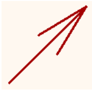
mk_VField Make Vector Field for plotting in 2D and in Prime 2

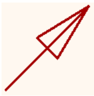
Arrowheads:

$ah\_std := (0 \quad -1+0.25i \quad 0 \quad -1-0.25i)^T$



$ah\_cls := (-1 \quad -1+0.25i \quad -1-0.25i \quad 0 \quad -1+0.25i)^T$

$ah\_str := (0 \quad -1+0.25i \quad -1-0.25i \quad 0)^T$



Narrower heads

$ah\_std\_narr := (0 \quad -1+0.15i \quad 0 \quad -1-0.15i)^T$

$ah\_cls\_narr := (-1 \quad -1+0.15i \quad -1-0.15i \quad 0 \quad -1+0.15i)^T$

$ah\_str\_narr := (0 \quad -1+0.15i \quad -1-0.15i \quad 0)^T$

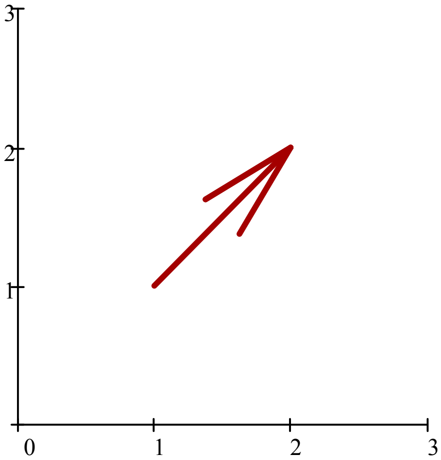
$\_ah := ah\_std$

$\_length := 50\%$ 50% ... length of arrowhead is 50% of vector lenght

Method to draw arrows was posted once by Tom Gutman

$$\_Vec(P,V) := \begin{array}{|l} r \leftarrow stack(P, P2 \leftarrow P+V) \\ r \leftarrow stack[P, P2 + \_ah\cdot((V\cdot\_length))] \quad \text{if } V \neq 0 \\ \text{return } stack(r, NaN) \end{array}$$

$TP := 1 + 1\cdot j \qquad TV := 1 + 1\cdot j$



Line thickness and color can be adjusted using the usual 2D formatting menu.

Parameter M can be:

1. A matrix consisting of complex numbers, representing x and y components of the vectors.
   These vectors are related to the points represented by the matrix indices. Thats the same behaviour as the Vector Field Pot in Mathcad 15.
2. A column or a row vector consisting of two matrices: the first consisting of the vectors represented by complex numbers as in 1. and the second consisting of the related points, also represented by complex numbers.

```
mk_VField(M) := | if IsArray(M[ORIGIN,ORIGIN])
                |   | M ← M^T  if cols(M) > 1
                |   | P ← M[ORIGIN+1]
                |   | M ← M[ORIGIN]
                | otherwise
                |   | P ← M
                |   | for r ∈ ORIGIN..ORIGIN + rows(M) − 1
                |   |   for c ∈ ORIGIN..ORIGIN + cols(M) − 1
                |   |     P[r,c] ← r + c·i
                | R ← "dummy"
                | for r ∈ ORIGIN..ORIGIN + rows(M) − 1
                |   for c ∈ ORIGIN..ORIGIN + cols(M) − 1
                |     R ← stack(R, _Vec(P[r,c], M[r,c]))
                | return submatrix(R, ORIGIN + 1, last(R), ORIGIN, ORIGIN)
```

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Example: Simple vector field

$vec(x,y) := x + j \cdot y$ function which returns the vector to be drawn at (x/y)

$f(x,y) := \begin{pmatrix} x \\ y \end{pmatrix}$ If you think of vectors rather than of complex numbers you may define

$vec(x,y) := f(x,y)^T \cdot \begin{pmatrix} 1 \\ j \end{pmatrix}$

$xmin := -10$ $xmax := 10$

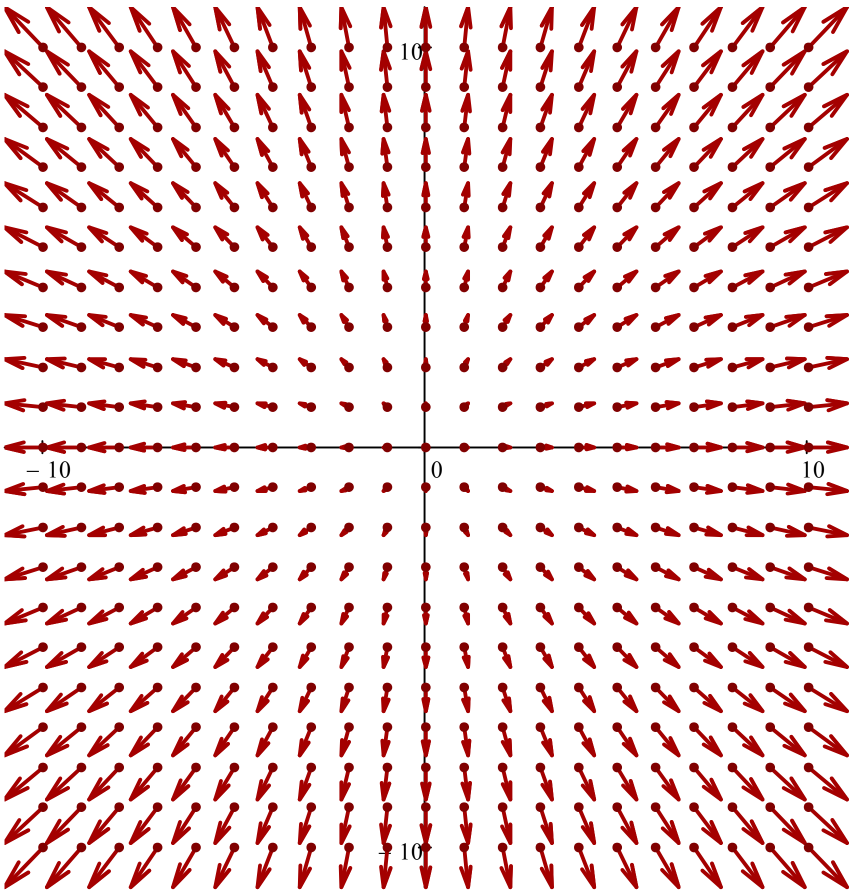
$ymin := -10$ $ymax := 10$ $scale := 0.1$

$ix := 0 .. xmax - xmin$ $iy := 0 .. ymax - ymin$

$V := 0$ $V_{ix,iy} := vec(ix + xmin, iy + ymin)$ $V := scale \cdot V$

$P := 0$ $P_{ix,iy} := ix + xmin + (iy + ymin) \cdot j$

$VF := mk\_VField\left(\begin{pmatrix} V \\ P \end{pmatrix}\right)$



Example: Simple vector field

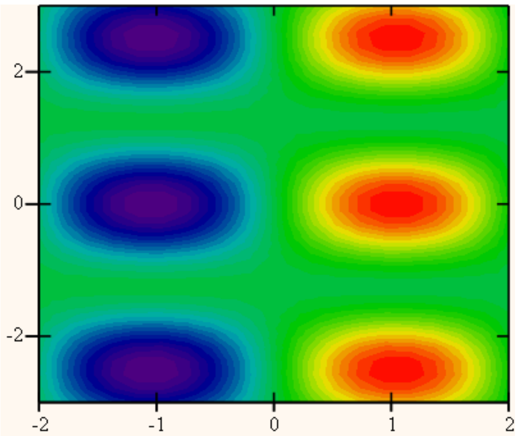
Example: Gradient Field

$f(x,y) := 22\sin\left(\frac{3}{2}x\right)\cdot\cos\left(\frac{5}{4}y\right)^2$

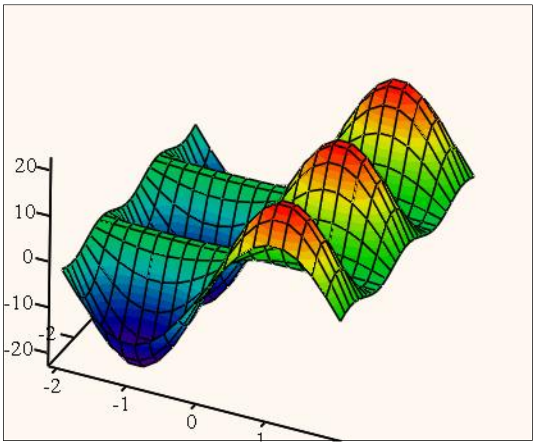
x range: $x0 := -2$ $x1 := 2$ x grid: $xg := (x1 - x0)\cdot 5$

y range: $y0 := -3$ $y1 := 3$ y grid: $yg := (y1 - y0)\cdot 5$

$f\_mesh := CreateMesh(f, x0, x1, y0, y1, xg, yg)$



f_mesh



f_mesh

$gradi(x,y) := \left(\nabla_{x,y} f(x,y)\right)^T \cdot \begin{pmatrix} 1 \\ j \end{pmatrix}$

creates a complex number the components of which are the values of the two partial derivatives.

$gradi(x,y) := \begin{pmatrix} \frac{\partial}{\partial x} f(x,y) & \frac{\partial}{\partial y} f(x,y) \end{pmatrix} \cdot \begin{pmatrix} 1 \\ j \end{pmatrix}$

As Prime does not support the gradient operator, we define it the lengthy way

$xwidth := \frac{x1 - x0}{xg - 1}$ $ywidth := \frac{y1 - y0}{yg - 1}$ width of the mesh points

$ix := 0 .. xg - 1$ $iy := 0 .. yg - 1$ indices

$xp := 0$ $xp_{ix} := x0 + ix \cdot xwidth$ $yp := 0$ $yp_{iy} := y0 + iy \cdot ywidth$ coordinates of the mesh points

$P := 0$ $P_{ix,iy} := xp_{ix} + j \cdot yp_{iy}$ related points (mesh points)

$V := 0$ $V_{ix,iy} := gradi\left(xp_{ix}, yp_{iy}\right)$ vectors to be drawn

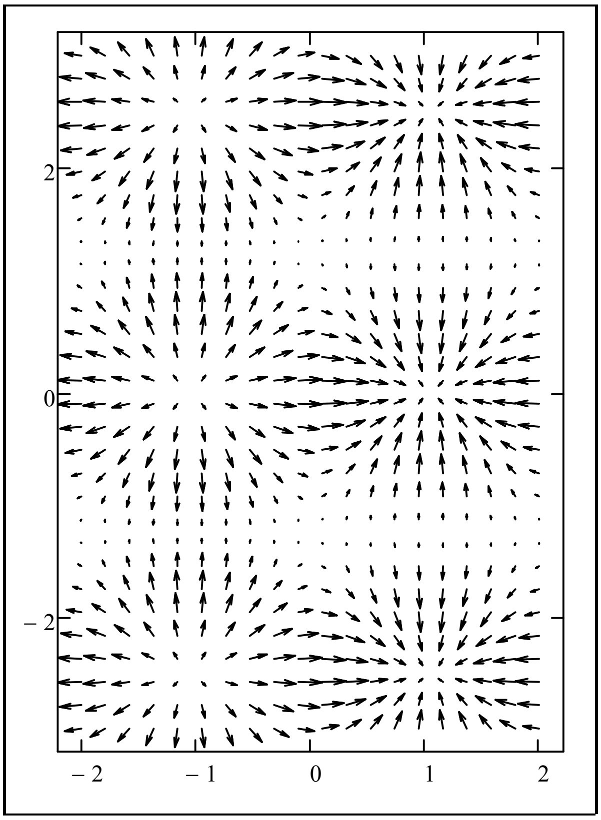
$V := V \cdot \min\left(\frac{xwidth}{\max\left(\overrightarrow{|Re(V)|}\right)}, \frac{ywidth}{\max\left(\overrightarrow{|Im(V)|}\right)}\right)$

scale V so, that no component of a vector is greater than the related mesh width.

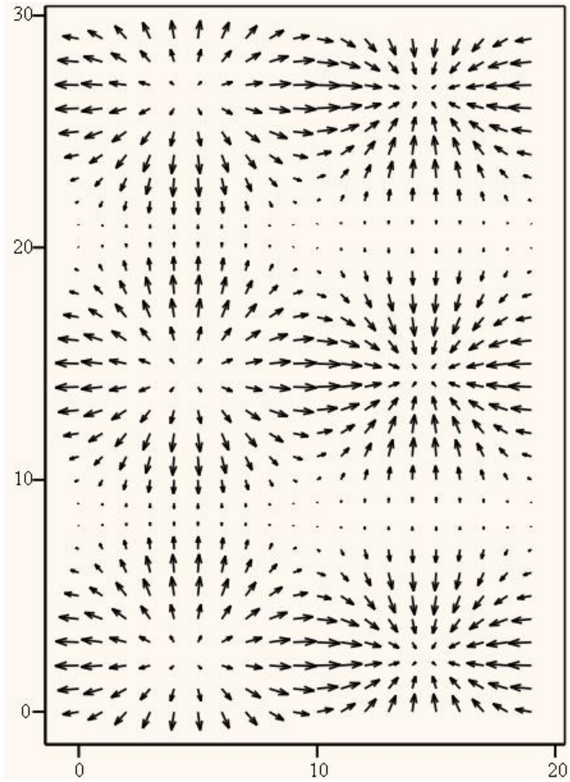
$VF := mk\_VField((V \;\; P))$

Gradient field of f(x,y):

2D graph using "mk_VField



Built-in Vector Field Plot of Mathcad 15. Note that the numbering does not reflect the actual coordinates.

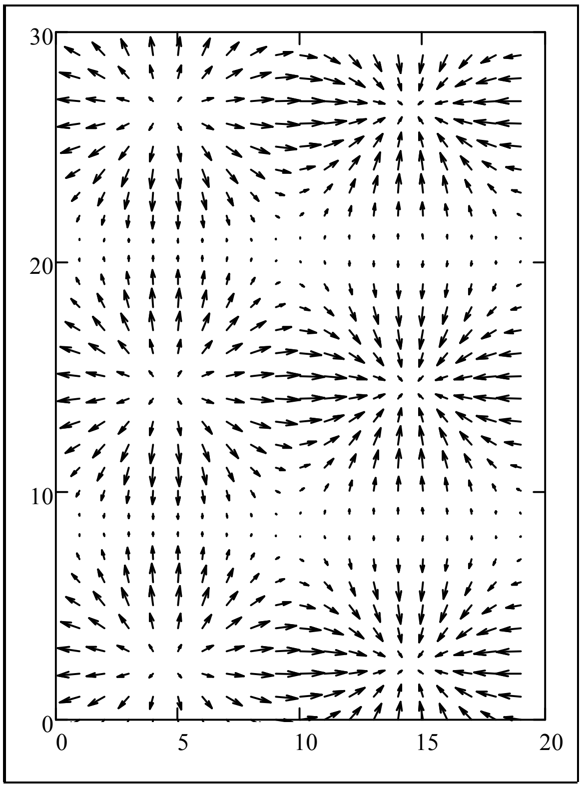


Trim for meshwidth 1 in both directions

$$V := V\cdot\min\left(\frac{1}{\max\left(\left|\overrightarrow{\mathrm{Re}(V)}\right|\right)},\frac{1}{\max\left(\left|\overrightarrow{\mathrm{Im}(V)}\right|\right)}\right)$$

We can duplicate the behaviour of MC 15 by simply not providing the matrix of point coordinates

$$VF2 := \mathrm{mk\_VField}(V)$$



Example: Gradient Field

Example: Slope field of a differential equation

Given

$$\frac{d}{dt}u(t) = \cos(u(t)) + \sin(t) \qquad u(-5) = a$$

$ul(a) := \mathrm{Odesolve}(t, 5)$

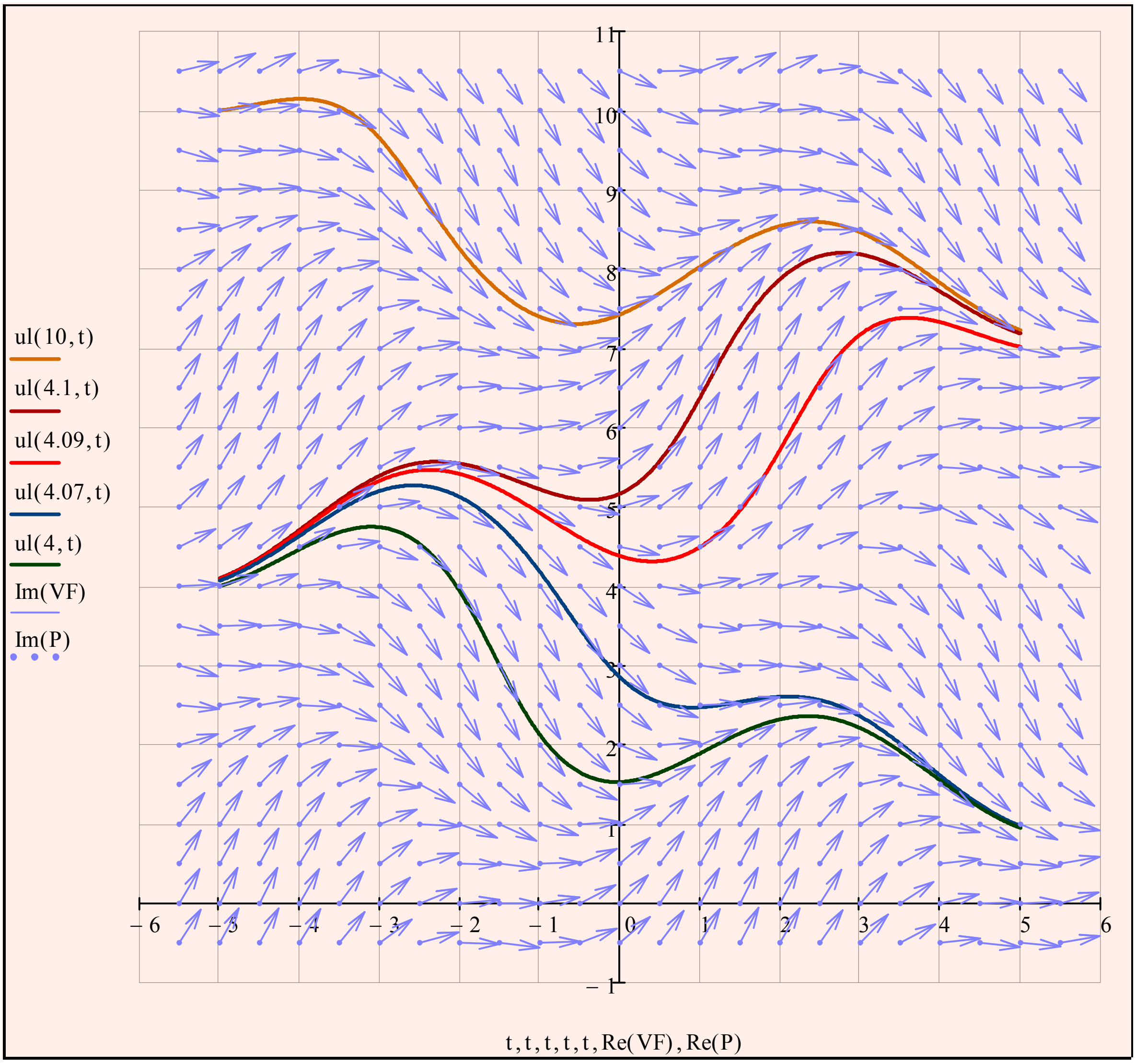
$u(z) := \cos(\mathrm{Im}(z)) + \sin(\mathrm{Re}(z))$

$i := 0..22 \qquad j := 0..22$

$P := 0 \qquad P_{i,j} := \frac{i}{2} - \frac{11}{2} + \left(\frac{j}{2} - \frac{1}{2}\right)\cdot j$

$$V := 0 \qquad V_{i,j} := \frac{1 + u(P_{i,j})\cdot j}{\left|1 + u(P_{i,j})\cdot j\right|} \qquad V := V\cdot \min\left(\frac{0.5}{\max\left(\overrightarrow{|\mathrm{Re}(V)|}\right)}, \frac{0.5}{\max\left(\overrightarrow{|\mathrm{Im}(V)|}\right)}\right)$$

$VF := \mathrm{mk\_VField}((V \;\; P))$



Example: Slope field of a differential equation