

CHAPTER 3 TRANSMISSION LINE CALCULATIONS

3.2 Reflection Coefficient Calculations

This document shows how you can use Mathcad's complex arithmetic and **root** function to carry out transmission line calculations. The examples include finding the reflection coefficient, load impedance, voltage standing wave ratio, and position of the voltage minimum and maximum along the transmission line. The final example finds the location and value of a shunt admittance for impedance matching. You enter a combination of the following parameters:

- **ZL**, normalized load impedance
- Γ , the reflection coefficient
- **VSWR**, the voltage standing wave ratio
- **z**, position on the transmission line

Background

Most transmission line operating parameters can be related to the reflection coefficient: impedance, admittance, voltage, and VSWR can be found as a function of distance from the load. These calculations are often carried out with the help of a Smith chart, but Mathcad's complex arithmetic capability and **root** function allow you to obtain the same results by direct calculation.

For more information on the graphical method, please see **3.3 The Smith Chart**.

Periodicity

Since all of these functions are periodic in distance along the transmission line, there may be more than one solution to each problem, depending on the actual length of the transmission line. When a solution is found, it is usually the one closest to the load, but not always. Other solutions will be one cycle (2π) or one half-wavelength away. It is wise to double check that the lengths calculated are not longer than the transmission line itself.

Impedance Matching

A note about impedance matching: it is physically difficult to create a series impedance for a transmission line, while it is relatively easy to construct a parallel admittance. For this reason, it is common to match impedances with a parallel admittance stub, as is done in this document.

Mathcad Implementation

The equations that follow define standing wave solutions for transmission line voltages, reflection coefficients, etc.

The following functions are defined here for use throughout the document. They transform between rectangular ($\mathbf{a} + \mathbf{j}\mathbf{b}$) notation and phasor form for complex numbers.

P gives the magnitude and phase (in degrees) of the complex number **A**. The output of the function **P** is a two-element row vector in which the first element is the magnitude of the complex input and the second element is the argument in degrees.

$$P(A) := \begin{bmatrix} |A| & \frac{\arg(A)}{\text{deg}} \end{bmatrix}$$

R(B) expresses the complex number **B** (magnitude, phase in radians) in the form $\mathbf{a} + \mathbf{j}\mathbf{b}$. The function **R** takes a similar row vector as input (with the argument in radians) and returns the equivalent complex number in rectangular form.

$$R(B) := B_{0,0} \cdot \cos(B_{0,1}) + 1j \cdot B_{0,0} \cdot \sin(B_{0,1})$$

For each computation, enter a value for a parameter by editing the corresponding definition.

Reflection Coefficient \Leftrightarrow Load Impedance

If the normalized load impedance is:

$$Z_{nL} := 1 + 2j$$

then the reflection coefficient is:

$$\Gamma_L := \frac{Z_{nL} - 1}{Z_{nL} + 1} = 0.5 + 0.5i$$

or in polar form,

$$P(\Gamma_L) = [0.707 \quad 45]$$

Conversely, if the reflection coefficient is

$$\Gamma_L := [.6 \quad 30] \quad \text{Gamma} := R(\Gamma_L)$$

and in rectangular form,

$$\text{Gamma} = 0.093 - 0.593i$$

The normalized load impedance is, then,

$$Z_{nL} := \frac{1 + \text{Gamma}}{1 - \text{Gamma}} = 0.545 - 1.009i$$

Voltage Standing Wave Ratio (VSWR)

If the reflection coefficient is

$$\Gamma_L := .8 + .2j$$

then the voltage standing wave ratio is

$$VSWR := \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = 10.404$$

If the normalized load impedance is

$$Z_{nL} := 1 + 2j$$

then the reflection coefficient is

$$\Gamma_L := \frac{Z_{nL} - 1}{Z_{nL} + 1}$$

and the

$$VSWR := \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = 5.828$$

Positions of the Voltage Minima and Maxima

If the normalized load impedance is

$$Z_{nL} := 1 + 2j$$

then the reflection coefficient is

$$\Gamma_L := \frac{Z_{nL} - 1}{Z_{nL} + 1} = 0.5 + 0.5i$$

and the positions of the voltage minimum and maximum **in wavelengths** are

$$z_{min} := -\frac{\arg(\Gamma_L) + \pi}{4 \cdot \pi} = -0.313$$

$$z_{max} := -\frac{\arg(\Gamma_L)}{4 \cdot \pi} = -0.063$$

The reflection coefficient and voltage at a position z **wavelengths** back from the load are, respectively,

$$\Gamma(z) := \Gamma_L \cdot e^{2 \cdot 1j \cdot 2 \cdot \pi \cdot z}$$

$$V(z) := e^{-1j \cdot 2 \cdot \pi \cdot z} + \Gamma_L \cdot e^{1j \cdot 2 \cdot \pi \cdot z}$$

$$d := 0, 0 - 0.01 + 0 .. -1.49 \quad \text{Range variable for distance}$$

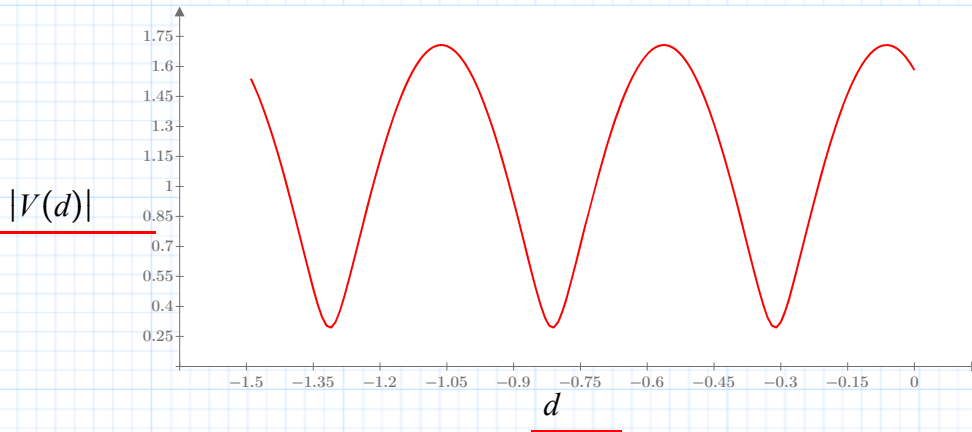


Fig. 3.2.1 |V| as function of distance from load



Additional minima and maxima can be found by adding 2π to the numerators of the expressions for **z_{max}** and **z_{min}**.

Load Impedance and Voltage

If the **VSWR** is given, it is straightforward to find the load impedance:

$$VSWR := 4 \quad z_{min} := -.2 \quad \text{wavelengths from the load}$$

The magnitude of the reflection coefficient at the load is

$$Mag\Gamma_L := \frac{VSWR - 1}{VSWR + 1}$$

and the phase of the reflection coefficient is

$$\Phi\Gamma_L := -(\pi + 4 \cdot \pi \cdot z_{min})$$

Therefore, the normalized load impedance is

$$Z_{nL} := \frac{1 + R \langle [Mag\Gamma_L \quad \Phi\Gamma_L] \rangle}{1 - R \langle [Mag\Gamma_L \quad \Phi\Gamma_L] \rangle} = 1.644 - 1.812i$$

Impedance Matching with a Shunt Admittance

If the normalized load impedance is

$$Z_{nL} := 2 + 1j$$

Then the reflection coefficient at the load is

$$\Gamma_L := \frac{Z_{nL} - 1}{Z_{nL} + 1}$$

and

$$\Gamma(z) := \Gamma_L \cdot e^{2 \cdot 1j \cdot 2 \cdot \pi \cdot z} \quad Z_n(z) := \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

The normalized admittance at a point z from the termination is

$$Y_n(z) := \frac{1}{Z_n(z)}$$

To match the impedance, find a position on the line where the admittance has real part **1** using Mathcad's root location function:

Make a guess for the location along the line:

$$d := -.2$$

$$z1 := \text{root}(\text{Re}(Y_n(d)) - 1, d)$$

The location is

$$z1 = -0.199 \quad \text{wavelengths from the load.}$$

The admittance at $z1$ is

$$Y_n(z1) = 1 + 1i$$

The shunt admittance to be added is the imaginary portion of Y_n :

$$\text{shunt} := -\text{Im}(Y_n(z1)) = -1$$

The length of the short-circuited stub with complex admittance **shunt** is

$$d := \frac{\text{atan}\left(\frac{1}{\text{shunt}}\right)}{2 \cdot \pi} = -0.125 \quad \text{wavelengths}$$

As a check, calculate the admittance of a short circuited stub of length

$$\Gamma_L := -1 \quad \Gamma(z) := \Gamma_L \cdot e^{2 \cdot 1j \cdot 2 \cdot \pi \cdot z} \quad Z_n(z) := \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

$$Y_{\text{stub}} := \frac{1}{Z_n(d)} = -5.551 \cdot 10^{-17} - 1i$$