

## 2 POWER SYSTEM PROTECTION

### Section 2.5b DC Motor Protection

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#### Section 2.5.3 DC Motor Simulation

Define the motor parameters for the systems.

$$I_a := 8 \text{ A} \quad \text{rated armature current}$$

$$L_a := 0.079 \text{ H} \quad \text{armature inductance}$$

$$R_a := 3.5 \text{ } \Omega \quad \text{armature resistance}$$

$$J := 0.02 \text{ kg} \cdot \text{m}^2 \quad \text{system rotational inertia}$$

$$\beta := 2.57 \cdot 10^{-4} \frac{\text{N} \cdot \text{m}}{\frac{\text{rad}}{\text{s}}} \quad \text{system damping constant}$$

$$K_i := 0.722 \frac{\text{V}}{\frac{\text{rad}}{\text{s}}} \quad \text{motor torque constant}$$

#### State Equation Functions

Performing iterative calculations with Mathcad requires vectorizing the equations. To simplify the notation, define the state equations as functions.

$$di_a(i_a, \omega, V) := -\frac{R_a}{L_a} \cdot i_a + \frac{K_i}{L_a} \cdot \omega + \frac{1}{L_a} \cdot V \quad (2.5.8)$$

$$d\omega_m(i_a, \omega, T) := \frac{K_i}{J} \cdot i_a + \frac{\beta}{J} \cdot \omega - \frac{1}{J} \cdot T \quad (2.5.9)$$

Define the time step used for the integration of the state equations. The time step should be at least 10 times smaller than the smallest time constant in the system to ensure numerical stability in the algorithm. Examine the electrical and mechanical time constants in Equations (2.5.8-9) to find a value of h. In this case select a time step of

$$h := 1 \text{ ms}$$

Define an armature voltage and load torque. Select a constant load torque. This load torque is typical of conveyor and hoisting loads.

$$V_i := 125 \text{ V}$$

$$T_L := 2.1 \text{ N} \cdot \text{m}$$

Define the maximum number of iterations used in the Adams-Bashford simulation. The number of iterations should be high enough for the system to reach steady state. Adjust the integration time step and the number of iterations to achieve steady-state response with a minimum number of iterations.

Set the maximum number of iterations

$$j_{max} := 500$$

Set the iteration range variable.

$$j := 2 .. j_{max}$$

Define the initial conditions of the system prior to the time of interest. Assume the motor starts from rest.

### Initial Conditions

$$i_{a_0} := 0 \text{ A} \quad \omega_{m_0} := 0 \frac{\text{rad}}{\text{s}} \quad t_2 := 0 \text{ s}$$

$$i_{a_1} := 0 \text{ A} \quad \omega_{m_1} := 0 \frac{\text{rad}}{\text{s}}$$

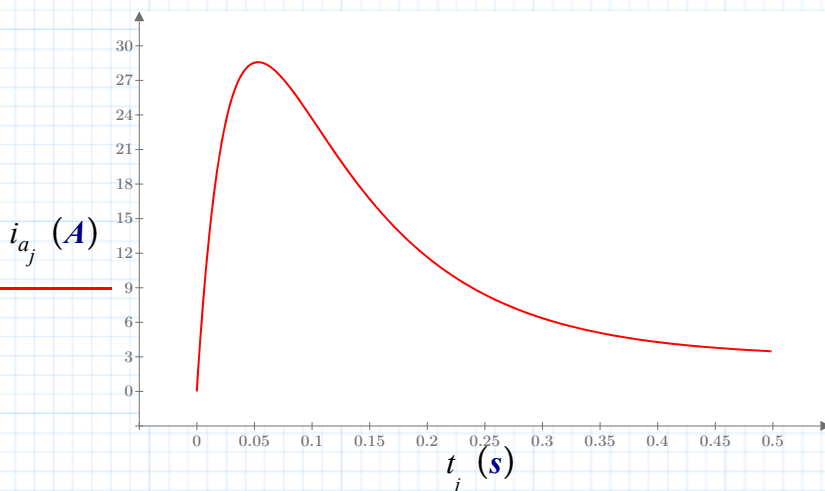
Define the vectors that hold the state variables.

$$\begin{bmatrix} i_{a_j} \\ \omega_{m_j} \\ t_j \end{bmatrix} := \begin{bmatrix} i_{a_{j-1}} + \left( 1.5 \cdot h \cdot di_a(i_{a_{j-1}}, \omega_{m_{j-1}}, V_t) - 0.5 \cdot h \cdot di_a(i_{a_{j-2}}, \omega_{m_{j-2}}, V_t) \right) \\ \omega_{m_{j-1}} + 1.5 \cdot h \cdot d\omega_m(i_{a_{j-1}}, \omega_{m_{j-1}}, T_L) + -0.5 \cdot h \cdot d\omega_m(i_{a_{j-2}}, \omega_{m_{j-2}}, T_L) \\ t_{j-1} + h \end{bmatrix}$$

Plot the response of the motor when the full supply voltage is applied.

Define the range to plot.

$$j := 0 .. j_{max}$$

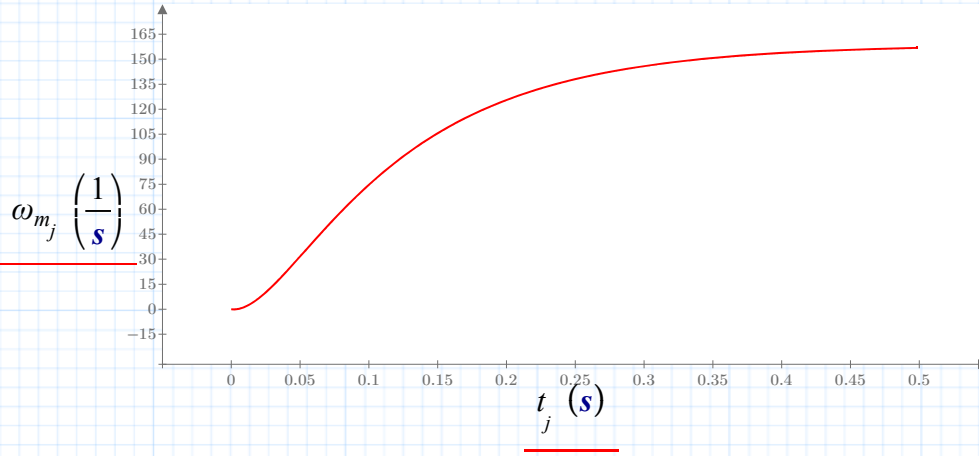


**Fig. 2.5.3 Separately excited DC Motor armature current response**

The steady-state armature current is

$$i_{a_{j_{max}}} = 3.478 \text{ A}$$

Plot the response of the motor speed for a full voltage start.



**Fig. 2.5.4 Separately Excited DC motor speed response**

The steady-state motor speed is

$$\omega_{m_{j_{max}}} = 156.804 \frac{\text{rad}}{\text{s}}$$

$$n := \omega_{m_{j_{max}}} \text{ rpm} = 16.42 \frac{1}{\text{s}^2}$$

## Section 2.5.4 Linear Torque Model Response

Apply a linear load to the same dc motor and simulate the system response. This simulation adds another variable to the array since the torque is a function of the motor speed. The modified solution vector includes the equation for torque.

Define the iteration counter.

$$j_{max} := 700$$

$$j := 2 \dots j_{max}$$

$$h := 0.0025 \text{ s}$$

Initialize the state variables.

$$i_{a_2} := 0 \text{ A}$$

$$\omega_{m_2} := 0 \frac{\text{rad}}{\text{s}}$$

$$T_{LL_2} := 0 \text{ N}\cdot\text{m}$$

$$t_2 := 0 \text{ s}$$

Define the torque load function.

$$T_{LI}(a, \omega) := a \cdot \omega$$

Set the linear torque constant.

$$b := 0.035 \frac{\text{N}\cdot\text{m}}{\frac{\text{rad}}{\text{s}}}$$

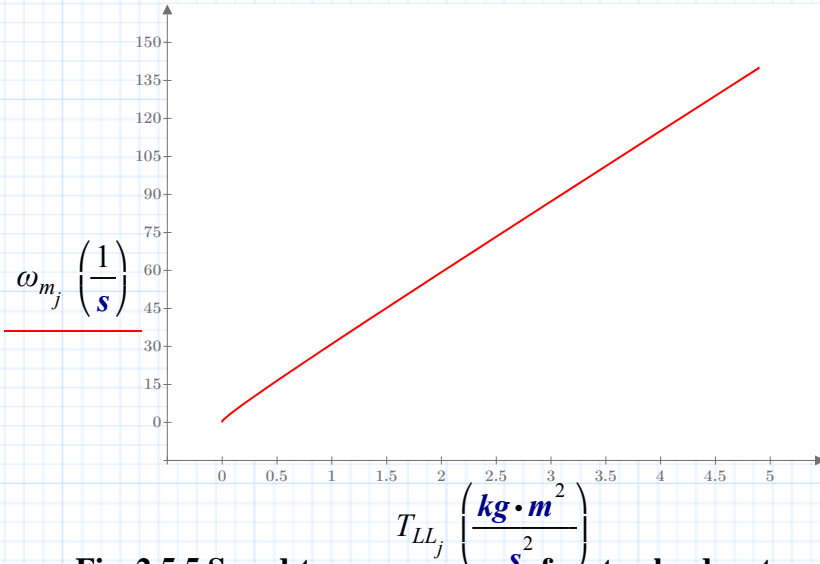
The state equations for the system are

$$\begin{bmatrix} i_{a_j} \\ \omega_{m_j} \\ T_{LL_j} \\ t_j \end{bmatrix} := \begin{bmatrix} i_{a_{j-1}} + \left( 1.5 \cdot h \cdot di_a(i_{a_{j-1}}, \omega_{m_{j-1}}, V_t) - 0.5 \cdot h \cdot di_a(i_{a_{j-2}}, \omega_{m_{j-2}}, V_t) \right) \\ \omega_{m_{j-1}} + \left( 1.5 \cdot h \cdot d\omega_m(i_{a_{j-1}}, \omega_{m_{j-1}}, T_{LL_{j-1}}) - 0.5 \cdot h \cdot d\omega_m(i_{a_{j-2}}, \omega_{m_{j-2}}, T_{LL_{j-2}}) \right) \\ T_{LI}(b, \omega_{m_{j-1}}) \\ t_{j-1} + h \end{bmatrix}$$

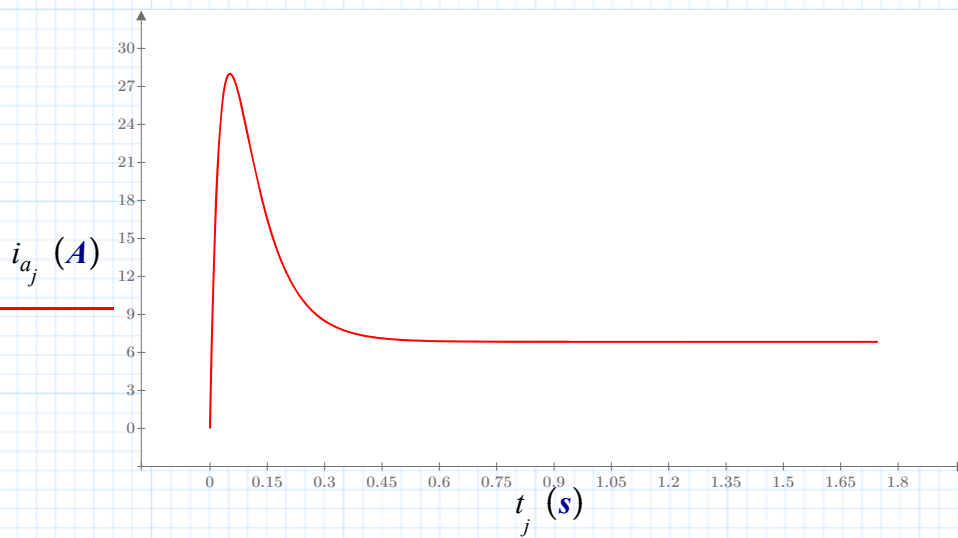
Plot the speed-torque trajectory of the linear load torque. Also plot the armature current and motor speed response for this classification of load.

Define the plot range.

$$j := 0 \dots j_{max}$$



**Fig. 2.5.5 Speed-torque response of motor-load system**



**Fig. 2.5.6 Armature current response (linear torque load)**

Steady-state values of armature current, motor speed and load torque are

$$i_{a_{j_{max}}} = 6.836 \text{ A}$$

$$T_{LL_{j_{max}}} = 4.9 \text{ N}\cdot\text{m}$$

$$\omega_{m_{j_{max}}} = 139.991 \frac{\text{rad}}{\text{s}}$$

$$\omega_{m_{j_{max}}} \cdot \text{rpm} = 14.66 \frac{1}{\text{s}^2}$$

## Section 2.5.5 DC Motor Starter Design

The previous simulations show that starting this dc motor using full armature voltage causes excessive armature current. The rated armature current was 8 amps. The transient armature was 3.5 times the rated value. In practice, additional starting resistance limits the armature inrush to values of no greater than 1-2 times the rated value of the motor. Figure 2.5.7 shows a typical starting scheme. Centripetal switches short the starting resistors,  $R_1$  to  $R_3$ , as the speed of the motor increases. The larger total resistance limits the inrush when the speed and back emf is low.

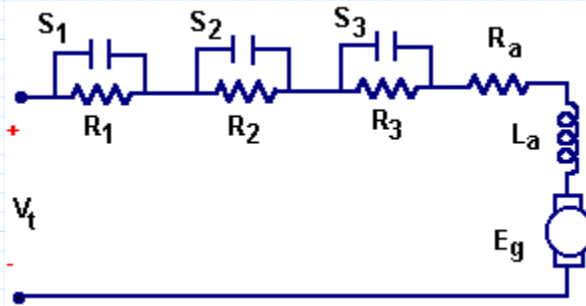


Fig. 2.5.7 DC motor starter schematic

Define the values of the starting resistance.

$$R_1 := 10 \, \Omega \quad \text{first stage of the starter}$$

$$R_2 := 10 \, \Omega \quad \text{second stage of the starter}$$

$$R_3 := 10 \, \Omega \quad \text{third stage of the starter}$$

$$R_a := 3.5 \, \Omega \quad \text{armature resistance}$$

When starting from rest, the armature resistance is

$$R_{T1} := R_1 + R_2 + R_3 + R_a = 33.5 \, \Omega$$

Define the settings of the speed switches.

$$S_1 := 30 \cdot \frac{\text{rad}}{\text{s}} \quad S_2 := 60 \cdot \frac{\text{rad}}{\text{s}} \quad S_3 := 100 \cdot \frac{\text{rad}}{\text{s}}$$

## Starter Resistance Function

Define a function that describes armature resistance as a function of motor speed. The unit-step (Heaviside) function,  $\Phi(x)$ , creates the step resistance function that simulates the motor starter operation.

$$R_T(\omega) := R_{T1} \cdot \Phi(\omega) - R_1 \cdot \Phi(\omega - S_1) - R_2 \cdot \Phi(\omega - S_2) - R_3 \cdot \Phi(\omega - S_3) \quad (2.5.10)$$

Plot the armature resistance as a function of speed. Let  $n$  vary from

$$n := 0 \cdot \frac{\text{rad}}{\text{s}}, 1 \cdot \frac{\text{rad}}{\text{s}} \dots 200 \cdot \frac{\text{rad}}{\text{s}}$$



Fig. 2.5.8 Motor starter resistance characteristics

## Modified State Equation

Modify the original state equation for  $di/dt$  to include the starting resistance function.

$$di_a(i, \omega, V) := -\frac{R_T(\omega)}{L_a} \cdot i + \frac{K_i}{L_a} \cdot \omega + \frac{1}{L_a} \cdot V \quad (2.5.11)$$

Re-define the iteration index.

$$j := 2 \dots j_{max} \quad h := 0.002 \cdot s$$

Initialize the state variables and the time index of the system.

$$i_{a_1} := 0 \cdot A \quad \omega_{m_1} := 0 \cdot \frac{\text{rad}}{\text{s}} \quad t_1 := 0 \cdot s$$

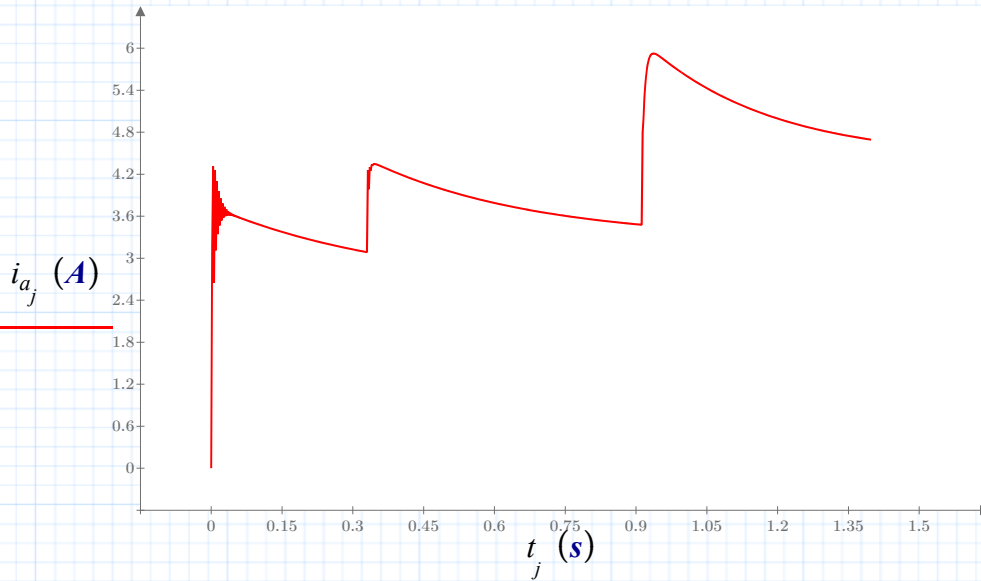
Compute the values for the system.

$$\begin{bmatrix} i_{a_j} \\ \omega_{m_j} \\ T_{LL_j} \\ t_j \end{bmatrix} := \begin{bmatrix} \left( i_{a_{j-1}} + 1.5 \cdot h \cdot di_a(i_{a_{j-1}}, \omega_{m_{j-1}}, V_t) \right) - 0.5 \cdot h \cdot di_a(i_{a_{j-2}}, \omega_{m_{j-2}}, V_t) \\ \left( \omega_{m_{j-1}} + 1.5 \cdot h \cdot d\omega_m(i_{a_{j-1}}, \omega_{m_{j-1}}, T_{LL_{j-1}}) \right) - 0.5 \cdot h \cdot d\omega_m(i_{a_{j-2}}, \omega_{m_{j-2}}, T_{LL_{j-2}}) \\ T_{L1}(b, \omega_{m_{j-1}}) \\ t_{j-1} + h \end{bmatrix}$$

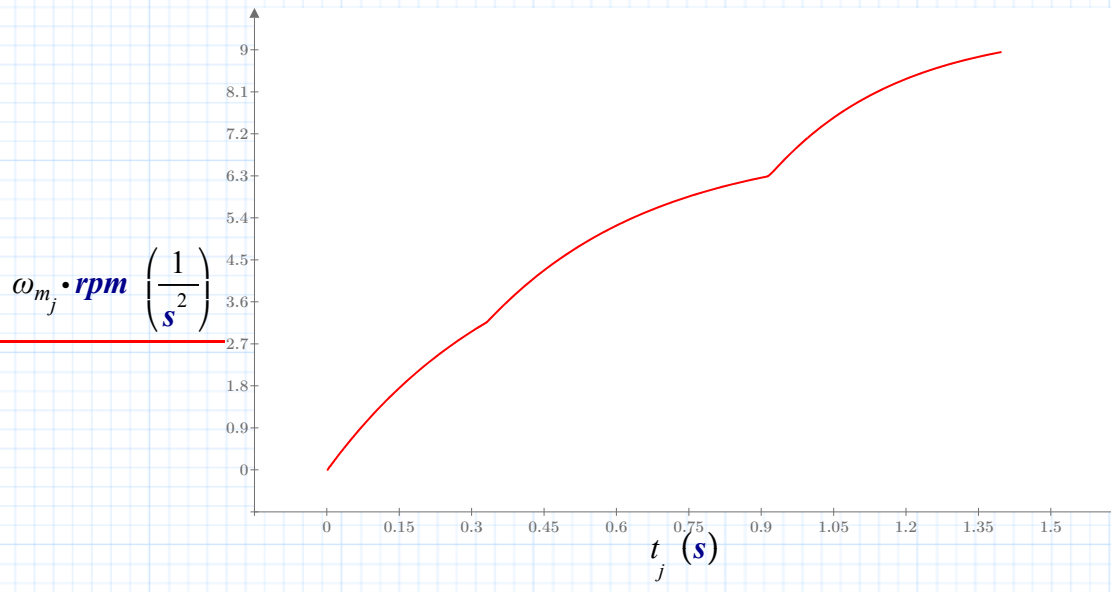
Plot the response of the system including the motor starter reaction.

Define the plot range.

$$j := 0 \dots j_{max}$$



**Fig. 2.5.9 Starter current response**



**Fig. 2.5.10 Motor starter speed response**

Final values of speed, torque, and armature current for the system using a step-resistance starter are

$$i_{a_{j_{max}}} = 4.693 \text{ A}$$

$$T_{LL_{j_{max}}} = 2.991 \text{ N}\cdot\text{m}$$

$$\omega_{m_{j_{max}}} \cdot rpm = 8.953 \frac{1}{s}$$



## Section 2.5.6 References

1. Chua, Leon O. and Lin, Pen-Min, *Computer Aided Analysis of Electric Circuits: Algorithms and Computational Techniques*, Prentice-Hall, Inc., 1975.