1 POWER DISTRIBUTION

Section 1.3a Load Flow Calculations

Section 1.3.1 Introduction

Load flow calculations are used to determine the voltage, current, and real and reactive power at various points in a power system under normal steady-state conditions. For power systems with a large number of buses, the load flow problem becomes computationally intensive. Therefore, for large power systems, the load flow is solved using specific programs based on iterative techniques, such as the Newton-Raphson method. Power systems of smaller size, however, require considerably less computational effort, and load flow algorithms can be developed which function easily on personal computers.

Mathcad provides the tools for solving load flow problems for relatively small power systems (approximately 10 buses) with reasonable solution times and convergence characteristics. This development is particularly useful for parametric studies. These may include investigation of alternative system topologies, or various values of reactive power compensation to achieve desired system performance. The interactive environment of Mathcad allows the user to quickly change system parameters and configuration, and observe the effect on the system operating conditions. Upon arriving at a satisfactory solution, the user may verify the result with a standard load flow program. This procedure will considerably reduce the effort and calculation time required to solve the problem, since for most standard load flow programs the input process is time consuming. The algorithm provided here can apply to distribution networks, and is useful for investigating line losses, bus voltages and line voltage drop under various loading conditions, compensation schemes and distribution network topology.

The approach used here for solving the load flow is based on the Newton-Raphson iterative method. The required input to the problem is the generated and load power at each bus and the voltage magnitude on generating buses. This information is acquired from load data and the normal system operating conditions. The solution provides the voltage magnitude and phase angle at all buses and the power flows and losses of the transmission lines. For an extended discussion of the Newton-Raphson method refer to *Elements of Power System Analysis*, 4th edition, by W.D. Stevenson, McGraw-Hill, 1982.

Section 1.3.2 Network Description

For load flow calculations, the system buses are classified into three types:

The slack bus: There is only one such bus in the system. Due to losses in the network, the real and reactive power cannot be known at all buses. Therefore, the slack bus will provide the necessary power to maintain the power balance in the system. The slack bus is usually a bus where generation is available. For this bus, the voltage magnitude and phase angle are specified (normally the voltage phase angle is set to zero degrees). The voltage phase angle of all other buses is expressed with the slack bus voltage phasor as reference.

The generating or PV-bus: This bus type represents the generating stations of the system. The information known for PV-buses is the net real power generation and bus-voltage magnitude. The net real power generation is the generated real power minus the real power of any local load.

The load or PQ-bus: For these buses, the net real and reactive power is known. PQ-buses normally do not have generators. However, if the reactive power of a generator reaches its limit, the corresponding bus is treated as a PQ-bus. This is equivalent to adjusting the bus voltage until the generator reactive power falls within the prescribed limits.

Distribution substations and feeders may be treated as generating buses in distribution networks.

The load flow equations are written in terms of the net power injection to each bus. With reference to Figure 1.3.1, the net power injection into the kth bus is the combination of generated and load power. The power flowing out of this bus must equal the net injected power. Therefore, the power balance equation at the kth bus is written as follows in terms of the system voltage.

$$P_k - jQ_k = \left[Y_{k,k} \cdot V_k + \sum_{i=1..N} \left(Y_{k,i} \cdot V_i \right) \right] \cdot \overline{V_k} \qquad \textbf{(1.3.1)}$$

where

N is the number of network buses,
Pk is the net real power injected into the kth bus,
Qk is the net reactive power injected into the kth bus,
Yk,i is the total admittance between bus k and i: this total can be found from the bus admittance matrix, Ybus, of the system,
Vi is the voltage of the ith bus.

The values of network admittances and voltages in Equation (1.3.1) are expressed in the pu system to facilitate the calculations. **To review the pu system definitions, see Section 1.1**.

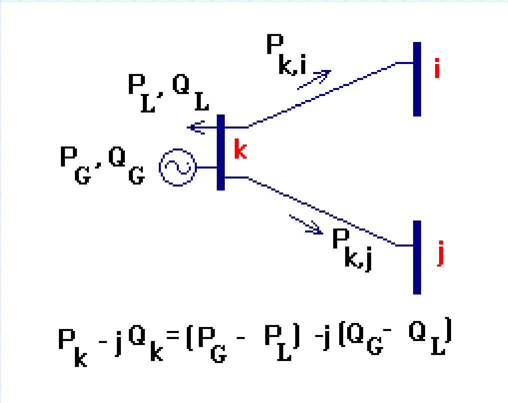

Fig. 1.3.1 Power flow through system

Equation (1.3.1) can be written separately for the real and reactive power.
Therefore,

$$P_k = \sum_n \left(\left| V_k \cdot V_n \cdot Y_{k,n} \right| \cdot cos\left(\theta_{k,n} + \delta_n - \delta_k \right) \right)$$

(1.3.2)

$$Q_k = \sum_n \left(\left| V_k \cdot V_n \cdot Y_{k,n} \right| \cdot sin\left(\theta_{k,n} + \delta_n - \delta_k \right) \right)$$

where θk,n is the angle of the admittance, Yk,n, and δj is the voltage phase angle at bus, j.

A real power equation is written for every PV- and PQ-bus and a reactive power equation is written for every PQ-bus. Thus, for a power system with N buses of which L are PQ-buses there are (N-1) real power equations (excluding the slack bus) and L reactive power equations (a total of N-1+L equations). The unknowns are the magnitude and phase angle of the L PQ-bus voltages and the phase angle of the (N-1-L) PV-bus voltages (a total of N-1+L unknowns).

The left-hand side of these equations are known and an iterative process is used for finding the unknown voltages and phase angles such that Equations (1.3.2) are balanced.

Section 1.3.3 The Newton-Raphson Method

The Newton-Raphson method provides a reliable approach for solving non-linear equations such as Equations (1.3.2). The main advantages of this method are its convergence characteristics and its speed. The procedure for applying the Newton-Raphson method is as follows:

From the network configuration and parameters the **bus-admittance matrix is constructed**. The elements of this matrix are used to calculate the power flows according to Equations (1.3.2).

Each network bus is assigned a type and, accordingly, information about the bus real and reactive power and bus voltage is collected.

From the above steps, the load flow equations can be assembled into the following form, with reference to Equations (1.3.2).

$$\begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} f_p(\delta, V) \\ f_q(\delta, V) \end{bmatrix} \qquad \textbf{(1.3.3)}$$

where

P is the vector of the known net real power injections at PV- and PQ-buses,
Q is the vector of the known reactive power injections at PQ-buses,
V is the vector of the unknown bus voltage magnitudes,
δ is the vector of the unknown bus voltage phase angles, and
fp, fq are functions defined according to Equations (3.1.2).

Solution of the load flow problem requires finding the values of V and δ such that the right-hand side of Equation (1.3.3) equals the known power injections at the network buses. For any estimation of V and δ, the difference between the known power injections, P and Q and the power injections calculated by Equation (1.3.3) is called the power mismatch.

$$\Delta S = \begin{bmatrix} P - f_p(\delta, V) \\ Q - f_q(\delta, V) \end{bmatrix} \qquad \textbf{(1.3.4)}$$

where ΔS is the net real and reactive power mismatch:

$$\Delta S = \begin{bmatrix} \Delta P \\ DQ \end{bmatrix}$$

The power mismatch is a measure of how close to the solution the estimations of V and δ are. A correction to these estimations is obtained using the Newton-Raphson method, resulting in an iterative calculation process.

$$\delta^{\langle j+1 \rangle} = \delta^{\langle j \rangle} + J^{-1} \cdot \Delta S^{\langle j \rangle}$$

$$\textbf{(1.3.5)}$$

$$V^{\langle j+1 \rangle} = V^{\langle j+1 \rangle} + J^{-1} \cdot \Delta S^{\langle j \rangle}$$

where the superscript, j, denotes variables calculated at the jth iteration step. J is the Jacobian matrix of Equations (1.3.3):

$$J = \begin{bmatrix} \frac{d}{d\delta} f_p & \frac{d}{dV} f_p \\ \frac{d}{d\delta} f_q & \frac{d}{dV} f_q \end{bmatrix} \qquad \textbf{(1.3.6)}$$

At the beginning of a new iteration, (j+1), the power mismatch is calculated using Equation (1.3.4) with the variables V and δ obtained from the previous iteration, j.

From the calculated mismatches, a new approximation to the system solution is found using Equation (1.3.5).

The iteration process continues until the power mismatch at the jth step is smaller than a preset number ε.

$$\sum_{n=2..N} \left[(\Delta P_n)^{\langle j \rangle} + (\Delta Q_n)^{\langle j \rangle} \right]^2 \le \varepsilon$$

To start the above iterative solution, an estimation of the unknown voltages and their phase angles is required. This first solution approximation is called initial guess. Typically, the initial guess for the voltage magnitudes is 1 pu and for their phase angles is 0 degrees (or radians).