Section 1.5a Power System Harmonic Analysis

Section 1.5.1 Introduction

Modern power systems serve a wide variety of loads. Almost all of these load types exhibit some non-linear response to sinusoidal excitation. Nonlinear loads include power electronic devices that switch the current to loads and generate high levels of harmonics. These harmonics interact with the system inductances, capacitances, and resistances and cause distortions to the voltage and current waveforms. These distortions interfere with the normal operation of the power system.

Harmonic currents created by variable-speed drives and power inverter systems interact with the system impedances to produce voltage distortions. Excessive voltage distortions cause malfunctions in sensitive electronic equipment and in the power conversion equipment itself. Also, the additional harmonic currents add to the existing fundamental frequency load current which, in turn, adds to the losses in the system. Excessive harmonic content in power circuits requires special transformers and over-sized conductors to prevent overloading by the harmonics.

The identification and mitigation of harmonic sources is especially important with the increased use of nonlinear loads in commercial and industrial power systems. Most of these loads are sensitive to high harmonic content in addition to being harmonic sources themselves. If non-linear loads will be added to an existing system, and they will generate harmonics in the system, it is important to examine the possible damaging effects on the system as a whole. This section lists possible harmonic sources, their frequency content, and a method of identifying harmonic interaction between a source and a system.

Section 1.5.2 Harmonic Sources

Harmonics are integer multiples of the fundamental frequency in an electrical network. In power systems, the fundamental frequency is 60 Hz. Power system loads that exhibit a nonlinear response to the sinusoidal excitation generate the harmonics by producing non-sinusoidal currents or voltages.

System harmonics are identified by their harmonic order in the frequency domain and by their phase sequence. (To learn more about system sequence impedances, see **Section 1.6b**)   The phasor domain representation classifies harmonics into three sequences--positive, negative, and zero. Equations (1.5.1) give the relationship between the harmonic order and the phase sequence.

$f := 60$ $h := 1 .. 10$ harmonic order

$f1_h := (3 \cdot h + 1) \cdot f$ positive sequence $f2_h := (3 \cdot h - 1) \cdot f$ negative sequence

$f0_h := (3 \cdot h) \cdot f$ zero sequence   **(1.5.1)**

Different types of loads generate different harmonics. These harmonics can be of all orders, h, in practical situations.

Harmonic Sources

Two factors contribute to harmonic generation on power systems: unbalances, and nonlinearity. Unbalances combined with nonlinear elements, such as power electronic devices, generate the zero and negative sequence harmonics. The unbalances can be temporary, such as system faults, or steady-state. Steady-state unbalance is due to unequal load distributions on the three phases of the power system. The nonlinearities in a power system come from the magnetic circuits in rotating machinery and transformers, and from power switching devices such as those in variable speed motor drives. (**To see the effect of transformer saturation on system voltage see Section 3.2b**).

Under normal circumstances, the nonlinearity of the magnetic circuits contributes only small amounts of harmonic content to a power system. Operating these devices outside their rated voltage values causes magnetic saturation in the devices which increases the harmonic production. In proper operation, these devices only produce a harmonic content of 3-5% of the devices rating. This content is usually below the industry guidelines outlined in IEEE standards, [1].

The major source of harmonics in power systems is power conversion equipment. This equipment includes high-voltage dc power transmission stations, ac-dc converters, and variable speed ac and dc motor drive systems. All these devices use power electronic devices to switch ac currents into dc currents. The resulting ac side currents are non-sinusoidal and have a high harmonic content.

Figure 1.5.1 shows the structure of a 6-pulse thyristor ac-dc power converter.

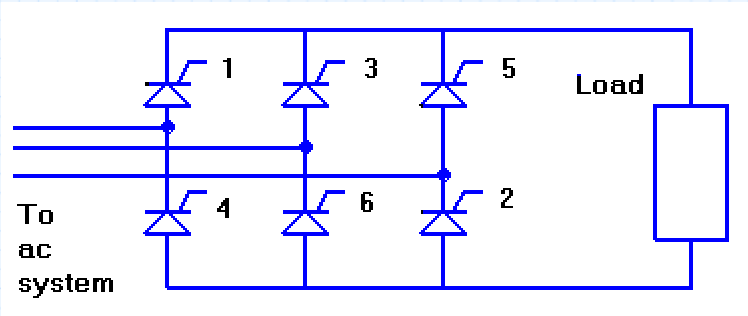

Figure 1.5.1 Six-pulse ac-dc power converter

The structure in Figure 1.5.1 is a six-pulse, phase-controlled ac-dc converter. The thyristors switch the ac current to produce a dc current on the load. Delaying the firing pulses on the gates of the 6 thyristors creates a variable dc output. Delaying the firing time of the gate pulse also causes the converter to have a lagging power factor when viewed from the ac side of the system. This device generates characteristic current harmonics on the ac and dc side of the circuit.

Characteristic Harmonics of Phase-Controlled Converters

The characteristic harmonics of thyristor-controlled converters are a function of the pulse number of the converter. A converter of pulse number, p, generates harmonic currents with characteristic orders given by Equations (1.5.2).

$q := 1..5$ integer range representing harmonic order

$p := 6$ pulse number of converter

$h_{dc_q} := p \cdot q$ characteristic dc side harmonics

$h_{ac1_q} := p \cdot q + 1$

characteristic ac side positive and negative sequence harmonics **(1.5.2)**

$h_{ac2_q} := p \cdot q - 1$

Characteristic harmonic orders produced by a 6-pulse thyristor converter.

$$h_{dc_q} = \begin{bmatrix} 6 \\ 12 \\ 18 \\ 24 \\ 30 \end{bmatrix} \qquad h_{ac1_q} = \begin{bmatrix} 7 \\ 13 \\ 19 \\ 25 \\ 31 \end{bmatrix} \qquad h_{ac2_q} = \begin{bmatrix} 5 \\ 11 \\ 17 \\ 23 \\ 29 \end{bmatrix}$$

The magnitude of the fundamental frequency converter current determines the magnitude of the current harmonics on the ac side of the converter. A Fourier analysis of the converter current waveforms shows that the magnitude of these harmonic currents decreases as the order of the harmonic increases. This relationship is given by Equations (1.5.3).

$I_1 := 1$ given full-load converter current in per unit at the fundamental frequency.

$I_{p_q} := \dfrac{I_1}{h_{ac1_q}}$ positive sequence harmonic current magnitude. **(1.5.3)**

$I_{n_q} := \dfrac{I_1}{h_{ac2_q}}$ negative sequence harmonic current magnitude.

Theoretical per unit harmonic current magnitudes for a 6-pulse converter

$$I_{p_q} = \begin{bmatrix} 0.143 \\ 0.077 \\ 0.053 \\ 0.04 \\ 0.032 \end{bmatrix} \qquad I_{n_q} = \begin{bmatrix} 0.2 \\ 0.091 \\ 0.059 \\ 0.043 \\ 0.034 \end{bmatrix}$$

Typical Harmonic Current Values

The harmonic current magnitude values produced from Equation (1.5.3) are higher than values measured on converters in operation. This increase is due to simplifying assumptions made in the analysis. These assumptions are

1. the converter voltages are three-phase, balanced, and sinusoidal;
2. the dc current is constant and is filtered with an inductor of infinite size;
3. the thyristors all conduct for exactly 1/6th of the period;
4. the system source impedance at the converter terminals is balanced.

When these factors are included, the typical values of per unit (pu) harmonic current are

$$h_{ac1_q} := \begin{bmatrix} 0.111 \\ 0.029 \\ 0.010 \\ 0.008 \end{bmatrix} \qquad h_{ac2_q} := \begin{bmatrix} 0.175 \\ 0.045 \\ 0.015 \\ 0.009 \end{bmatrix}$$

Including the non-ideal converter factors also produces uncharacteristic harmonics on the ac side of the converter. If the converter operation is unsymmetrical, uncharacteristic harmonics can appear on the ac side and interact with the system impedance to excite resonant modes in the power system. All practical converters produce some uncharacteristic harmonics. Therefore, care must be taken when applying converters to existing power systems.

Uncharacteristic harmonics (UCHs) can interact with converter controls to produce unstable control operation. The UCHs also cause shifts in the zero crossing of the voltage waveforms which causes unequal firing of the converter thyristors. This firing produces even more uncharacteristic harmonics.

For more information on the harmonic interaction with power converters see reference [2].

Section 1.5.3 Harmonic Interaction with Systems

Both characteristic and uncharacteristic current harmonics interact with the system inductances, capacitances and resistance to produce harmonic distortion in the voltage waveform. Determining the impact of a proposed converter installation on a power system requires that the harmonic impedances of a system be found. Plotting the impedances found from this study against frequency will detect any inherent system resonance that the converter injections may excite. Applying harmonic filters to the offending harmonic currents will prevent any serious operational problems with the installation.

To determine the harmonic impedance of a system requires the repeated construction of the system Zbus matrix for a range of frequencies (To see a Mathcad implementation of the Zbus matrix, see **Section 2.1b**). Since converters generate a limited number of harmonic frequencies that diminish rapidly in magnitude as the order increases, the highest frequency of interest can be 1800 Hz. This frequency corresponds to the 30th harmonic of 60 Hz. At frequencies higher than 1800 Hz, the current magnitude is so small that the effects are negligible in most cases

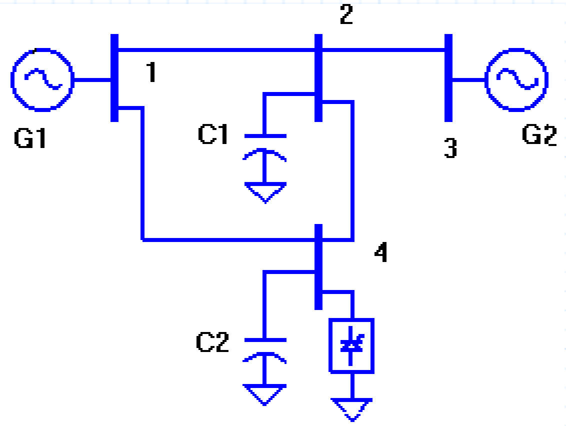

Fig. 1.5.2 Harmonic test system

Figure 1.5.2 shows a typical distribution system. Installation of a six-pulse converter on bus 4 is planned. The harmonic impact on this section of the system is required. Generators G1 and G2 represent the Thevenin equivalent of the network beyond bus 1 and bus 2 respectively. This formulation is ideal for quick analysis of the local impact of the installation of harmonic sources in distribution systems.

Problem Formulation

To determine the impact of installing the converter, the harmonic impedances must first be determined. Determining the harmonic impedances requires the repetitive computation of the Zbus matrix as a function of frequency. Use of a generalized circuit element simplifies the computation of the Zbus matrices required. Figure 1.5.3 shows the schematic for the generalized circuit element used to represent all branches in the system of Figure 1.5.2.

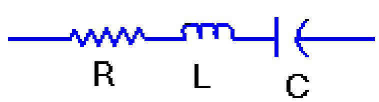

Fig 1.5.3 Generalized circuit element

With this representation, the branch admittance for each circuit element is given by Equation (1.5.4). The skin effect associated with resistance in Equation (1.5.14) is ignored in this analysis to simplify the formulation of the problem. The accuracy of the solution remains largely unaffected since the inductive and capacitive elements are much larger than the resistance in practical power line configurations. (For more information on the skin effect, see **Section 1.6a**.)

$$Y_{ij} = \frac{1}{R + 1j \cdot \omega \cdot L + \frac{1}{1j \cdot \omega \cdot C}} \qquad \textbf{(1.5.4)}$$

If the circuit element that connects two nodes does not include all the parameters of the generalized model, then it is eliminated. To eliminate resistance and inductance from a circuit branch, enter a zero for that parameter. To eliminate capacitance from a branch, enter a large value of capacitance to cause the last term in the denominator of Equation (1.5.4) to vanish.

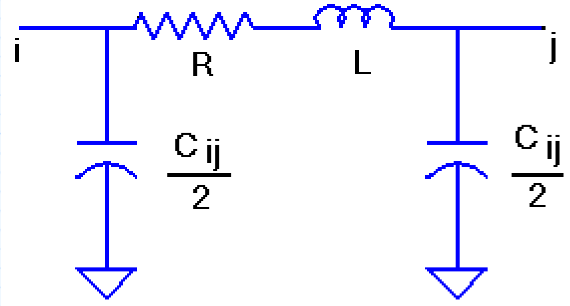

Fig. 1.5.4 Power line Pi model

Figure 1.5.4 shows the power line model for harmonic analysis at frequencies below 1800 Hz. To adapt the generalized branch model for use with the line model, the unused parameters must be eliminated. Branch ij omits the series capacitance, so the value of C in the branch model has a large value. The values of R and L take the values found from line parameter calculations. The shunt capacitor values connect nodes i and j to the reference node, which is ground (To learn more about line-impedance calculations, see **Section 1.6a**).

This branch omits both series R and L by giving these parameters a zero value in the generalized branch model. The value of C is taken from the Pi model calculations. To convert power loads to impedances, use the bus voltages from the load flow solution in **Section 1.3a**, and the following relationship.

$$Z = \frac{V^2}{S}$$

Construction of the Admittance Matrix

Matrix methods existing that permit the construction of the Ybus matrix from the values of the branch elements and the circuit node connections. This method requires two matrices, the branch admittance matrix, Yb, and the system connection matrix, A.

The Yb matrix is a n x n matrix that contains all the values of admittance computed from Equation (1.5.4) along the diagonal. The value of n is the number of elements in the system. Each branch element in the system has an entry on the diagonal of the Yb matrix. To see a Mathcad example, see **Section 1.3b**.

The A matrix holds the connection information for the system. The A matrix is a n x m array where n is the number of branch elements in the system and m is the number of buses in the system.

The elements in the rows of the A matrix are 1, -1, or 0. These values represent the direction of currents entering and leaving the branch element connecting the buses in the system. All branch elements that connect non-reference system buses have an entry of 1 and -1 in the row of the A matrix. The 1 and -1 represent the connection of the element between the two nodes in the system. Branch elements connecting a system bus to the reference bus (bus shunts) have an entry of 1 in the column which represents the bus connection. All other entries in the row are 0.

Construction of the Yb and A matrices allows the computation of the bus impedance matrix, Zbus, from Equation (1.5.5).

$$Z_{bus} = \left(A^{T} \cdot Y_b \cdot A\right)^{-1} \qquad \textbf{(1.5.5)}$$

The diagonal element m of Zbus is the Thevenin's equivalent impedance of the system seen from bus m. Plotting this impedance as a function of frequency identifies system resonances. The other elements of column m are the mutual couplings between bus m and all other buses in the system. Injecting a current into bus m will produce voltages on the other system buses proportional to these impedances. Multiplying a current injection by the mutual impedances finds the voltages at all other system buses due to the current. This procedure allows computation of harmonic voltages given estimates of harmonic currents and the impedance values for the harmonics of interest.

Harmonic Distortion Computation Factors

A number of formulae exist for quantifying the effects of harmonic currents and voltages on power systems. Two widely used methods are the current and voltage harmonic factors from [1]. Equations (1.5.6) and (1.5.7) give the relationships for these factors.

Given

$q := 2..8$ harmonic orders

$I_1 := 1.0$ fundamental current (pu)

$I_{h_q} := \frac{I_1}{q}$ pu harmonic currents

Current Harmonic Factor is

$$HF_I := \sqrt{\frac{\sum_q \left(I_{h_q}\right)^2}{I_1}} = 0.726 \qquad \textbf{(1.5.6)}$$

This factor is a measure of the amount of harmonic distortion in the current of a system. The lower the harmonic factor, the less harmonic distortion exists on a system.

For the given values of pu voltage and harmonic voltages

$V_1 := 1.0$ fundamental voltage (pu)

$V_{h_q} := \frac{V_1}{q}$ pu harmonic voltages

Voltage Harmonic Factor is

$$HF_V := \sqrt{\frac{\sum_q \left(V_{h_q}\right)^2}{V_1}} = 0.726 \qquad \textbf{(1.5.7)}$$

This factor indicates the degree of voltage distortion in a power system. The lower the value of this index, the less harmonic voltage is present in a system.