Section 2.5a DC Motor Protection

Section 2.5.1 DC Motor Modeling

DC motors require gradual application of the armature voltage to prevent excessive armature current transients. Low armature resistance and zero back emf, Eg, allow large inrush currents to flow. Inrush current creates magnetic forces that mechanically stress the field windings and armature windings of the machine. The rapid application of armature voltage also produces an acceleration that causes the armature speed to exceed the rated value momentarily. This is called speed over-shoot. Damage to the mechanical load can occur when the over-shoot is excessive. This application outlines a method for simulating the transient response of a separately excited dc motor with various load torque characteristics.

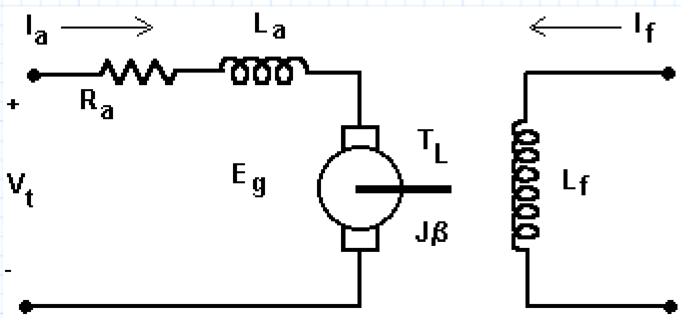

Fig. 2.5.1 Separately excited DC motor model

Figure 2.5.1 shows the model that represents the separately excited dc motor. The field current is constant so the effects of the field inductance are ignored. Varying the armature terminal voltage will control the speed and torque of the machine. The mechanical load torque, TL, depends on the type of mechanical load driven by the machine.

The armature resistance, Ra, the armature inductance, La, and the back emf, Eg, represent the armature section of the motor model. The rotational inertia, J, is the combined inertia of the load and armature. The damping coefficient, β, accounts for bearing losses and windage effects in the model.

Steady-State Relationships

The motor torque constant, Ki, relates the field current to the back emf. Field current magnitude determines the value of this constant. In the MKS system, the units on the torque constant are V-sec/radian. Measuring the generated voltage at different speeds for a given value of field current gives the experimental value of Ki for a separately excited machine. The following formulae provide an analytical method of determining the value of Ki if the motor construction details are known.

$$K_i = K_e \cdot \phi_f$$

where ϕf is the flux produced by the field current.

$$K_e = \frac{P \cdot z}{2 \cdot \pi \cdot a}$$

where P is the number of machine poles, z is the total number of conductors in the armature, and a is the number of parallel current paths.

The constant Ki relates the armature current to the developed torque by

$$T_{em} = K_i \cdot I_a$$

where Ia is the armature current.

The constant Ki also relates the back emf to the motor speed by

$$E_g = K_i \cdot \omega_m$$

where ωm is the motor speed.

Dynamic Model of the dc Motor

The state equations of this system are

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} \cdot i_a + -\frac{K_i}{L_a} \cdot \omega_m + \frac{1}{L_a} \cdot V_t$$ armature current **(2.5.1)**

$$\frac{d\omega_m}{dt} = \left(\frac{K_i}{J}\right) \cdot i_a + -\frac{\beta}{J} \cdot \omega_m - \frac{1}{J} \cdot T_L$$ armature speed **(2.5.2)**

where the state variables are armature current, ia, and armature speed, ωm.

Numerical Integration Formulae

Numerical integration of the state equations gives the transient response of the motor. The Adams-Bashford second-order method [1] provides a simple, quick method of solving the state equations of the motor. Substituting the state variables of the motor system into the Adams-Bashford formulae gives

$$i_{a_k} = i_{a_{k-1}} + \frac{3}{2} \cdot h \cdot I_{a_{k-1}} - \frac{1}{2} \cdot I_{a_{k-2}}$$ **(2.5.3)**

$$\omega_{m_k} = \omega_{m_{k-1}} + \frac{3}{2} \cdot h \cdot W_{m_{k-1}} - \frac{1}{2} \cdot W_{m_{k-2}}$$ **(2.5.4)**

The integration time interval, h, must be at **least 10 times smaller than the smallest system time constant** to insure that the solution is numerically stable. Examining the electrical and mechanical time constants of the system helps determine the acceptable values of the time step for a system.

Section 2.5.2 Mechanical Load Modeling

Motor mechanical loads can take several forms. The load characteristics are generally modeled by

TL = c constant torque,
TL = aωm linear torque, or
TL = aωm2 parabolic torque (also known as "fan loading")

As functions of motor speed, these become

$T_{Lc}(c) := c$ **(2.5.5)**

$T_{L1}(a,\omega) := a \cdot \omega$ **(2.5.6)**

$T_{L2}(a,\omega) := a \cdot \omega^2$ **(2.5.7)**

Plot these characteristics over a typical speed range.

$\omega := 0 \cdot \frac{rad}{s}, 10 \cdot \frac{rad}{s} .. 200 \cdot \frac{rad}{s}$

Define the function constants and the system units. The MKS system of units is used in this simulation.

$a := .00018 \cdot \frac{N \cdot m}{\left(\frac{rad}{s}\right)^2}$ constant for parabolic torque characteristic

$b := 0.035 \cdot \frac{N \cdot m}{\frac{rad}{s}}$ constant for linear torque characteristic

$c := 5 \cdot N \cdot m$ constant torque value

Plot the three types of motor load functions using the values of the constants a, b, and c over the range of motor speed defined above.

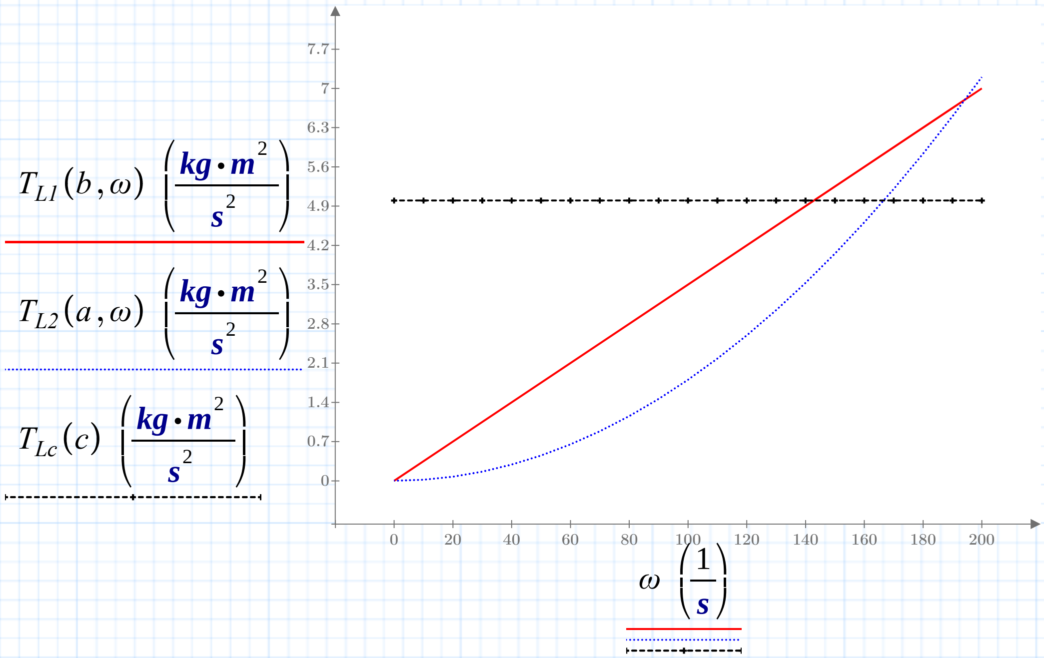

Fig. 2.5.2 Typical motor mechanical torque loads