

APPLICATIONS IN CIRCUIT ANALYSIS

Section 6 Two-Port Networks

This document converts among four sets of two-port parameters: **y**, **h**, **z**, and **ABCD**. Parameters in any one of these sets can be converted to any other set by casting them in a two-by-two matrix. Conversions are given by a transformation that accepts the given matrix, and returns the converted parameters in the same form. You provide:

- **y**, **z** or **ABCD**, the matrix parameters,
- **i1** and **v2**, or **i2** and **v1**, the known currents and voltages.

References

The conversion formulas and definitions used are given in Kaufman and Wilson, *Theory and Problems of Electronics Technology*, McGraw-Hill (New York, 1982). See also William H. Hayt and Jack E. Kemmerly, *Engineering Circuit Analysis*, McGraw-Hill (New York, 1978).

Background

A two port network is any circuit which can be connected through more than one pair of nodes, as follows:

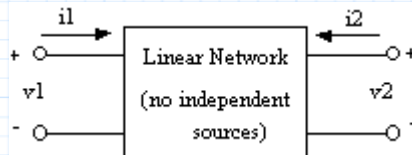


Fig. 6.1 A two-port network

Four sets of parameters are commonly used to describe two-port networks: **y**, **z**, **h**, and **ABCD** (or **a**) parameters. These parameters are expressions of the voltage and current laws for circuits; they account for the complexity added by the additional pair of addressable nodes. Two-port parameters are used to solve systems of simultaneous equations which describe the circuit. Each parameter relates known voltage and current quantities to unknown quantities. Any parameter may be transformed into another, thus completely specifying the circuit.

The four parameters examined in this document are the short-circuit admittance parameters, the open-circuit impedance parameters, the hybrid parameters relating input current and output voltage, and **ABCD** parameters relating short and open-circuit characteristics.

Mathcad Implementation

To use the conversion functions in this document, enter the matrix of parameters and apply the appropriate conversion function, as illustrated in the examples. Each function takes a two-by-two matrix as its input and returns another two-by-two parameter matrix as output. These functions allow you to convert a given set of two-port parameters into the set that is most convenient for a particular application.

The voltages and currents at the two ports of the network are given in terms of the parameter matrices **y**, **h**, **z**, and **a** by the following relations:

y parameters:

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = y \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

z parameters:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = z \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

h parameters:

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = h \cdot \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

$$h = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix}$$

where

h_i = short-circuit input impedance

h_f = short-circuit forward-transfer current ratio

h_r = open-circuit reverse-transfer voltage ratio

h_o = open-circuit output admittance

ABCD parameters

$$a = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} v_s \\ i_s \end{bmatrix} = a \cdot \begin{bmatrix} v_r \\ i_r \end{bmatrix}$$

A = inverse open-circuit voltage gain

B = short-circuit reverse-transfer impedance

C = open-circuit forward-transfer admittance

D = inverse short-circuit current gain

In each of the following, the function name indicates the conversion it carries out; for example **YZ(y)** converts from **y** parameters to **z** parameters, and **AH(a)** converts from **ABCD** parameters to **h** parameters.

Conversions from y to z, h, and ABCD

$$YZ(y) \equiv \frac{1}{\|y\|} \cdot \begin{bmatrix} y_{1,1} & -y_{0,1} \\ -y_{1,0} & y_{0,0} \end{bmatrix}$$

$$YH(y) \equiv \frac{1}{y_{1,1}} \cdot \begin{bmatrix} 1 & -y_{0,1} \\ y_{1,0} & \|y\| \end{bmatrix}$$

$$YA(y) \equiv \frac{1}{y_{1,0}} \cdot \begin{bmatrix} -y_{1,1} & -1 \\ -\|y\| & -y_{0,0} \end{bmatrix}$$

Conversions from z to y, h, and ABCD

$$ZY(z) \equiv \frac{1}{\|z\|} \cdot \begin{bmatrix} z_{1,1} & -z_{0,1} \\ -z_{1,0} & z_{0,0} \end{bmatrix}$$

$$ZH(z) \equiv \frac{1}{z_{1,1}} \cdot \begin{bmatrix} \|z\| & z_{0,1} \\ -z_{1,0} & 1 \end{bmatrix}$$

$$ZA(z) \equiv \frac{1}{z_{2,1}} \cdot \begin{bmatrix} z_{0,0} & \|z\| \\ 1 & z_{1,1} \end{bmatrix}$$

Conversions from h to z, y, and ABCD

$$HZ(h) \equiv \frac{1}{h_{2,2}} \cdot \begin{bmatrix} \|h\| & h_{0,1} \\ -h_{1,0} & 1 \end{bmatrix}$$

$$HY(h) \equiv \frac{1}{h_{0,0}} \cdot \begin{bmatrix} 1 & -h_{0,1} \\ h_{1,0} & \|h\| \end{bmatrix}$$

$$HA(h) \equiv \frac{-1}{h_{1,0}} \cdot \begin{bmatrix} \|h\| & h_{0,0} \\ h_{1,1} & 1 \end{bmatrix}$$

Conversions from ABCD to y, h, and z

$$AY(a) \equiv \frac{1}{a_{0,1}} \cdot \begin{bmatrix} a_{1,1} & -\|a\| \\ -1 & a_{0,0} \end{bmatrix}$$

$$AH(a) \equiv \frac{1}{a_{1,1}} \cdot \begin{bmatrix} a_{0,1} & \|a\| \\ -1 & a_{1,0} \end{bmatrix}$$

$$AZ(a) \equiv \frac{1}{a_{1,0}} \cdot \begin{bmatrix} a_{0,0} & \|a\| \\ 1 & a_{1,1} \end{bmatrix}$$

y parameters into z and ABCD parameters

Enter the y parameters in the matrix y:

$$y := \begin{bmatrix} 2.55 \cdot 10^{-3} & 6.00 \cdot 10^{-5} \\ .3 & 2.30 \cdot 10^{-6} \end{bmatrix}$$

Calculate z parameters:

$$YZ(y) = \begin{bmatrix} -0.128 & 3.334 \\ 1.667 \cdot 10^4 & -141.713 \end{bmatrix}$$

Calculate ABCD parameters:

$$YA(y) = \begin{bmatrix} -7.667 \cdot 10^{-6} & \\ & \ddots \end{bmatrix}$$

Given i_1 and v_2 , find i_2 and v_1 using h parameters

Set values for i_1 and v_2

$$i_1 := 2 \qquad v_2 := 20$$

Calculate i_2 and v_1 using the y parameters above to calculate the h parameters.

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} := YH(y) \cdot \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \qquad \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 8.69 \cdot 10^5 \\ 2.607 \cdot 10^5 \end{bmatrix}$$



It may be necessary or desirable to describe the network in terms of complex impedances, which will necessarily enter into network parameters. It is possible to define any parameter matrix as a function of the frequency, but Mathcad will insist that you define the value of the frequency in future calculations. For example:

$$y(\omega) := \begin{bmatrix} 2.55 \cdot 10^{-3} + \frac{1}{1j \cdot \omega} & 6.00 \cdot 10^{-5} \\ .3 & 2.30 \cdot 10^{-6} + \frac{1}{1j \cdot \omega} \end{bmatrix}$$

is consistent with Mathcad notation, but to display $YH(y)$, you must define:

$$\omega := 1 \cdot 10^4$$

$$YH(y(\omega)) = \begin{bmatrix} 229.878 + 9.995i \cdot 10^3 & -0.014 - 0.6i \\ 68.964 + 2.998i \cdot 10^3 & -0.002 - 0.18i \end{bmatrix}$$

You can use the following transformation to convert voltages and currents into their magnitudes and phases.

$$P(A) := \begin{bmatrix} |A| & \frac{\arg(A)}{\text{deg}} \end{bmatrix}$$

Also, because of Mathcad's ability to handle large matrix calculations, it is easy to implement parameter transformations for multi-port networks. Additional matrix elements can be added by selecting **Mathcad Ribbon: Matrices/Tables Tab: Insert/Delete Row/Column buttons**.