CHAPTER 2 TRANSIENT HEAT CONDUCTION

2.1 Lumped Parameter Model and the Thermal Network Method

All materials can store heat. Therefore, when a temperature or heat flux change is imposed it takes some time to reach steady state. During this time, we must perform a transient analysis to determine temperatures and heat flows. For systems with negligible thermal resistance, we may perform a simplified analysis.

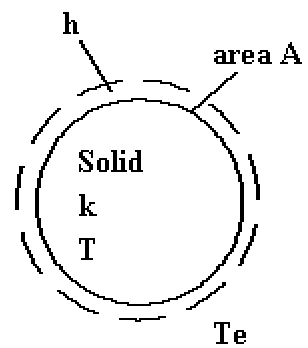
The Biot number (Bi) is a dimensionless number, equal to the ratio of the internal thermal resistance (1/k) to the surface thermal resistance (1/ h·L). This number determines whether lumped parameter analysis is applicable.

$$Bi = \frac{h \cdot L}{k}$$

where L is a characteristic length (L = Volume / Area).

If B_i is small (< 0.1), we can assume with reasonable accuracy that the body is isothermal, and lumped parameter analysis can be performed.

Consider the cooling of a resistance element in an electric heater at initial temperature T_o exposed to an environment at T_e. Assume that the element is a cylindrical wire.



Objective: *Determine T(t)*

Initial condition: T(t=0)= To, Environment temp. = T_e

Given data:

$L := 0.5\ m$ wire length

$D := 0.001\ m$ diameter

$k := 374\ \frac{W}{m \cdot \Delta°C}$ thermal conductivity

$c := 383 \ \frac{J}{kg \cdot \Delta°C}$ specific heat capacity

$\rho := 8930 \ \frac{kg}{m^3}$ density

$h := 10 \ \frac{W}{m^2 \cdot \Delta°C}$ heat transfer coefficient

$A := \pi \cdot D \cdot L$ surface area

$V := \pi \cdot \frac{D^2}{4} \cdot L$ wire volume

$L := \frac{V}{A}$ characteristic length

$Bi := h \cdot \frac{L}{k} = ?$

$T_o := 150 \ \Delta°C$ $T_e := 40 \ \Delta°C$

Energy balance (Biot number < 0.1):

change in internal energy during time dt = net heat flow from body during dt

$$-C \cdot dT = \left(T - T_e\right) \cdot \frac{dt}{R}$$

where

$C := c \cdot \rho \cdot V$ thermal capacitance of body

$R := \frac{1}{A \cdot h}$ surface resistance

Therefore

$$\frac{d\left(T - T_e\right)}{T - T_e} = -\frac{dt}{R \cdot C}$$

Integrating both sides and applying the initial condition with

$i := 0..5$

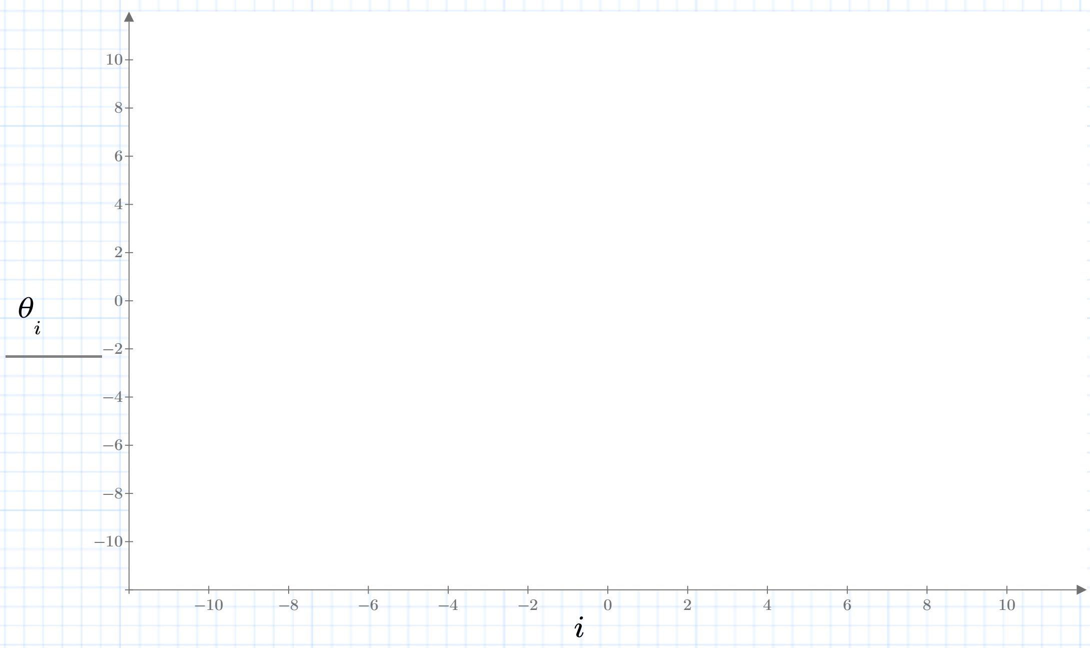
$t_i := i \cdot R \cdot C$ $\qquad t_1 = ?$ $\qquad$ time constant

Let

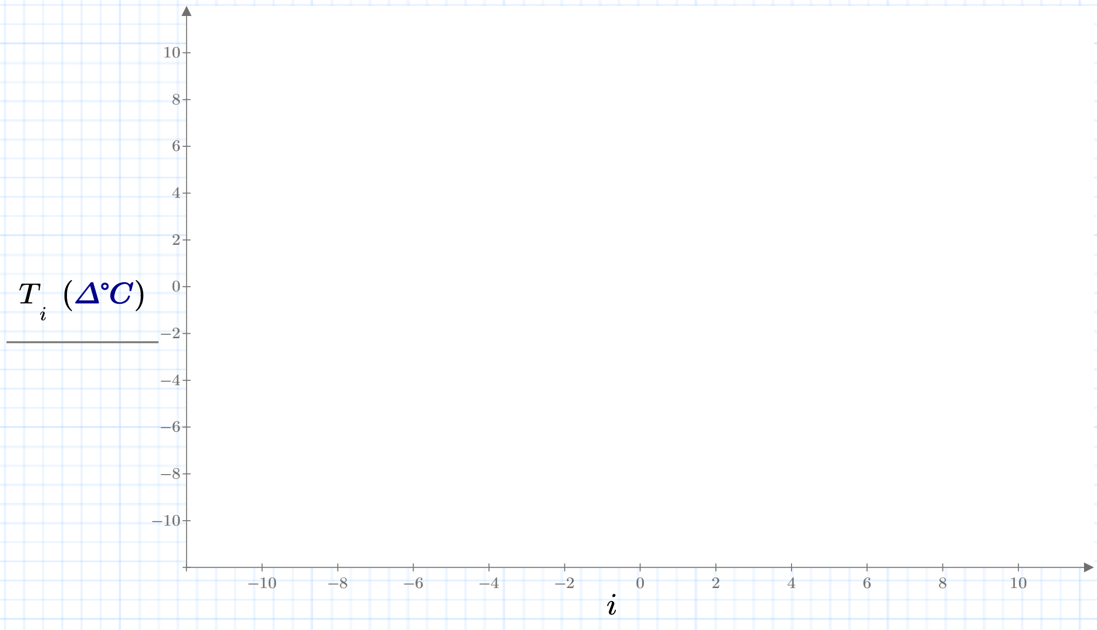
$\theta = \dfrac{T - T_e}{T_o - T_e}$ $\qquad$ dimensionless temperature

Solution:

$\theta_i := \exp\left(-\dfrac{t_i}{R \cdot C}\right)$



$$T_i := \theta_i \cdot (T_o - T_e) + T_e$$



Note that 63% of the change occurs after one time interval (from i=0 to i=1), i.e. a temperature drop of 0.63·(T_o - T_e)

$$\theta_0 - \theta_1 = 18.075\ 1\%$$

$$\frac{T_0 - T_1}{T_0 - T_5} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\ 1\%$$

Thermal Network Model

The body can be modeled with an isothermal capacitance C in parallel with a resistance R (equal to 1/A·h) both connected to the environment at temperature T_e.
Note that the capacitance (or capacitances) in a thermal network are always modeled as connected between a reference temperature (usually the environment temperature T_e) and their own temperature T. Heat flow into the capacitance means flow from T to T_e.

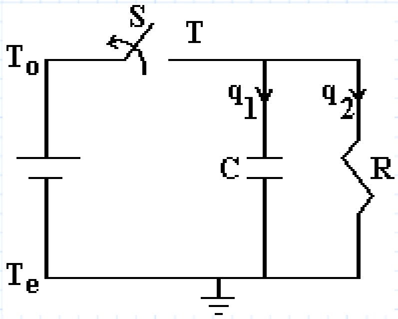
The initial condition T(t=0) = T_o can be simulated by a "battery" T_o-T_e connected through a switch S to the capacitance, and also connected to the reference node.

The constitutive equation for a thermal resistance is simply

$$q = \frac{T_{hot} - T_{cold}}{R}$$

Similarly, for a capacitance we have

$$q=C\cdot\frac{d}{dt}\left(T-T_e\right)$$ where T_e is constant



Switch S opens at t = 0. Energy balance at T:

Heat flow into C + Heat flow to R = 0

or $q_1+q_2=0$

$$C\cdot\frac{d}{dt}(T-Te)+\frac{T-T_e}{R}=0$$

$$\frac{d}{dt}(T-Te)+\frac{T-T_e}{R\cdot C}=0$$

The same solution as before is obtained (initial condition $T(0)=T_o$).

In **Chapter 9**, we will see how a transient thermal network of a room is developed and solved to determine heating/cooling loads and room temperatures.