \frac{P_u}{1.2}$ $\tau_i = 193.925 \ s$

$G_c(s) := K_p \cdot \left(1 + \frac{1}{\tau_i \cdot s}\right)$ PI controller transfer function

Unit step change in the setpoint:

$T_{sp}(s) := \frac{1}{s}$

$G_{sp}(s) := \frac{G_c(s) \cdot G_m(s) \cdot Z_{11}(s)}{1 + G_c(s) \cdot G_m(s) \cdot Z_{11}(s) \cdot G_s(s)}$ setpoint transfer function for this case

$T_{ai}(s) := G_{sp}(s) \cdot T_{sp}(s)$ room temperature response in Laplace domain

Time-domain Response Using Numerical Inverse Laplace Transform Method:

Poles z_i and residues K'_i for inversion are given below for M = 10 and N = 8.

$M' := 5$ $i := 1, 2 .. M'$

$z_1 := 11.83009373916819 + 1.593753005885813j$

$z_2 := 11.22085377939519 + 4.792964167565670j$

$z_3 := 9.933383722175002 + 8.033106334266296j$

$z_4 := 7.781146264464616 + 11.36889164904993j$

$z_5 := 4.234522494797000 + 14.95704378128156j$

$K'_1 := 16286.62368050479 - 139074.7115516051j$

$K'_2 := -28178.11171305163 + 74357.58237274176j$

$K'_3 := 14629.74025233142 - 19181.80818501836j$

$K'_4 := -2870.418161032078 + 1674.109484084304j$

$K'_5 := 132.1659412474876 + 17.47674798877164j$

$l := 1, 2 .. 80$ $\Delta t := 30\ s$

$t_l := l \cdot \Delta t$ time array

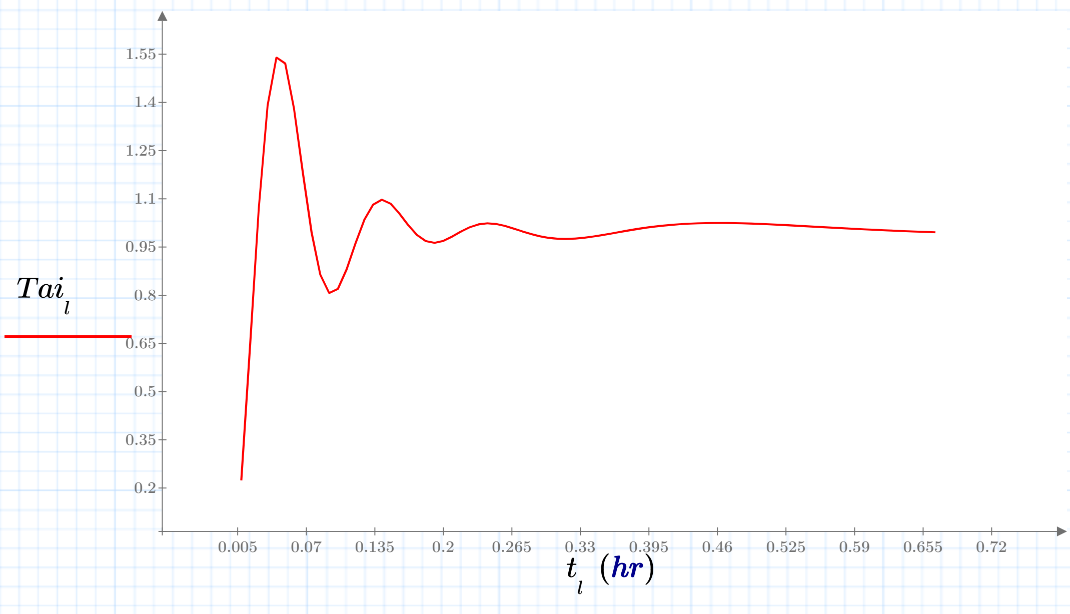
$$s_{i,l} := \frac{z_i}{t_l}$$

The Laplace transfer function to be inverted is

$$V_{i,l} := T_{ai}\left(s_{i,l}\right)$$

The time domain response is given by

$$Tai_0 := 0 \qquad Tai_l := \frac{1}{-t_l} \cdot \left(\sum_i \mathrm{Re}\left(K'_i \cdot V_{i,l}\right) \right)$$



The graph shows the room temperature change due to step setpoint change of 1 degC.

$$\max(Tai) = 1.54$$

References

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