

CHAPTER 5 FEEDBACK CIRCUITS AND STABILITY CRITERIA

5.1 Bode Plots and Nichols Charts

Given a transfer function, this document generates Bode plots, and finds the unity gain crossover, gain margin, phase margin, and resonant frequency for the circuit. Gain and phase margin are also shown on a plot of log magnitude-phase plot. This application also draws a Nichols chart, showing a contour of constant magnitude closed-loop gain. You supply:

- $G(s)$, the transfer function which describes your system,
- f , the frequency range to plot.

For calculation of RLC system functions, please see **Section 4: Network Analysis Using an Admittance Matrix**.

Background

Linear feedback circuits are commonly defined by a variety of interdependent quantities: resonant frequency, gain and phase margin, and the poles and zeros of the system. These interdependent quantities can be examined together to design the system for stability and maximum operating range. There are a variety of ways to view these quantities; in this section we will examine Bode plots and Nichols charts, relating them to the resonant frequency and operating margins of the system.

A magnitude-phase angle plot may provide more useful information than Bode plots, because in some cases the exact frequency at crossover is of less interest than the gain and phase margins; Magnitude-phase plots specify both gain and phase margins on a single plot, which can then be used to identify the operating range of the system.

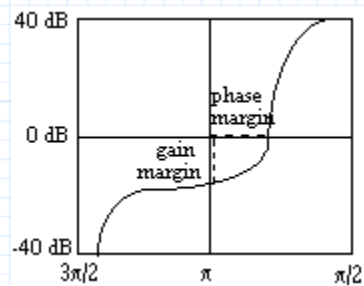


Fig. 5.1.1 Magnitude-Phase plot

Nichols Charts

Nichols charts are a special case of magnitude-phase angle plots which show curves of constant closed-loop gain along with curves of open-loop gain. If the closed-loop gain passes to the right of the open-loop point (0 dB, -180 degrees), then the system is stable.

Mathcad Implementation

This document carries out several common transfer function calculations. In each section, you enter values for one or more parameters, and possibly a guess value to be used by the **root** function.

The following functions will be used throughout the document to calculate magnitude (in dB) and phase shift (in degrees) from the transfer function **G**.

Magnitude:

$$K := 2 \quad G(s) := \frac{K}{s \cdot (1+s) \cdot \left(1 + \frac{s}{3}\right)}$$

$$db(G, \omega) := 20 \cdot \log(|G(1j \cdot \omega)|)$$

Phase shift:

$$ps(G, \omega) := \arg(G(1j \cdot \omega)) - \text{if}(\arg(G(1j \cdot \omega)) \geq 0, 360 \text{ deg}, 0 \text{ deg})$$

Modify definitions for **K** and **G** to specify the **open-loop transfer function** which describes your system.

$$K := 2 \quad G(s) := \frac{K}{s \cdot (1+s) \cdot \left(1 + \frac{s}{3}\right)}$$

Next, choose a **frequency range** for the plots (use powers of 10 for convenient plotting):

lowest frequency: $\omega_{start} := 0.01$ highest frequency: $\omega_{end} := 100$

number of points: $N := 50$

step size: $r := \log\left(\frac{\omega_{start}}{\omega_{end}}\right) \cdot \frac{1}{N}$

range for plot: $i := 0 .. N$

range variable: $\omega_i := \omega_{end} \cdot 10^{i \cdot r}$ $s_i := 1j \cdot \omega_i$ ω

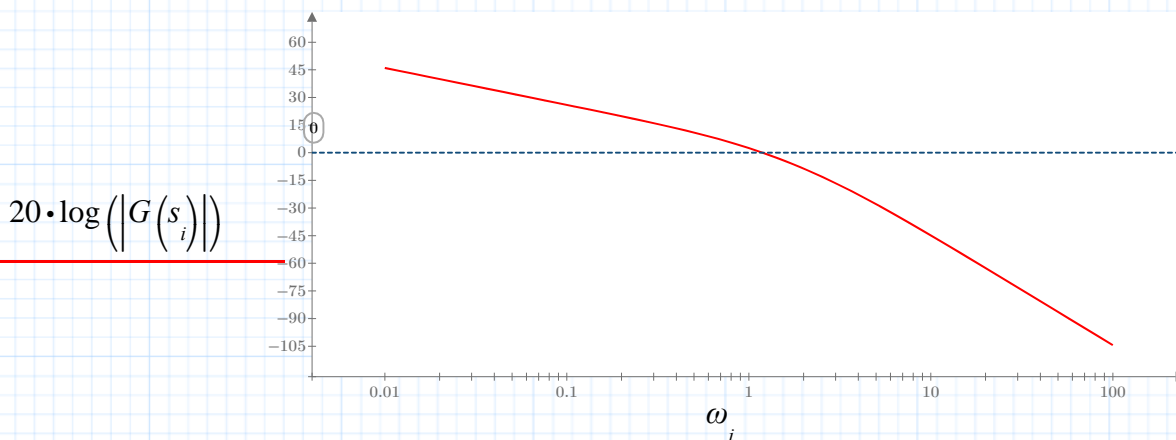


Fig. 5.1.2 Open-loop gain magnitude plot

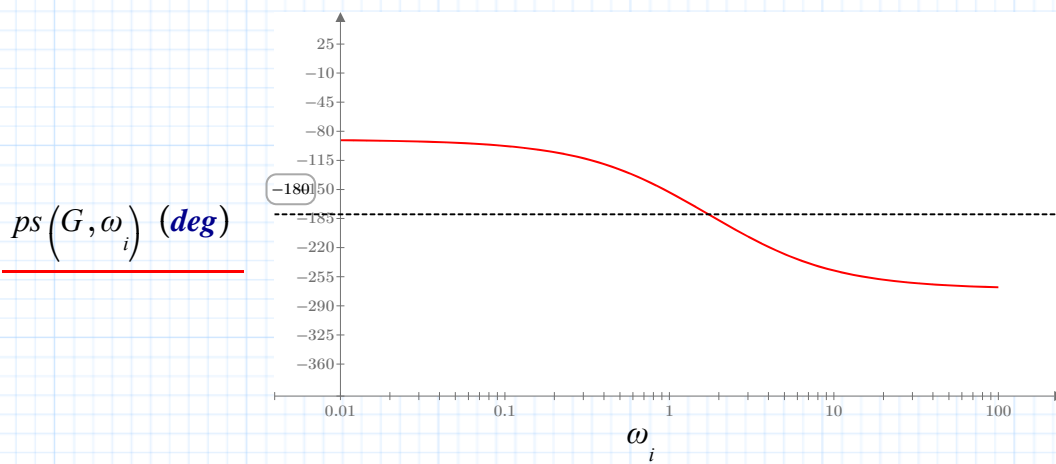


Fig. 5.1.3 Open-loop gain phase shift

Unity Gain Crossover and Phase Margin

Use the first plot to make a rough guess for the frequency at which the magnitude of $\log|G(s)|$ is 0 (gain = 1), and enter this guess in the definition of ω_c below. Mathcad uses the **root** function to find the exact value of the crossover frequency, and then uses the phase shift function defined above to calculate the phase margin at this frequency.

Guess for **crossover frequency**: $\omega_c := 1$

Solve for the gain crossover frequency:

$$\omega_c := \text{root}(db(G, \omega_c), \omega_c) = 1.193$$

Calculate the **phase margin**:

$$pm := ps(G, \omega_c) + 180 \text{ deg} = 18.265 \text{ deg}$$

Gain Margin

Now using the phase angle plot, estimate the frequency at which the phase shift crosses 180 degrees:

$$\omega_{gm} := 1.8$$

Solve for ω at the phase shift point of 180 degrees:

$$\omega_{gm} := \text{root}(ps(G, \omega_{gm}) + 180 \text{ deg}, \omega_{gm}) = 1.732$$

Calculate the **gain margin**:

$$gm := -db(G, \omega_{gm}) = 6.021$$

A magnitude-phase angle plot combines the two plots above, and marks the two crossover frequencies as reference points. The gain and phase margins can be read from this plot directly.

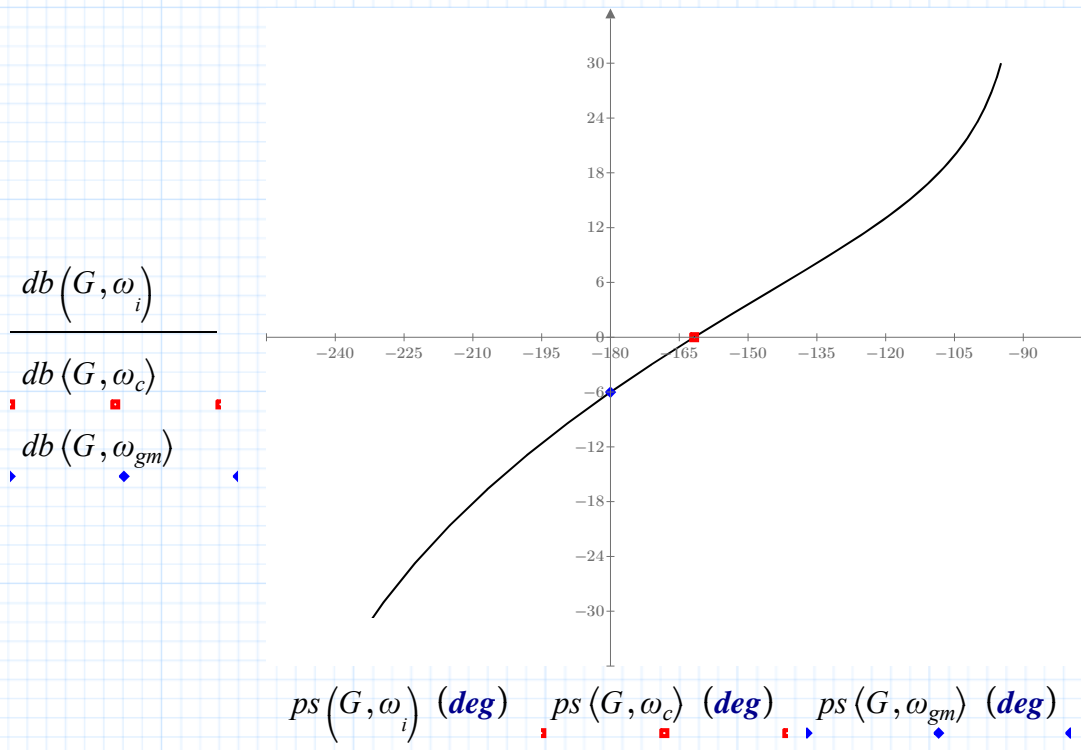


Fig. 5.1.4 Magnitude-Phase plot of open-loop gain

Resonant Frequency and Resonance Peak

Define the closed loop transfer function and plot the closed loop gain as a function of frequency.

$$H(s) := \frac{G(s)}{1 + G(s)}$$

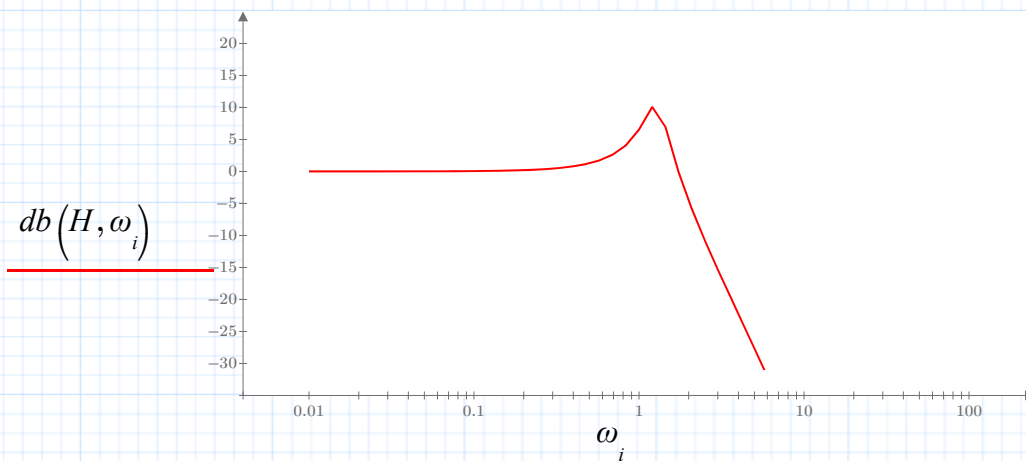


Fig. 5.1.5 Frequency response of closed-loop gain

Estimate the resonant frequency from the plot: this supplies a guess value for the **root** function. To assist you in making the guess, enter an estimate for the frequency at the peak in one of the x-axis marker placeholders. When the guess looks reasonably close, enter it as the value of **ω_r** below:

$$\omega_r := 1.2$$

Solve for the resonant frequency (where the derivative of the magnitude is zero):

$$\omega_r := \text{root} \left(\frac{d}{d\omega_r} |H(1j \cdot \omega_r)|, \omega_r \right) = 1.256$$

If the expression for **ω_r** is not converging properly, try a different guess, to help the root function converge. In general, be sure to check your result with the gain plot above.

Next, calculate resonance peak in dB:

$$M_p := db(H, \omega_r) = 10.393$$

Nichols Chart

This plot resembles Fig. 5.1.4, but also shows a contour of constant closed-loop gain. The functions **Hmag** and **Hphase** generate the necessary magnitude and phase values for plotting on the magnitude-phase plane. The plotting routine is somewhat complex: functionally it uses the symmetry of the contour to help define points for Mathcad's plotting routine.

Definitions for plotting constant closed-loop gain contours:

$$M := 10^{0.05 \cdot Mdb} \quad M := \text{if}(M = 1, 1.001, M)$$

$$H_{mag}(M, \phi, s) := \frac{-M^2 \cdot \cos(\phi) + s \cdot M \cdot \sqrt{M^2 \cdot \cos(\phi)^2 - (M^2 - 1)}}{M^2 - 1}$$

$$H_{phase} := \text{if} \left(M > 1, \left(-\arccos \left(\sqrt{\frac{M^2 - 1}{M^2}} \right) - 180 \text{ deg} \right), -270 \text{ deg} \right)$$

Notice that the expression for **Hmag** is really the root of a quadratic equation. There are two solutions, and the variable **s** determines which is used. This corresponds to the positive and negative halves of the contour on a magnitude-phase plot.

Set the angle at which to start the contour: $\phi_0 := \text{if}(M > 1, -180 \text{ deg}, -360 \text{ deg})$

The variable **d** tells Mathcad which root of the quadratic expression for the magnitude to use in plotting different sections of the contour.

$$d := \text{if}(M < 1, 1, -1)$$

Define step sizes for plotting the contour of constant **H**. For convenience, we define two steps of opposite sign to assist in plotting different portions of the contour.

$$\delta 1 := 0.05 \cdot (H_{\text{phase}} - \phi_0) = -0.922 \text{ deg}$$

$$\delta 2 := \delta 1 \cdot d = 0.922 \text{ deg}$$

Now fill vectors **Mag** and **P** with magnitudes and corresponding phases of the closed-loop gain, **H**:

Vector array numbers:

$$k := 0..20 \quad n := 0..80$$

Magnitude vectors (y-axis):

$$\text{Mag}_k := 20 \cdot \log(H_{\text{mag}}(M, \phi_0 + k \cdot \delta 1, -d))$$

$$\text{Mag}_{k+20} := 20 \cdot \log(H_{\text{mag}}(M, H_{\text{phase}} + k \cdot \delta 2, -1))$$

$$\text{Mag}_{k+40} := \text{Mag}_{40-k}$$

$$\text{Mag}_{60+k} := \text{Mag}_{20-k}$$

Phase vectors (x-axis):

$$P_k := \phi_0 + k \cdot \delta 1$$

$$P_{k+20} := H_{\text{phase}} + k \cdot \delta 2$$

$$P_{k+40} := -180 \text{ deg} + k \cdot \delta 2$$

$$P_{k+60} := -360 \text{ deg} - H_{\text{phase}} + k \cdot \delta 1$$

Now define the magnitude (in dB) of the contour you would like to plot by redefining the variable **Mdb**. This number may be positive or negative. It's defined globally so that you can see its effect on the plot.

$$Mdb \equiv 10$$

Notice that: $M_p = 10.393$

If you wish, you can also mark a frequency value on the plot: enter this value as the definition for **f**.

$$f := \omega_c$$

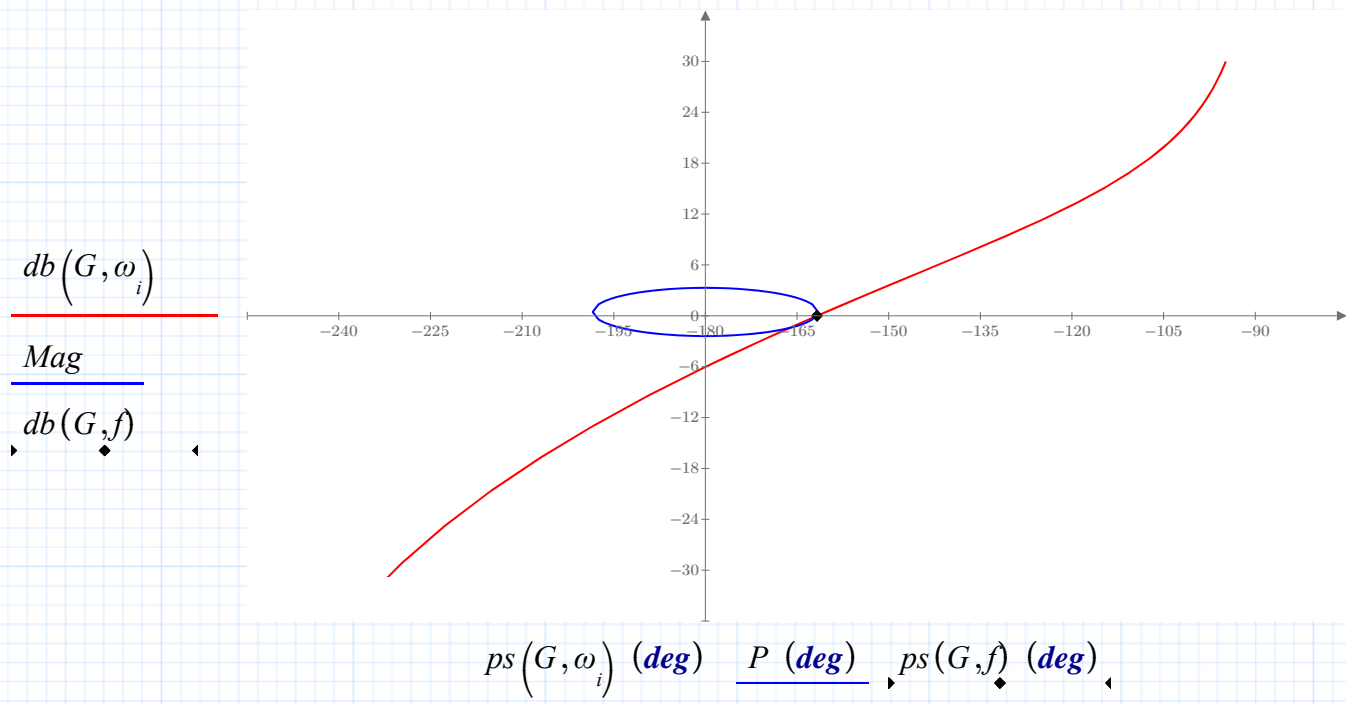


Fig. 5.1.6 Nichols chart