Section 1.6a Power Line Parameters

Section 1.6.1 Introduction

The function of distribution and transmission lines in power systems is to convey electric energy from its sources of generation to end users. Electric lines have three parameters that affect their ability to effectively move large amounts of energy. These parameters are resistance, inductance, and capacitance. Although distribution and transmission lines are constructed from low resistance conductors, power transmission distances produce significant inductive and capacitive effects even at 60 Hz. Conductor resistance also adds to power losses in the system which decreases system efficiency. All the electrical parameters affect the line's ability to transmit power in the same way; they increase the voltage drop across the system, and limit the active power that the line can transmit before system stability is sacrificed.

Line Parameters

The metal used in a conductor determines the resistance of a power transmission line. Aluminum, copper and their alloys are the most commonly used conductors. Both the dc and the ac resistance are in some way determined by the metal parameters. Resistance is determined by the resistivity of the conductor, and is a direct function of the conductor temperature. **Tables of conductor characteristics** include the dc and ac resistance for conductor temperatures at 25°C and 50°C. These values represent typical light and heavy load operating conditions experienced by the conductor.

In addition to the conductor temperature, the ac resistance is also effected by the penetration of higher frequency power into the conductor. This is known as the skin effect, and it is a function of both the metal and the frequency. In addition to the ac resistance, its important to look at non-linear impedances on the line: the inductive and capacitive effects.

Magnetic effects in and around the conductor produce inductive voltage drops. An inductive voltage drop is the voltage difference induced by a magnetic field in the conductor and by magnetic coupling with other current carrying conductors on the same structure. Line calculations account for both of these inductive effects.

Charged conductors separated by an insulator, such as air, exhibit capacitive effects. The capacitive effects are due to the electric field surrounding the conductor and the fields produced by other conductors in the vicinity. The capacitive effects are also a function of the voltage level used on the structure. Higher voltage lines exhibit larger values of charging current than lower voltage lines.

The following sections provide methods for computing the resistance, inductance, and capacitance for transmission and distribution lines. The procedures make use of tabulated values for common conductor sizes used in industry, and are included in a **table** at the end of this section.

Section 1.6.2 Resistance of Power Lines

The resistivity of the conductor, usually aluminum or copper, determines the resistance of the line. Equation (1.6.1) gives the dc resistance of a conductor.

$$R_{dc}(\rho, l, A) := \frac{\rho \cdot l}{A} \qquad \textbf{(1.6.1)}$$

where Rdc is the dc resistance of the conductor (in ohms), ρ is the resistivity of the conductor material, l is the length of the conductor, and A is the area of the conductor.

The units of the resistivity, ρ, depend on the system of units. The units for area, A, and length, l, commonly used in power transmission are as follows. The unit of area used for conductors is the circular mil (**cmil**). One circular mil corresponds to the area of a circle that has a diameter of 1 mil. A mil is 0.001 in. The area of a conductor in cmil is the square of its diameter in mils. To find the area in square mils, multiply cmil by π/4.

Make these global unit definitions for area in **cmils**.

$$cmil \equiv (0.001 \cdot in)^2$$

Using the definition for area in **cmils** the resistivity for copper conductor at 20°C is,

$$\rho := 10.66 \cdot \frac{\Omega \cdot cmil}{ft}$$

For aluminum conductor, the resistivity at 20°C is

$$\rho := 17.00 \cdot \frac{\Omega \cdot cmil}{ft}$$

The resistivity of metallic conductors varies with the temperature of the conductor. The conductor resistivity increases as the temperature increases, so the dc resistance of a metallic conductor also increases. The dc resistance of conductors is tabulated for 25 and 50°C conductor temperatures. The dc resistance, ra, (in ohms/mi), is given in **Table 1.6.1** for most common types of conductor.

Skin Effect

Skin effect increases the ac resistance of conductors. The skin effect refers to a nonuniform current distribution in a conductor cross section, causing more current to flow on the outer area, or skin, of the conductor. This effect usually occurs at high frequencies. The skin effect in power systems results from the frequency of operation caused by internal magnetic effects and large conductor areas. The magnetic flux in the conductor causes a non-uniform current distribution in the conductor cross-section. The skin effect at 60 Hz becomes significant when the conductor size is larger than 566 million circular mils (MCM), as can also be seen in the table.

The conductor table (**Table 1.6.1**) gives the ac resistance including the skin effect of common conductors for the frequencies of 25, 50, and 60 Hz.

A second conductor table for small gauge conductors (**Table 1.6.2**) is also shown, but the skin effect will be less important here as the diameter of the cable begins to be smaller than the skin penetration depth.

For frequencies other than those in the table, a correction factor, K, from [1] corrects the dc resistance to account for the skin effect. This is useful when the calculation of losses due to large magnitudes of harmonic current is necessary.

The correction factor, K, is a function of the parameter X. Equation (1.6.1a) relates X to the dc resistance and the frequency.

$$X(\mu, f, r_{dc}) := 0.063598 \cdot \sqrt{\frac{\mu \cdot f}{r_{dc}}} \qquad \textbf{(1.6.1a)}$$

where μ is permeability of the conductor (1.0 for non-magnetic materials), f is the frequency in Hertz, and rdc is the dc resistance in ohms/mile.

Approximation of the X-K Relationship

The National Bureau of Standards Bulletin No. 169 gives a tabular form of the relationship between X and K for values of X up to 100. This table is on pages 226-8 of the standard. Equation (1.6.1b) is 6th order polynomial curve fit for values of X between 0.1 and 3.9.

$$d := 0 .. 6$$ index of the coefficients

$$a_d := \begin{bmatrix} 1.00042 \\ -0.00866 \\ 0.03150 \\ -0.04287 \\ 0.03138 \\ -0.00691 \\ 4.86752 \cdot 10^{-4} \end{bmatrix}$$ coefficients of the polynomial

Approximate relationship of X to K.

$$K(x) := \sum_d \left(a_d \cdot x^d\right) \qquad \textbf{(1.6.1b)}$$

Equation (1.6.1c) from [1] gives the ac resistance of a conductor at the frequency of interest as a function of X.

Given

$X := 1.6$ and $r_{dc} := 0.385 \cdot \frac{\Omega}{mi}$

$r_{ac} := K(X) \cdot r_{dc}$ **(1.6.1c)**

$$r_{ac} = \begin{bmatrix} 16.59 \\ -0.144 \\ 0.522 \\ -0.711 \\ 0.52 \\ -0.115 \\ 0.008 \end{bmatrix} \frac{\Omega}{mi}$$

Section 1.6.3 Inductance Calculation for Lines

Equation (1.6.2) from [1] includes the effect on the internal magnetic field of the conductor and the effects of neighboring conductors to give the total inductive reactance of a conductor in ohms/mile.
For the given values,

$f := 60 \cdot Hz$ $GMR := 0.00255 \cdot ft$ $D_{eq} := 7.5 \cdot ft$

$$x_L(f, GMR, D_{eq}) := 0.2794 \cdot \left(\frac{f}{60 \cdot Hz}\right) \cdot \log\left(\frac{1 \cdot ft}{GMR}\right) \cdot \frac{\Omega}{mi} + 0.2794 \cdot \left(\frac{f}{60 \cdot Hz}\right) \cdot \log\left(\frac{D_{eq}}{1 \cdot ft}\right) \cdot \frac{\Omega}{mi} \quad \textbf{(1.6.2)}$$

$$x_L(f, GMR, D_{eq}) = 0.969 \ \frac{\Omega}{mi}$$

where f is the system frequency (in Hz), GMR is the geometric mean radius of the conductor (in feet), and Deq is the geometric mean distance (GMD) between surrounding conductors and the conductor of interest

The value for GMR is listed in **Table 1.6.1**. The first term of Equation (1.6.2) is the same as the value of xa in Table 1.6.1. The tabulated value, xa, is the inductive reactance due to the internal magnetic effects of the conductor. This reactance also includes the effects of the flux at 1 foot spacing. The second term of Equation (1.6.2) is the inductive reactance due to the surrounding conductors and is called xd. It is found from the geometric mean distance, Deq, between the conductors.

$$x_a(f, GMR) := 0.2794 \cdot \left(\frac{f}{60 \cdot Hz}\right) \cdot \log\left(\frac{1 \cdot ft}{GMR}\right) \cdot \frac{\Omega}{mi} \quad \textbf{(1.6.3)}$$

$$x_d(f, D_{eq}) := 0.2794 \cdot \left(\frac{f}{60 \cdot Hz}\right) \cdot \log\left(\frac{D_{eq}}{1 \cdot ft}\right) \cdot \frac{\Omega}{mi} \quad \textbf{(1.6.4)}$$

By using the values of Table 1.6.1, Equation (1.6.2) simplifies to

$$x_L = x_a + x_d \quad \textbf{(1.6.5)}$$

Computing the Geometric Mean Distance

Using Equation (1.6.2) requires the computation of the geometric mean distance (GMD), Deq, between the phase conductors. Figure 1.6.1 shows the conductor spacing for a three phase transmission system. Suppose the distances between conductors are:

$D_{ab} := 10.5 \cdot ft$ distance from A phase to B phase

$D_{bc} := 7.25 \cdot ft$ distance from B phase to C phase

$D_{ca} := 5.75 \cdot ft$ distance from C phase to A phase

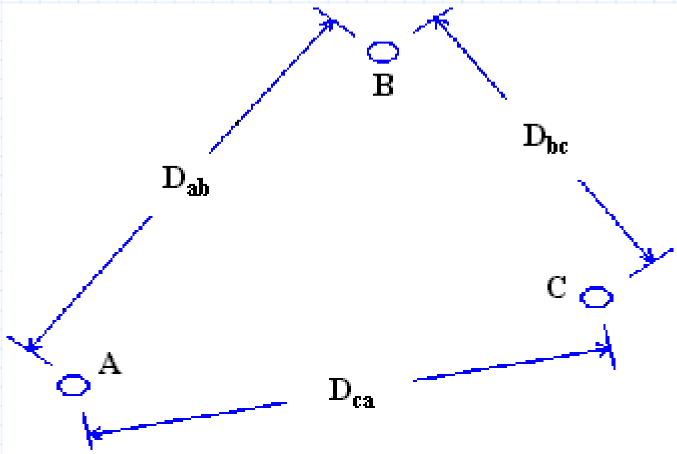

Fig. 1.6.1 Three phase line spacing

Then the relationship for Deq is

$$D_{eq} := \left(D_{ab} \cdot D_{bc} \cdot D_{ca}\right)^{\frac{1}{3}} = 7.593\ ft \qquad \textbf{(1.6.6)}$$

The reactance spacing facor, xdd, accounts for the inductive effects of neighboring conductors and is a function of the GMD of the conductor spacing. It is found from the relationships:

$$x_{dd} := 0.2794 \cdot \log\left(\frac{D_{eq}}{1 \cdot ft}\right) \cdot \frac{\Omega}{mi}$$ at 60 Hz

$$x_{dd} := 0.2328 \cdot \log\left(\frac{D_{eq}}{1 \cdot ft}\right) \cdot \frac{\Omega}{mi}$$ at 50 Hz

$$x_{dd} = 0.246\ \frac{\Omega}{mi}$$

You can activate the equation for 50 Hz if that is appropriate in your system. The value of Deq is the GMD for the structure.

Activating Equations

The figures which follow indicate typical pole-top configurations. From the geometry of these structures, you can calculate Deq.

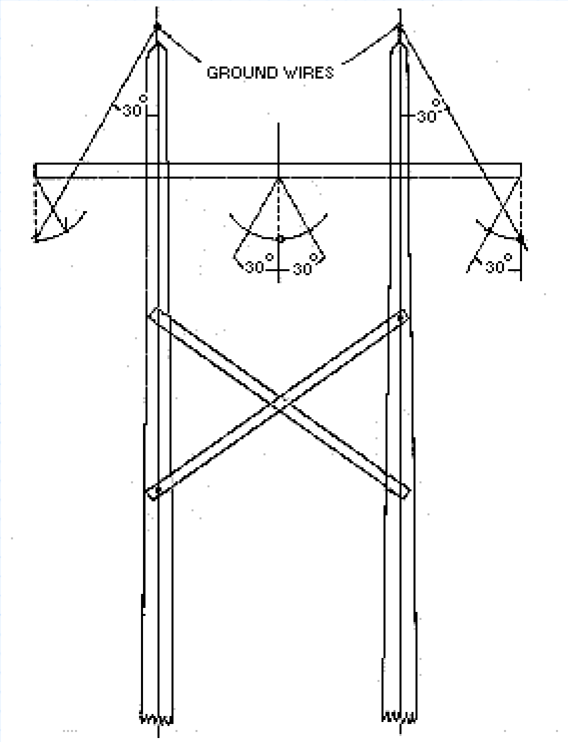

Fig. 1.6.2(a) H-Structure poe top configuration

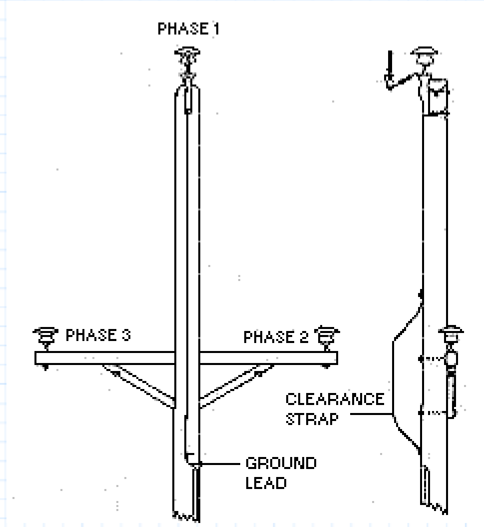

Fig. 1.6.2(b) Single-circuit, three-phase line pole top

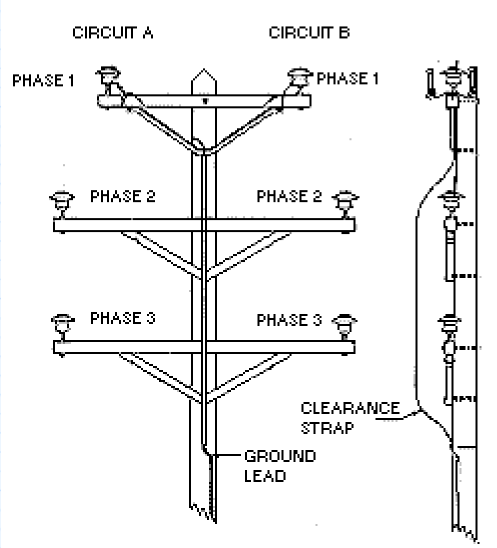

Fig. 1.6.2(c) Two-circuit, three-phase line pole top

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Section 1.6.4 Capacitance Calculations for Lines

The capacitance of a transmission or distribution line is due to the potential difference between conductors. The conductors of a line are charged just like the plates of a capacitor, a result of the different potentials of the phases in the line. Another capacitance is due to the potential difference between the conductor and the ground. The capacitance is a constant for a given conductor type and spacing.

Ac voltage applied to a transmission line causes a time varying electric field. The varying field produces a current to flow in the line even when the load is disconnected. This current is called **charging current**. A single-phase capacitive reactance between the ground reference and the conductor models the charging current on a per phase basis.

The shunt capacitive reactance is given by Equation (1.6.7) from [1].
Given

$$f = 60\ \frac{1}{s} \qquad r := 0.026375 \cdot ft \qquad D_{eq} = 7.593\ ft$$

$$x_C\left(f, r, D_{eq}\right) := \frac{4.10 \cdot Hz}{f} \cdot 10^{6} \cdot \log\left(\frac{1 \cdot ft}{r}\right) \cdot \Omega \cdot mi + \frac{4.10 \cdot Hz}{f} \cdot 10^{6} \cdot \log\left(\frac{D_{eq}}{1 \cdot ft}\right) \cdot \Omega \cdot mi$$

$$x_C\left(f, r, D_{eq}\right) = \left(1.68 \cdot 10^{5}\right)\ \Omega \cdot mi \qquad \textbf{(1.6.7)}$$

where Deq is the GMD of the structure spacing (feet), r is the conductor radius (feet), f is the system frequency (Hz), and xc is the capacitive reactance (ohms-mi).

Equation (1.6.7) is analogous to the relationship for inductive reactance in Section 1.6.3. The first term represents the effects of the electrostatic field to a radius of 1 foot from the conductor. The second term includes the effects of the electric flux from 1 foot to the adjacent conductors. The first term value only depends on the system frequency and the conductor radius, r. The second term of the equation depends on the value of Deq. Equation (1.6.6) computes the value of Deq in Equation (1.6.7).

Table Values of Shunt Capacitance

Using the values of Table 1.6.1, the equation for shunt capacitive reactance simplifies to

$$x_C = x_{ca} + x_{cd}$$

where xca is the reactance from the conductor surface to a 1 foot radius, and xcd is the reactance due to the presence of neighboring conductors.
The value of xca is given in the table as x'a. Equations (1.6.8) find the value of xcd.

$$x_{cd}\left(D_{eq}\right) := 0.06831 \cdot \log\left(\frac{D_{eq}}{1 \cdot ft}\right) \cdot 10^{6} \cdot \Omega \cdot mi \qquad \text{at 60 Hz}$$

(1.6.8)

$$x_{cd}\left(D_{eq}\right) := 0.08198 \cdot \log\left(\frac{D_{eq}}{1 \cdot ft}\right) \cdot 10^{6} \cdot \Omega \cdot mi \qquad \text{at 50 Hz}$$

The value of xcd is the shunt capacitance due to the presence of other conductors on the same structure. The total capacitive reactance is the sum of the electric field from the conductor radius to a distance of 1 foot and the effects of the local conductors electric fields.