3 ELECTRICAL TRANSIENTS

Section 3.2b Transformer Energization

Section 3.2.3 Transformer Saturation Modeling

The transformer circuit of Figure 3.2.1 can be modified to include the nonlinear effects of the core characteristic. This modification can be accomplished by replacing the ideal magnetizing inductor in Figure 3.2.1 with a nonlinear impedance. The flux-current characteristic of this impedance is chosen to match the magnetic core characteristic of the transformer.

It is convenient to represent the transformer characteristic using a number of linear segments, as was demonstrated in the previous graphs. This method preserves the essential properties of the saturation and reduces the simulation complexity. An example using three segments to represent the saturation characteristic is presented next.

A three-segment representation of the transformer saturation is shown by the dashed lines in **Figure 3.2.2**. Segment **A'A** represents the unsaturated portion of the transformer characteristic. The slope of this segment is chosen to equal the unsaturated transformer magnetizing inductance. Segments **B'A'** and **AB** represent the saturation portion of the negative and positive regions of the transformer characteristic, respectively. The slope of these segments are chosen to equal the air core inductance of the transformer. The point on the flux-current plane where the saturation segments intersect the linear segment of the characteristic is defined as the knee point of saturation. The knee point defines the minimum flux and magnetizing current for which saturation is appreciable. The characteristic representation is calculated as follows:

From name plate data, we obtain

$I_{mo} := 6\ A$	steady-state magnetizing current, under rated voltage
$X_s := 8\ \Omega$	transformer leakage reactance
$V_s := 7000\ V$	rated primary voltage

Other typical values for transfomer impedance are given in **Table 3.2.1**. The knee point of saturation can be expressed as the pu magnitude of excitation voltage for which saturation starts. Typically this is 10% above rated voltage.

$K_\sigma := 1.1$ transformer voltage at saturation knee-point in pu

Determine the inductance of the magnetizing branch of the transformer from the steady state magnetizing current. Assume the transformer operates at rated voltage (1 pu).

$\omega := 377\ \frac{rad}{s}$ source frequency

$$L_m := \frac{\sqrt{\frac{2}{3}} \cdot V_s}{\omega \cdot I_{mo}} = 2.527\ H$$

Determine the air core inductance of the transformer from the leakage reactance. The air core inductance is 10-20 times greater than the leakage inductance.

$$L_s := \frac{X_s}{\omega} \cdot 10 = 0.212 \; \boldsymbol{H}$$

Determine the core linkage flux at the knee point.

$$\lambda\sigma_\sigma := \frac{\sqrt{\frac{2}{3}} \cdot V_s \cdot K_\sigma}{\omega} = 16.676 \; \boldsymbol{Wb}$$

Determine the magnetizing current at the knee point.

$$I_{m\sigma} := \frac{\lambda\sigma_\sigma}{L_m} = 6.6 \; \boldsymbol{A}$$

Define a ramp function of x using the step function provided by Mathcad. This function will produce the piece-wise linear plot of the transformer magnetic characteristic.

$$r(x) := x \cdot \Phi(x)$$

Define the function for flux. The value of the magnetizing current corresponding to maximum unsaturated flux shifts the ramp characteristic of the plot.

$$\lambda(x) := L_m \cdot \left(r\left(x + I_{m\sigma}\right) - r\left(x - I_{m\sigma}\right)\right) - L_s \cdot \left(r\left(-x - I_{m\sigma}\right) - r\left(x - I_{m\sigma}\right)\right) - \lambda\sigma_\sigma$$

Plot the magnetizing characteristic with respect to magnetizing current.

$$I_m := -20 \; \boldsymbol{A}, -19.5 \; \boldsymbol{A} .. 20 \; \boldsymbol{A}$$

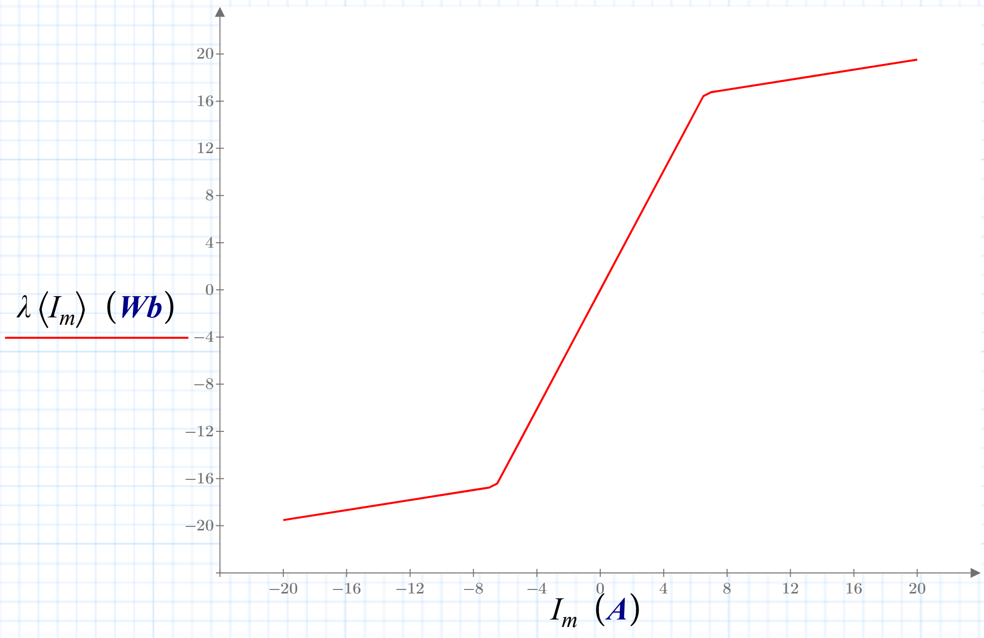

Fig. 3.2.4 Piece-wise linear saturation representation

The method presented above can be extended to more linear segments by appropriately including more knee points in the characteristic. However, for the study of inrush current phenomena, three segments give a satisfactory approximation.

The piece-wise linear representation of the transformer characteristic will be used below to derive the system simulation.

Section 3.2.4 Simulation of Transformer Energization

In this section, an example is given that demonstrates the effect of the transformer energization on the system voltage. A single phase system representation is used to simplify the calculations. The reader can extend the equations to a three-phase system representation.

A numerical integration method is used to obtain the system solution for the transformer energization. For convenience, the inverse relationship between the flux and magnetizing current is used here. Therefore,

defining the magnetizing current as a function of the flux:

$$I_m(x) := \frac{1}{L_m} \cdot \left(r\left(x + \lambda\sigma_\sigma\right) - r\left(x - \lambda\sigma_\sigma\right)\right) - \frac{1}{L_s} \cdot \left(r\left(-x - \lambda\sigma_\sigma\right) - r\left(x - \lambda\sigma_\sigma\right)\right) - I_{m\sigma}$$

Define the system voltage as a function of time.

$Vs := 7000\ V$ voltage amplitude in pu

$$V(t) := \sqrt{\frac{2}{3}} \cdot Vs \cdot \cos\left(\omega \cdot t + \phi o_o\right)$$

The phase angle in the above function corresponds to the instant on the system voltage waveform at which transformer energization occurs. This phase angle, along with the trapped flux, determines the severity of the inrush. Choose

$\phi o_o \equiv -10\ deg$

Define the trapped flux in the transformer core

$\lambda o_o := 5\ Wb$

The external system is represented by a series RL equivalent in series with the system voltage. Where

$R_L := 3.5\ \Omega$ line resistance

$X_L := 25\ \Omega$ line inductive reactance

These values correspond to the following pu inductance:

$L := \frac{X_L}{\omega} = 0.066\ H$ inductance

A shunt capacitor is also considered at the transformer terminals. This capacitor represents compensation or is used to adjust for system resonance. Assume

$X_C := 200\ \Omega$ shunt capacitive reactance

corresponding to

$C := \frac{1}{\omega \cdot X_C} = \left(1.326 \cdot 10^{-5}\right)\ F$ pu capacitance

The combination of the shunt capacitor and system inductance result in a resonant frequency of

$$f_r := \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}} = 169.71\ \frac{1}{s}$$

The system resonance occurs at a frequency that is near the harmonic frequencies of the inrush current of the transformer. Therefore, a harmonic interaction can be expected upon energization between the system and the transformer nonlinearity.

The secondary side of the transformer is considered to be open (no load). This representation is preferred since it results in pessimistic calculations of the system response. Adding load will increase the system damping and, therefore, will significantly reduce the effects of the inrush current.

The differential equations for the system are assembled following the procedure and examples of **Section 3.1a**.

Define the system differential equations.

$$Di\left(I_L, V_C, t\right) := \frac{-R_L \cdot I_L - V_C + V(t)}{L}$$ line current derivative

$$Dv_c\left(I_L, V_C\right) := \frac{I_L - I_m\left(V_C\right)}{C}$$ capacitor voltage derivative

Notice that in the expression of the capacitor voltage derivative, the transformer saturation is represented by the magnetizing current injection.

Define the time step of the integration. This time step must be at least 10 times smaller than the smallest time constant in the system to prevent numerical oscillations from developing in the solution.

$$dt := .3 \cdot \mathbf{ms}$$

Define maximum simulation time.

$$T := 0.20\ \mathbf{s}$$

$$h_1 := 1.5 \cdot dt \qquad h_2 := 0.5 \cdot dt$$

$$N := \text{floor}\left(\frac{T}{dt}\right)$$

$$k := 2 .. N$$ iteration counter

Define the initial conditions for the system.

Initialize time.

$$t_0 := 0.0\ \mathbf{s}$$

$$t_1 := 0.0\ \mathbf{s}$$

Compute time at each interval.

$$t_k := k \cdot dt$$

Initialize the capacitor voltage.

$$v_{c_0} := 0.0\ V$$

Assume fault condition in the system prior to energization.

$$v_{c_1} := 0.0\ V$$

Initialize the line current.

$$i_0 := 0.0\ A$$

$$i_1 := 0.0\ A$$

The initial value of core flux is the residual flux defined above.

$$\lambda_0 := \lambda o_o$$

$$\lambda_1 := \lambda o_o$$

Vectorize the state equations and solve the system of equations. The solution gives the values of transformer current, capacitor voltage, and transformer flux for the evaluation interval.

$$\begin{bmatrix} i_k \\ v_{c_k} \\ \lambda_k \end{bmatrix} := \begin{bmatrix} i_{k-1} + h_1 \cdot Di\left(i_{k-1}, v_{c_{k-1}}, t_{k-1}\right) - h_2 \cdot Di\left(i_{k-2}, v_{c_{k-2}}, t_{k-2}\right) \\ v_{c_{k-1}} + h_1 \cdot Dv_c\left(i_{k-1}, \left(\lambda_{k-1}\right)\right) - h_2 \cdot Dv_c\left(i_{k-2}, \left(\lambda_{k-2}\right)\right) \\ \lambda_{k-1} + h_1 \cdot \left(v_{c_{k-1}}\right) - h_2 \cdot \left(v_{c_{k-2}}\right) \end{bmatrix}$$

The system response is shown below.

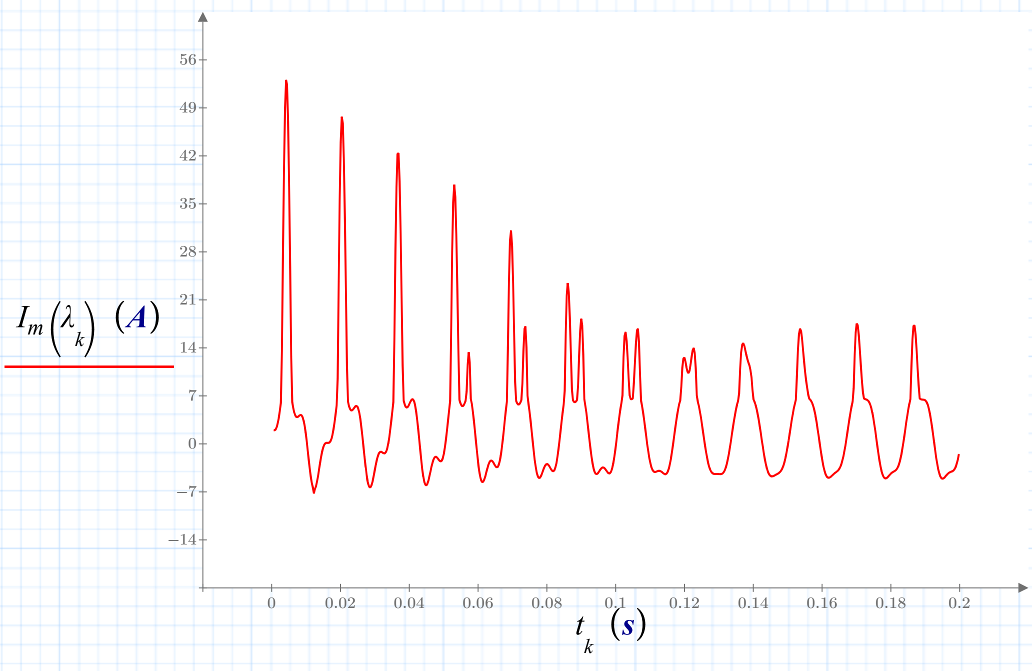

Fig. 3.2.5 Transformer inrush

This graph shows the effects of the dc offset and nonlinear magnetic characteristic. The first cycle is severely offset by a dc component which is damped out in about three cycles.

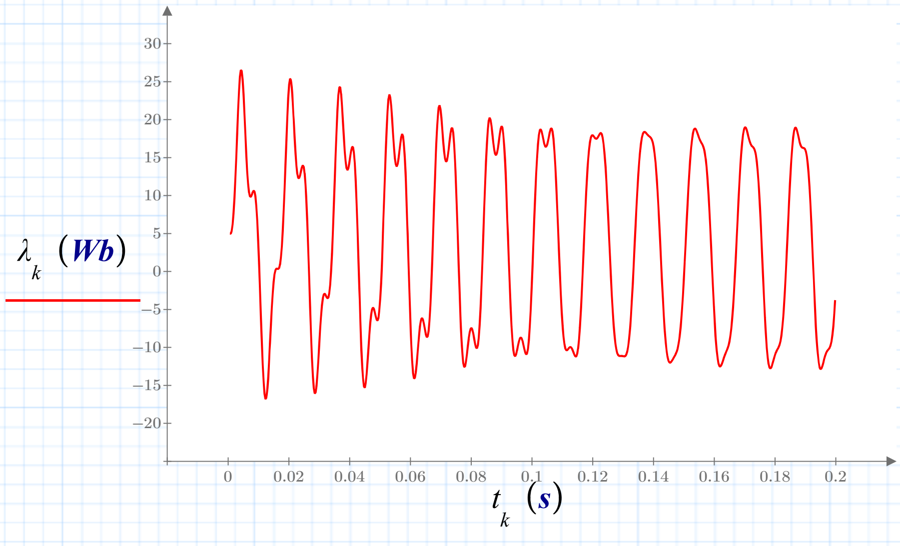

Fig. 3.2.6 Transformer flux

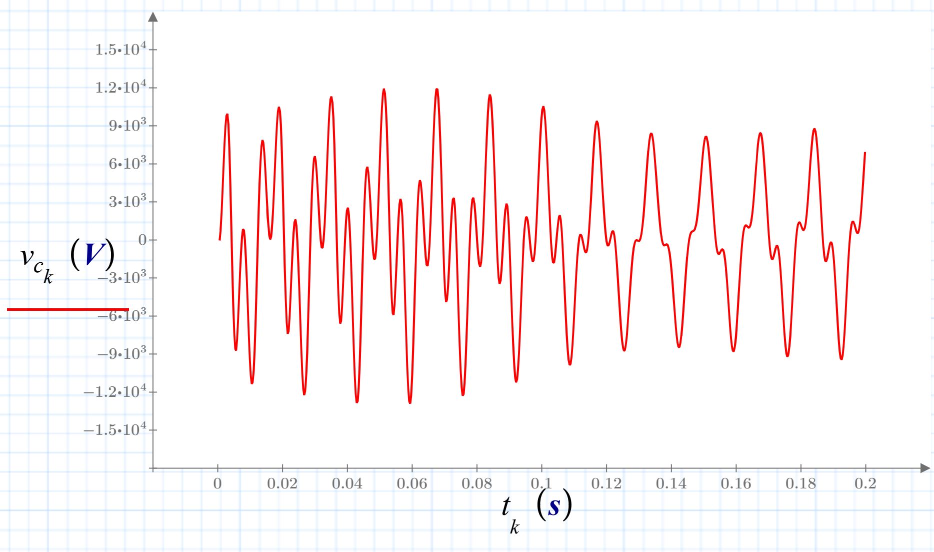

Fig. 3.2.7 Transformer bus voltage

On the previous graph, observe the effect of the non-sinusoidal inrush current on the bus voltage. The result is a harmonic overvoltage.

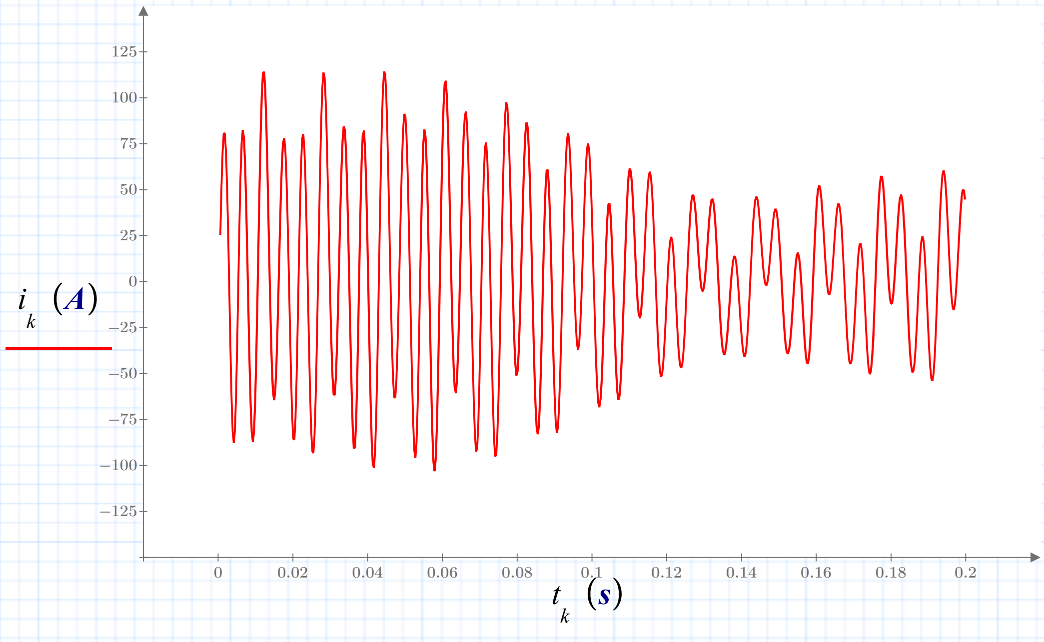

Fig. 3.2.8 Line current

It is possible to compensate for some of these effects by adding a preinsertion resistor. For more information on this process, see **Section 3.2c**.