CHAPTER 2 TRANSIENT HEAT CONDUCTION

2.3 Semi-Infinite Slab: Radiant Heat Flux on Floor

A high-intensity infrared ceiling heating system is employed to keep a factory floor warm. If the heating system is switched on with the floor initially at temperature T_i, determine the radiant heat flux intensity on the floor required to warm the floor surface to a temperature T_s after time t. What is the floor temperature at the bottom surface of the concrete slab?

Assume that the concrete floor slab is 20 cm thick and that the semi-infinite model applies.

Concrete properties (assume medium density):

$\rho := 1500 \ \frac{kg}{m^3}$ density

$k := 1.2 \ \frac{W}{m \cdot \Delta°C}$ thermal conductivity

$c := 800 \cdot \frac{J}{kg \cdot \Delta°C}$ specific heat capacity

$\alpha := \frac{k}{\rho \cdot c}$ thermal diffusivity

$T_i := 7 \ \Delta°C$ initial floor temperature

$T_s := 18 \ \Delta°C$ final floor surface temperature after time t

$t := 3 \ hr$

For a semi-infinite slab with constant surface heat flux q_s after time t = 0, we have

$$q_s := \frac{(T_s - T_i) \cdot k}{2 \cdot \sqrt{\frac{\alpha \cdot t}{\pi}}} = 112.566 \ \frac{W}{m^2}$$

We will determine the temperature at different depths for a semi-infinite model to see how accurate it is for the present case:

$x := 0.0\ m, 0.1\ m .. 0.5\ m$

$$T(x) := \left[2 \cdot \frac{q_s}{k} \cdot \sqrt{\frac{\alpha \cdot t}{\pi}} \cdot \exp\left(-\frac{x^2}{4 \cdot \alpha \cdot t} \right) - \frac{q_s \cdot x}{k} \cdot \left(1 - \operatorname{erf}\left(\frac{x}{2 \cdot \sqrt{\alpha \cdot t}} \right) \right) \right] + T_i$$

Examination of the temperature variation with depth at t = 3 hours shows that the heat flux on the surface has no significant effect below a depth of 0.2 m. Therefore, the present model is satisfactory. Note that if there was a significant heating of the lower boundary of the slab then heat loss through this boundary would have to be included in the analysis.