CHAPTER 3 TRANSMISSION LINE CALCULATIONS

3.1 Impedance as a Function of z, ω

This document plots transmission line impedance as a function of frequency at a given distance from the termination. Using either characteristic impedance and phase velocity, or distributed shunt capacitance and distributed series inductance, the complex impedance is calculated. The plots show the real and imaginary parts of the impedance and the locus of points on the complex plane.

You provide **either**

- Z0, the characteristic impedance of the line, and
- **vp**, the phase velocity

or

- **C**, the distributed shunt capacitance, and
- **L**, the distributed series inductance

Background

Characteristic impedance on a transmission line is defined as the ratio of voltage to current on a line with no reflected waves (i.e., with a matched load.) But this tells us nothing about the periodic behavior of voltage and impedance on the line if the load is not matched, which is more commonly the case.

Reactance

Input impedance has both a real and an imaginary part, both of which will vary periodically as a function of distance from the load and as a function of frequency. The imaginary part is known as the reactance. A system with positive reactance behaves like an inductor, and a system with negative reactance behaves like a capacitor. Because the impedance fluctuates periodically between these two cases as a function of length, it is possible to match the load at different points down the transmission line with additional inductance or capacitance. This subject is examined in more detail in

3.2: Reflection Coefficient Calculations

3.3: The Smith Chart.

Mathcad Implementation

The equations that follow define impedance characteristics for a transmission line over a range of operating frequencies. A plot will be generated showing the real and imaginary parts of the impedance, and a locus of points in the complex plane.

Constants

$neper \equiv 1$

First, enter values for either the distributed shunt capacitance and series inductance **or** the characteristic impedance and phase velocity. Enter a zero value for each unused parameter; Mathcad will determine internally which set of parameters is to be used, and calculate the complex impedance based on the numbers inserted.

Transmission Line Parameters

$Z_o := 50\ \Omega$ characteristic impedance

$v_{ph} := 2 \cdot 10^8\ \frac{m}{s}$ phase velocity

Or

$C := 0\ \frac{F}{m}$ distributed shunt capacitance

$L := 0\ \frac{H}{m}$ distributed series inductance

Next, enter the parameters describing the system: the impedance of the load, and the attenuation constant along the line. Specify the distance from termination at which the impedance will be calculated.

System Parameters

$Z_L := 30\ \Omega$ impedance at termination

$\alpha := 0\ neper \cdot m^{-1}$ attenuation constant

$z := 5\ m$ distance from termination

Mathcad now calculates the complex impedance from the transmission line parameters given above. The **if** statements used here check which set of values were given at setup.

Characteristic Impedance:

$$Z_o := \mathbf{if}\left(C = 0\ \boldsymbol{F} \cdot \boldsymbol{m}^{-1}, Z_o, \sqrt{\frac{L}{C}}\right)$$

Phase Velocity:

$$v_{ph} := \mathbf{if}\left(C = 0\ \boldsymbol{F} \cdot \boldsymbol{m}^{-1}, v_{ph}, \frac{1}{\sqrt{L \cdot C}}\right)$$

Complex impedance is calculated via the complex wavenumber. The final expression will be a function of frequency, and will represent the impedance at the specified distance from termination, **z**.

$$k(f) := \alpha + 1\mathrm{i} \cdot \frac{2 \cdot \pi \cdot f}{v_{ph}}$$

$$Z(f,z) := Z_o \cdot \frac{Z_L + Z_o \cdot \tanh(k(f) \cdot z)}{Z_o + Z_L \cdot \tanh(k(f) \cdot z)}$$

Finally, specify a frequency interval and step size for the plots. A plot of the real and imaginary parts of the impedance as functions of frequency, and position appear below.

Plotting Parameters

$f_{min} := 3\ \boldsymbol{MHz}$ start frequency

$f_{max} := 25\ \boldsymbol{MHz}$ end frequency

$nstep := 100$ number of steps

$$f := f_{min}, \frac{f_{max} - f_{min}}{nstep} + f_{min} .. f_{max}$$

A plot of the real and imaginary parts of the normalized impedance vs. frequency response is generated below.

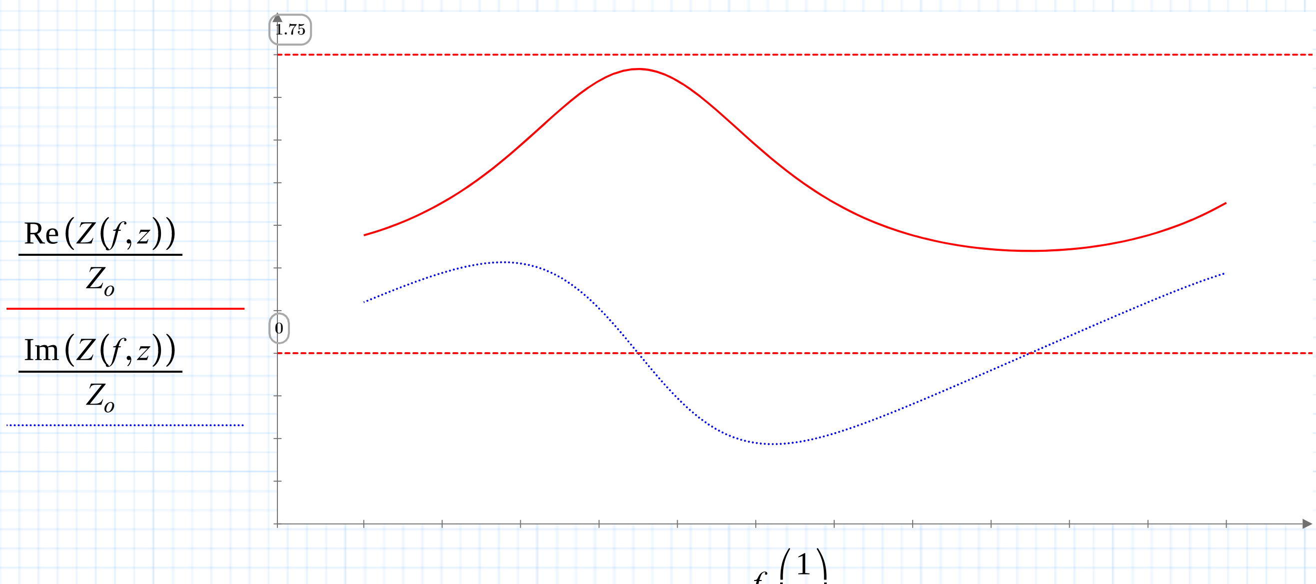


Fig. 3.1.1 Real and imaginary impedance

Complex impedance can also be viewed as a locus of points on the complex plane as a function of frequency:

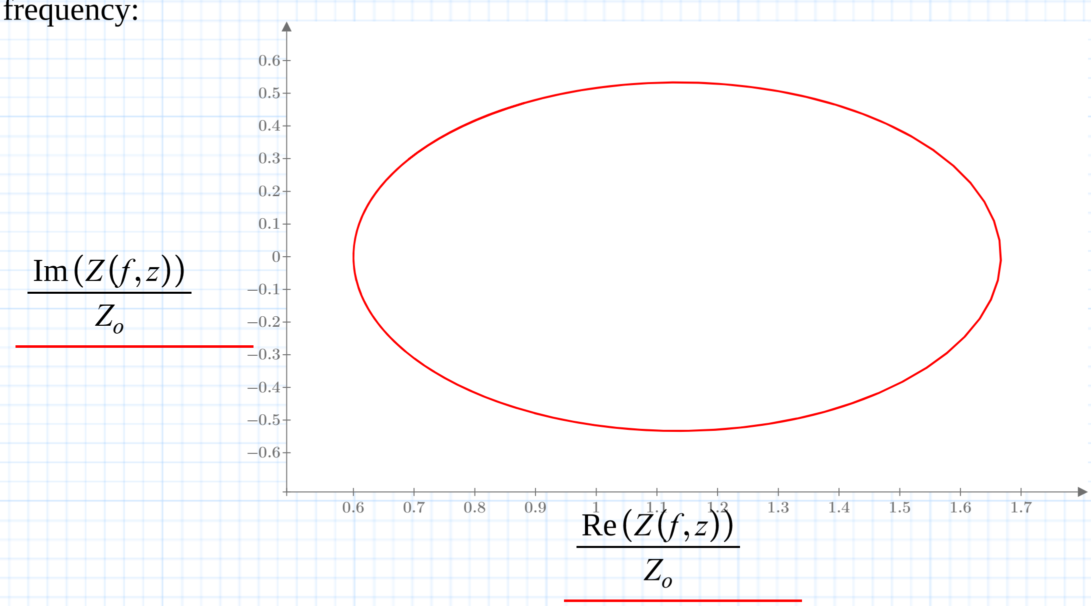
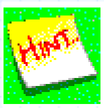


Fig. 3.1.2 Z(f) on the complex plane



It may be useful to set the frequency constant and look at the impedance as a function of distance from termination. Here distance is defined in terms of wavelengths.

$f := 5\ \mathbf{MHz}$ $\qquad \lambda := \dfrac{v_{ph}}{f}$ $\qquad \lambda = 40\ \mathbf{m}$

$step := 0.01 \cdot \lambda$ $\qquad z := 0\ \mathbf{m}, 0\ \mathbf{m} - step \,..\, -99\ \mathbf{m}$

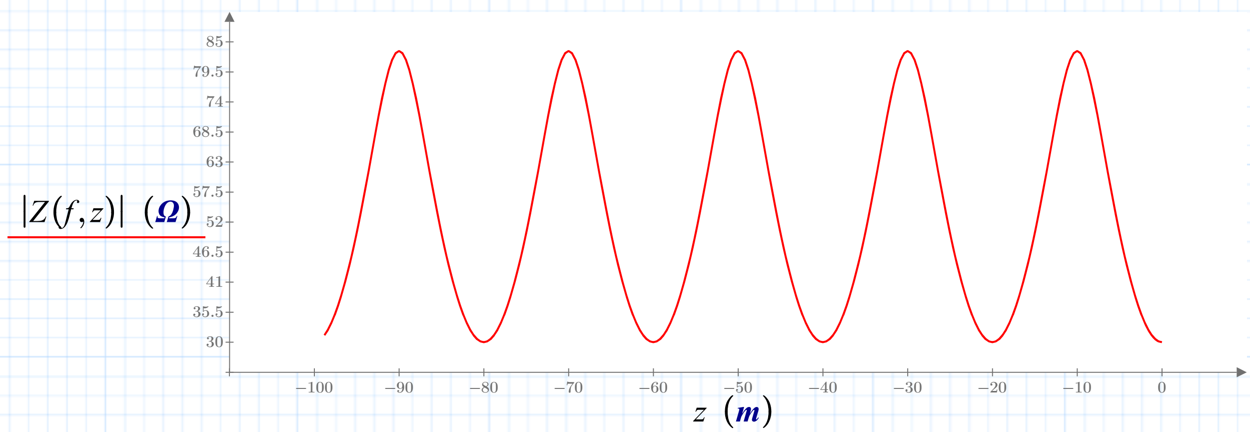


Fig. 3.1.3 Impedance as a function of distance

Notice that the impedance will repeat along the transmission line every half wavelength.