## 1 POWER DISTRIBUTION

# Section 1.6b Power Line Parameters

### Section 1.6.5 Sequence Impedances of Lines

Unbalanced power systems require analysis by the method of symmetrical components. This method breaks down the unbalanced three phase system into three sets of balanced components. These components are called the positive, negative, and zero sequence values. All equipment connected in a power system has an impedance model for each of the three sequence networks. For transmission and distribution lines, the positive and negative sequence impedances have the same values. The series impedance values in terms of the resistance and reactance values in Sections 1.6.2 and 1.6.3 are

$$z_1 = r_a + 1\mathrm{j} \cdot \left(x_a + x_d\right)$$ positive sequence

$$z_2 = r_a + 1\mathrm{j} \cdot \left(x_a + x_d\right)$$ negative sequence

Given the parameter values for 266 MCM aluminum cable-steel reinforced

$$r_a := 0.352\ \frac{\Omega}{mi} \qquad GMR := 0.00684\ ft$$

$$f := 60\ Hz \qquad D_{eq} := 7.89\ ft$$

See Table 1.6.1

$$x_a(f, GMR) := 0.2794 \cdot \left(\frac{f}{60 \cdot Hz}\right) \cdot \log\left(\frac{1 \cdot ft}{GMR}\right) \cdot \frac{\Omega}{mi}$$

$$x_d\left(f, D_{eq}\right) := 0.2794 \cdot \left(\frac{f}{60\ Hz}\right) \cdot \log\left(\frac{D_{eq}}{1\ ft}\right) \cdot \frac{\Omega}{mi}$$

See Table 1.6.2

$$z_1 := r_a + 1\mathrm{j} \cdot \left(x_a(f, GMR) + x_d\left(f, D_{eq}\right)\right)$$

$$z_1 = (0.352 + 0.856\mathrm{i})\ \frac{\Omega}{mi}$$

The value of z1 is the positive sequence series impedance of the line per mile of distance. Negative sequence impedance, z2, is numerically equal to the positive sequence impedance.

## Positive and Negative Sequence Shunt Capacitance

The shunt capacitive reactance is given by

$x_{c1} = x_{ca} + x_{cd}$ Positive sequence

$x_{c2} = x_{ca} + x_{cd}$ Negative sequence

Given the radius of the conductor, the frequency, and the geometric mean distance between the neighboring conductors, Equation (1.6.7) calculates both terms of the positive sequence capacitive reactance. Note that the equation uses actual conductor radius in feet and not the GMR as in the inductive reactance calculations.

Given the values of

$r := 0.026375\ ft$ $f := 60\ Hz$

$D_{eq} := 12.5\ ft$

$$x_C(f, r, D_{eq}) := \frac{4.10 \cdot Hz}{f} \cdot 10^6 \cdot \log\left(\frac{1 \cdot ft}{r}\right) \cdot \Omega \cdot mi + \frac{4.10 \cdot Hz}{f} \cdot 10^6 \cdot \log\left(\frac{D_{eq}}{1 \cdot ft}\right) \cdot \Omega \cdot mi$$

$$x_{c1} := x_C(f, r, D_{eq}) = (1.828 \cdot 10^5)\ \Omega \cdot mi$$

The negative sequence value of shunt capacitive reactance is numerically equal to the positive sequence value.

## Zero Sequence Resistance and Inductive Reactance

Zero sequence impedance opposes the current produced when an unbalance, such as a ground fault, occurs on a power system. Zero sequence currents are in phase and equal in magnitude. They return to the source via the system neutral, earth grounds, and over-head (OH) ground wires. Zero sequence currents can flow through any combination of these paths. If none of these return paths are present, no zero sequence current flows and the zero sequence impedance is infinite.

The return path usually consists of the earth and over-head ground wires in parallel. The return path affects both resistance and inductive reactance of the zero sequence. So the impedance is computed with a single formula. Considering that the return current flows through an equivalent conductor at some distance below the surface simplifies the analysis of the problem. The equivalent depth of the return current is given by Equation (1.6.9) from [1].

$$D_e(f, \rho) := 2160 \cdot \sqrt{\frac{\rho}{f}} \cdot ft \cdot Hz \qquad \textbf{(1.6.9)}$$

where De is the equivalent depth of the return, and ρ is the earth resistivity.

Two impedance values characterize the zero sequence effects on lines. The effect of the current flowing in a single conductor with ground return is the self-impedance. Equation (1.6.10) from [1] gives the zero sequence self-impedance for a three phase line with ground return.

$$z_o\left(r_c, f, D_e, GMR\right) := \left(3 \cdot r_c + \frac{0.00477}{Hz} \cdot f + 1j \cdot \frac{0.01397}{Hz} \cdot \log\left(\frac{D_e}{GMR}\right)\right) \cdot \frac{\Omega}{mi} \qquad \textbf{(1.6.10)}$$

where rc is the equivalent resistance of the three phase conductors in parallel, De is the image conductor depth, f is the system frequency, and GMR is the conductor geometric mean radius.

The effects of the current flowing through adjacent conductors with ground return is called the mutual impedance. Equation (1.6.11) gives the relationship.

$$z_{om}\left(f, D_e, d\right) := \left(\frac{0.00477}{Hz} \cdot f + 1j \cdot \frac{0.01397}{Hz} \cdot f \cdot \log\left(\frac{D_e}{d}\right)\right) \cdot \frac{\Omega}{mi} \qquad \textbf{(1.6.11)}$$

where d is the distance in feet between parallel conductors.

Equation (1.6.10) is valid for a three-phase line with ground return and a single OH ground wire. Equation (1.6.11) gives the mutual impedance between two ground wires.

## Zero Sequence Impedance Using Conductor Tables

Use of the conductor tables simplifies the calculation of zero sequence line impedances. The conductor tables provide the values of ra, xa, and xd. Adding the additional quantities of re and xe provides the ground return effects. Equations (1.6.12) and (1.6.13) define these quantities.

$$r_e(f) := \frac{0.00477}{Hz} \cdot f \cdot \frac{\Omega}{mi} \qquad \textbf{(1.6.12)}$$

$$x_e(f, \rho) := \frac{0.006985}{Hz} \cdot f \cdot \log\left(4.6656 \cdot 10^6 \cdot \frac{Hz}{\Omega \cdot m} \cdot \left(\frac{\rho}{f}\right)\right) \cdot \frac{\Omega}{mi} \qquad \textbf{(1.6.13)}$$

Different configurations of ground wires and grounding require different formulae to compute the self and mutual impedances. For a single circuit with ground return but no OH ground wires, Equations (1.6.14) give the zero sequence impedances.

Given the spacing between each conductor

$d_{ab} := 5\ ft$ $\qquad d_{bc} := 6\ ft$ $\qquad d_{ca} := 5.5\ ft$

$f := 60\ Hz$ $\qquad r_a := 0.332\ \frac{\Omega}{mi}$ $\qquad \rho := 100 \cdot \Omega \cdot m$

$$x_{do} := \frac{1}{3} \cdot \left(x_d\left(f, d_{ab}\right) + x_d\left(f, d_{bc}\right) + x_d\left(f, d_{ca}\right)\right)$$

$$z_o := \left(\left(r_a + r_e(f)\right) + 1j \cdot \left(x_e(f, \rho) + x_a(f, GMR) - 2 \cdot x_{do}\right)\right) = (0.618 + 3.08i)\ \frac{\Omega}{mi} \qquad \textbf{(1.6.14)}$$

The mutual impedance *per phase* for a single circuit with ground return but no OH ground wire is given by Equation (1.6.15).

$$z_{om}\left(f, D_e, GMD\right) := \left(\frac{0.00477}{Hz} \cdot f + 1j \cdot \frac{0.01397}{Hz} \cdot f \cdot \log\left(\frac{D_e}{GMD}\right)\right) \cdot \frac{\Omega}{mi} \qquad \textbf{(1.6.15)}$$

For circuits with one ground wire and earth return, use Equation (1.6.16).

$$z_{og} := \left(3 \cdot r_a + r_e(f)\right) + 1j \cdot \left(x_e(f, \rho) + 3 \cdot x_a(f, GMR)\right) = (1.282 + 4.703i)\ \frac{\Omega}{mi} \qquad \textbf{(1.6.16)}$$

## Zero Sequence Shunt Capacitance

Ground return adds another term to the shunt capacitive reactance calculation from above. This additional term accounts for the electric field effects between the conductor and the image conductor. Equation (1.6.17) gives the relationship for the zero sequence capacitance factor.

$$x_{ce}(f, h) := \frac{12.30 \cdot Hz}{f} \cdot \log\left(2 \cdot \frac{1}{ft} \cdot h\right) \cdot \Omega \cdot mi \qquad \textbf{(1.6.17)}$$

where h is the conductor height above ground.

Using the values from the tables, the zero sequence capacitive reactance is given by Equations (1.6.18).

For a conductor spacing of

$d_{ab} := 6.25\ ft$ $\qquad d_{bc} := 7.5\ ft$ $\qquad d_{ca} := 6.25\ ft$

An average conductor height of

$h := 22.5\ ft$

From the conductor tables,

$$x_{ca} := 0.560 \cdot 10^6 \cdot \Omega \cdot mi$$

$$x_{cd}\left(D_{eq}\right) := 0.06831 \cdot \log\left(\frac{D_{eq}}{1 \cdot ft}\right) \cdot 10^6 \cdot \Omega \cdot mi$$

$$x_{cd0} := \frac{1}{3} \cdot \left(x_{cd}\left(d_{ab}\right) + x_{cd}\left(d_{bc}\right) + x_{cd}\left(d_{ca}\right)\right)$$

**(1.6.18)**

$$x_{c0} := -1j \cdot \left(x_{ca} + x_{ce}(f, h) - 2 \cdot x_{cd0}\right) = -4.477i \cdot 10^5\ \Omega \cdot mi$$

### Section 1.6.6 Mathcad Formulation

Compute the line series impedance and shunt capacitive reactance for the line structure in Figure 1.6.3. The conductor is 336 MCM ACSR.

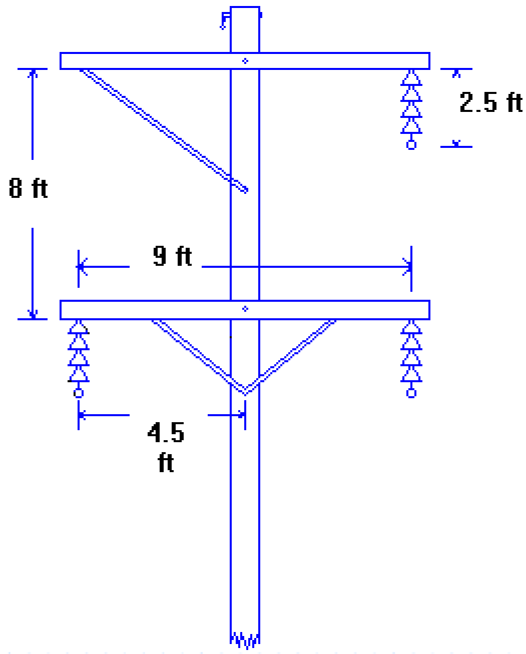


**Fig. 1.6.3 Tangent line structure**

From the conductor table for 336 MCM ACSR,

| | |
|---|---|
| $d := 0.741 \cdot in$ | conductor diameter |
| $GMR := 0.0255 \cdot ft$ | conductor geometric mean radius |
| $r_a := 0.278 \cdot \frac{\Omega}{mi}$ | dc resistance per mile of conductor |
| $h := 35 \cdot ft$ | average conductor height above ground |

## Example Solution

Calculate the radius in feet for the capacitive reactance.

$$r := \frac{d}{2} \cdot \left(\frac{1 \cdot ft}{12 \cdot in}\right)$$

Calculate the distances between the conductors from the geometry of Figure 1.6.2.

$$d_{bc} := 8 \cdot ft \qquad d_{ca} := 9 \cdot ft$$

$$d_{ab} := \sqrt{d_{bc}^{\;2} + d_{ca}^{\;2}} = 12.042\ ft$$

The geometric mean radius is

$$GMD := \left(d_{ab} \cdot d_{bc} \cdot d_{ca}\right)^{\frac{1}{3}} = 9.535\ ft$$

Compute the series inductive reactance using Equation (1.6.2).

$$x_L\left(f, GMR, D_{eq}\right) := 0.2794 \cdot \left(\frac{f}{60 \cdot Hz}\right) \cdot \log\left(\frac{1 \cdot ft}{GMR}\right) \cdot \frac{\Omega}{mi} + 0.2794 \cdot \left(\frac{f}{60 \cdot Hz}\right) \cdot \log\left(\frac{D_{eq}}{1 \cdot ft}\right) \cdot \frac{\Omega}{mi}$$

$$x_L(f, GMR, GMD) = 0.719\ \frac{\Omega}{mi}$$

$$r_a = 0.278\ \frac{\Omega}{mi}$$

The positive and negative sequence impedances are

$$z_1 := r_a + 1\mathrm{j} \cdot x_L(f, GMR, GMD) = (0.278 + 0.719\mathrm{i})\ \frac{\Omega}{mi}$$

Compute the shunt capacitive reactance.

$$x_c := -1\mathrm{j} \cdot \left(x_C(f, r, GMD)\right) = -1.701\mathrm{i} \cdot 10^5\ \Omega \cdot mi$$

The positive and negative sequence shunt capacitive reactance is equal to the value of xc.

## Zero Sequence Impedance and Shunt Reactance Calculation

The structure in Figure 1.6.2 has a single overhead ground wire that is grounded at each pole. The zero sequence impedance calculations will reflect these additional ground return paths.

The average earth resistivity over the line right-of-way is

$$\rho := 58 \cdot \Omega \cdot m \qquad f := 60 \cdot Hz$$

Equations (1.6.12) and (1.6.13) give the ground return components of the resistance and the inductive reactance.

$$r_e(f) = 0.286\ \frac{\Omega}{mi} \qquad x_e(f,\rho) = 2.789\ \frac{\Omega}{mi}$$

Compute the inductive reactance for a one foot conductor spacing.

$$x_a(f, GMR) = 0.445\ \frac{\Omega}{mi}$$

Combine the values above to get the self-impedance for the zero sequence.

$$z_{og} := \left(3 \cdot r_a + r_e(f)\right) + 1\mathrm{j} \cdot \left(x_e(f,\rho) + 3 \cdot x_a(f, GMR)\right) = (1.12 + 4.124\mathrm{i})\ \frac{\Omega}{mi}$$

Compute the value of zero sequence shunt capacitive reactance. Calculate the effects of the image conductor.

$$x_{ce}(f,h) = 0.378\ \Omega \cdot mi$$

Compute the electric field effects of the neighboring conductors for the spacing factor.

$$d_{ab} = 12.042\ ft \qquad d_{bc} = 8\ ft \qquad d_{ca} = 9\ ft$$

$$x_{cd0} := \frac{1}{3} \cdot \left(x_{cd}\left(d_{ab}\right) + x_{cd}\left(d_{bc}\right) + x_{cd}\left(d_{ca}\right)\right)$$

Equations (1.6.18) give the zero sequence shunt capacitive reactance.

$$x_{c0} := -1\mathrm{j} \cdot \left(x_{ca} + x_{ce}(f,h) - 2 \cdot x_{cd0}\right) = -4.262\mathrm{i} \cdot 10^{5}\ \Omega \cdot mi$$

## Section 1.6.7 References


**1**. Central Station Engineers, *Electrical Transmission and Distribution Reference Book*, ABB Power T&D Co., Raleigh, NC.