CHAPTER 1 STEADY-STATE HEAT CONDUCTION

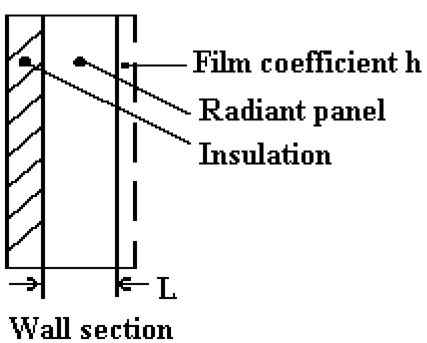
1.3 Walls with Internal Heat Generation

Heat may be generated within a wall. For example, electric radiant panels, which often form the interior layer of a ceiling or a wall to provide radiant heat, contain electric resistance elements which generate heat. This heat can be approximated as internal heat generation.

We often may assume one-dimensional heat conduction. If a steady-state analysis is performed then the relevant energy balance equation is

$$k \cdot d^2 \frac{T}{dx^2} + Qg = 0$$

where k is the thermal conductivity of the wall layer and Qg is the rate of internal heat generation. Consider, for example, a radiant panel of area 1 square meter and thickness 13 mm made of gypsum board with electric resistance elements built into it with a total power output 250 W assumed uniformly generated within the panel.



$h := 12 \frac{W}{m^2 \cdot \Delta°C}$ $\qquad$ $T_{Room} := 20\ \Delta°C$

$A := 1\ m^2$ $\qquad$ unit area

$L := 0.013\ m$ $\qquad$ panel thickness

$Volume := A \cdot L$

$x := 0.001\ m, 0.002\ m .. 0.013\ m$ $\qquad$ distance from back side of panel

$k := 0.16 \frac{W}{m \cdot \Delta°C}$ $\qquad$ $Qg := \frac{250\ W}{Volume}$

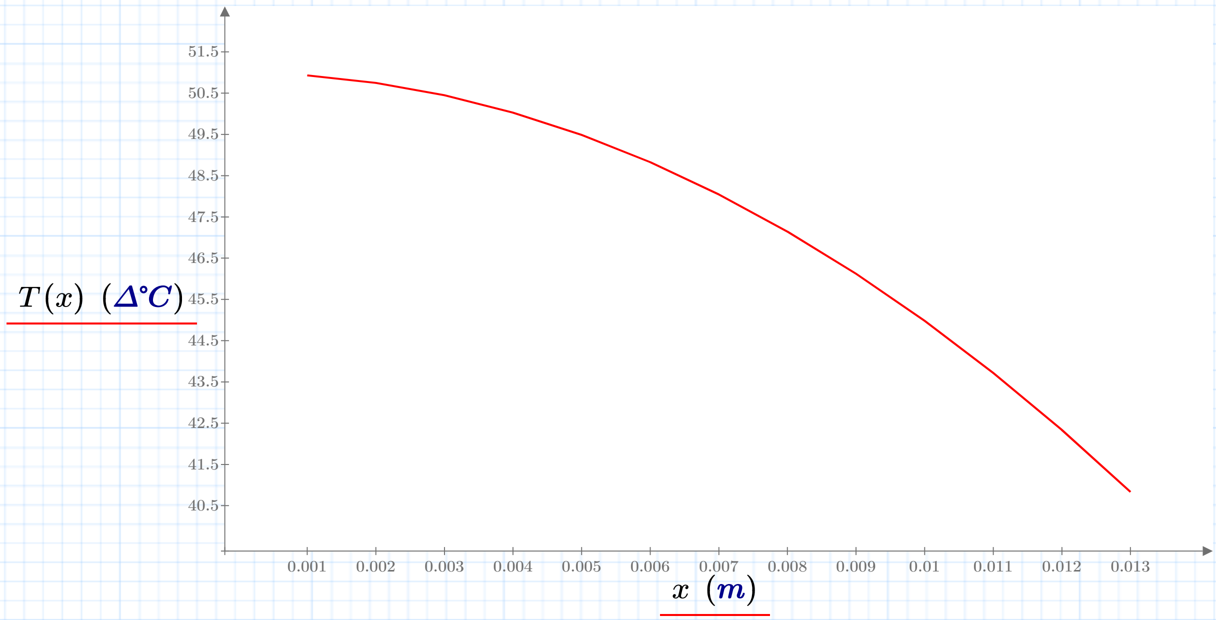
Boundary conditions

1. Adiabatic at x = 0: $\frac{d}{dx} T = 0$

2. Convective at x = L: $-k \cdot \frac{d}{dx} T = h \cdot (T - T_{Room})$

$$T(x) := T_{Room} + \left(\frac{Qg \cdot L}{h} + Qg \cdot \frac{L^2 - x^2}{2 \cdot k} \right)$$

Examination of the graph on the right reveals that the maximum temperature is at the interface between the insulation and the panel (x=0).



Therefore, maximum temperature is $T(0\ m) = 50.99\ \Delta°C$