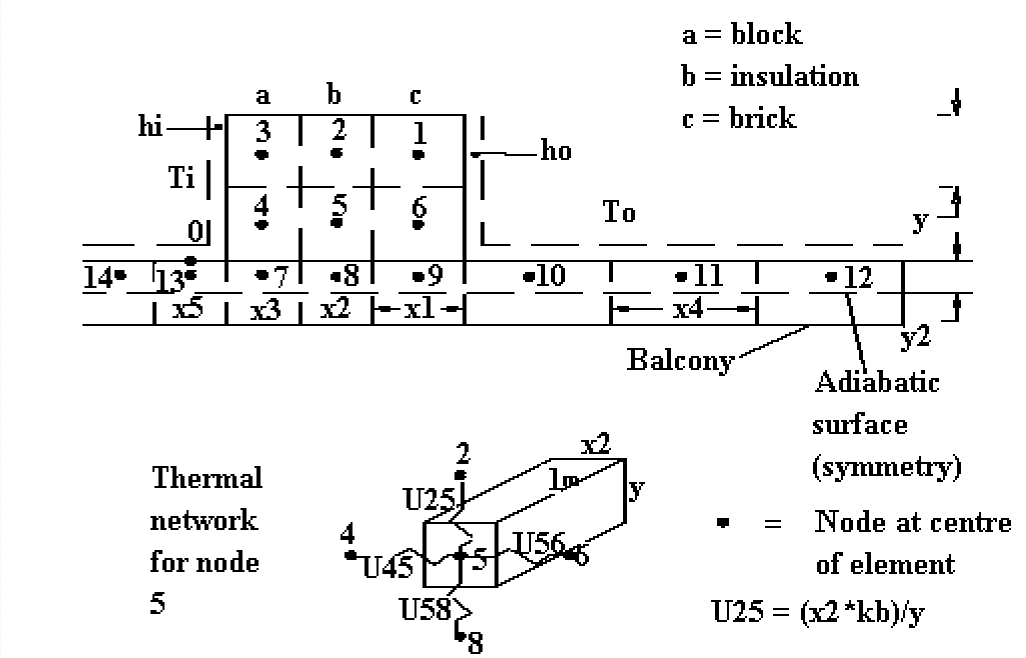
CHAPTER 3 HEAT CONDUCTION IN BUILDINGS WITH THE FINITE DIFFERENCE METHOD

3.1 Steady-State Two-Dimensional Analysis of Thermal Bridges

Thermal bridges, that is thermal short-circuits in the building envelope, can be analyzed with a two-dimensional thermal network to determine heat loss and low temperatures which may cause condensation. Consider, for example, the thermal bridge formed by a balcony which is an extension of a concrete floor slab.

This wall section has been subdivided into 14 elements. Each node is located at the center of an element. The thermal resistances representing two-dimensional conduction in an element of width x2 and height y (depth perpendicular to x-y plane is assumed equal to 1) are determined as shown below:



Properties of wall (thermal conductivity):

$k_a := 1 \ \dfrac{W}{m \cdot \Delta°C}$ block

$k_b := 0.03 \ \dfrac{W}{m \cdot \Delta°C}$ insulation

$k_c := 1.5 \ \dfrac{W}{m \cdot \Delta°C}$ brick

$k_d := 1.7 \ \dfrac{W}{m \cdot \Delta°C}$ concrete

Basic assumptions in setting-up two-dimensional finite-difference model:

1. At a distance of 60 cm from the floor (top surface of elements with nodes 1-3), the temperature distribution is one-dimensional .

2. Assume an adiabatic boundary condition in the center of the floor slab.

Element dimensions:

$x1 := 0.1\ m$ $\quad x2 := 0.05\ m$ $\quad x3 := 0.1\ m$

$x4 := 0.4\ m$ $\quad x5 := 0.3\ m$

$y := 0.3\ m$ $\quad y2 := 0.1\ m$

$h_i := 9\ \dfrac{W}{m^2 \cdot \Delta°C}$ $\quad h_o := 30\ \dfrac{W}{m^2 \cdot \Delta°C}$

$T_o := -10\ \Delta°C$ $\quad T_i := 20\ \Delta°C$

$L := 1\ m$ assume unit width.

Calculation of conductances Uij between nodes i and j:

$$U1o := \frac{1}{\dfrac{x1}{2 \cdot k_c \cdot y} + \dfrac{1}{y \cdot h_o}} \qquad U12 := \frac{1}{\dfrac{x1}{2 \cdot k_c \cdot y} + \dfrac{x2}{2 \cdot k_b \cdot y}}$$

$$U56 := U12$$

$$U23 := \frac{1}{\dfrac{x3}{2 \cdot k_a \cdot y} + \dfrac{x2}{2 \cdot k_b \cdot y}} \qquad U3i := \frac{1}{\dfrac{x3}{2 \cdot k_a \cdot y} + \dfrac{1}{y \cdot h_i}}$$

$$U45 := U23 \qquad U34 := k_a \cdot \frac{x3}{y}$$

$$U25 := k_b \cdot \frac{x2}{y} \qquad U16 := k_c \cdot \frac{x1}{y}$$

$$U6o := U1o \qquad U4i := U3i$$

$U13\_14 := k_d \cdot \frac{y2}{x5}$

$U7\_13 := \frac{k_d \cdot 2 \cdot y2}{x5 + x3}$

$U78 := \frac{k_d \cdot y2 \cdot 2}{x3 + x2}$

$U89 := \frac{k_d \cdot y2 \cdot 2}{x2 + x1}$

$U9\_10 := \frac{k_d \cdot y2 \cdot 2}{x1 + x4}$

$U10\_11 := U13\_14$

$U11\_12 := U13\_14$

$$U47 := \frac{1}{\frac{y}{2 \cdot k_a \cdot x3} + \frac{y2}{2 \cdot k_d \cdot x3}}$$

$$U58 := \frac{1}{\frac{y}{2 \cdot k_b \cdot x2} + \frac{y2}{2 \cdot k_d \cdot x2}}$$

$$U69 := \frac{1}{\frac{y}{2 \cdot k_a \cdot x1} + \frac{y2}{2 \cdot k_d \cdot x1}}$$

$$U10o := \frac{1}{\frac{y2}{2 \cdot k_d \cdot x4} + \frac{1}{x4 \cdot h_o}}$$

$U11o := U10o$

$$U12o := U10o + \frac{1}{\frac{x4}{2 \cdot y2 \cdot k_d} + \frac{1}{y2 \cdot h_o}}$$

$U0\_13 := \frac{x5 \cdot k_d}{y2}$

$U0i := x5 \cdot h_i$

$$U14i := \frac{1}{\frac{y2}{x5 \cdot k_d} + \frac{1}{x5 \cdot h_i}}$$

$$U7\_13 := \frac{1}{\frac{x5}{2 \cdot k_d \cdot y2} + \frac{x3}{2 \cdot k_d \cdot y2}}$$

$U13i := U14i$

The energy balance for all nodes can be written in matrix form for N nodes as:

$$[U]_{NxN} \cdot [T]_N = [Q]_N$$

where the elements of the conductance matrix [U] and source vector [Q] are determined as follows:

1. Diagonal element U(i, i) is equal to the sum of conductances connected to node i.

2. Off-diagonal element U(i, j) is equal to the total conductance between nodes i and j times -1.

3. The source vector element Q(i) is equal to the sum of the heat sources into node i plus equivalent heat sources due to specified temperatures (e.g. U1i ·Ti in the above example).

Initialize the elements of the conductance matrix U:

$i := 0, 1 .. 14 \qquad j := 0, 1 .. 14 \qquad U_{i,j} := 0 \dfrac{W}{\Delta°C \cdot m}$

Diagonal elements of [U]:

$$U_{0,0} := U0\_13 + U0i$$

$$U_{1,1} := U12 + U16 + U1o$$

$$U_{2,2} := U23 + U12 + U25$$

$$U_{3,3} := U3i + U23 + U34$$

$$U_{4,4} := U47 + U45 + U4i + U34$$

$$U_{5,5} := U25 + U45 + U58 + U56$$

$$U_{6,6} := U56 + U6o + U16 + U69$$

$$U_{7,7} := U47 + U78 + U7\_13$$

$$U_{8,8} := U58 + U78 + U89$$

$$U_{9,9} := U69 + U89 + U9\_10$$

$$U_{10,10} := U10o + U9\_10 + U10\_11$$

$$U_{11,11} := U11o + U10\_11 + U11\_12$$

$$U_{12,12} := U11\_12 + U12o$$

$$U_{13,13} := U0\_13 + U13\_14 + U78$$

$$U_{14,14} := U14i + U13\_14$$

Off-diagonal elements of U:

$U_{1,2} := -U12$ $U_{2,3} := -U23$ $U_{3,4} := -U34$

$U_{2,5} := -U25$ $U_{1,6} := -U16$ $U_{13,14} := -U13\_14$

$U_{7,13} := -U7\_13$ $U_{7,8} := -U78$ $U_{8,9} := -U89$

$U_{9,10} := -U9\_10$ $U_{10,11} := -U10\_11$

$U_{11,12} := -U11\_12$ $U_{4,7} := -U47$

$U_{5,8} := -U58$ $U_{6,9} := -U69$

Since the conductance matrix U is symmetric, it can be shown that

$$U_{i,j} := \text{if}\left(i > j, U_{j,i}, U_{i,j}\right)$$

Initialize source vector elements:

$$Q_j := 0 \ \frac{W}{m}$$

$Q_0 := U0i \cdot T_i$ $Q_1 := U1o \cdot T_o$ $Q_6 := U6o \cdot T_o$

$Q_3 := U3i \cdot T_i$ $Q_4 := U4i \cdot T_i$ $Q_{10} := U10o \cdot T_o$

$Q_{11} := U11o \cdot T_o$ $Q_{12} := U12o \cdot T_o$ $Q_{14} := U14i \cdot T_i$

$$T := U^{-1} \cdot Q$$

Heat loss:

$$q := U1o \cdot \left(T_1 - T_o\right) + U6o \cdot \left(T_6 - T_o\right) + U10o \cdot \left(T_{10} - T_o\right) + U11o \cdot \left(T_{11} - T_o\right) + U12o \cdot \left(T_{12} - T_o\right)$$

$$q = 13.969 \ \frac{W}{m}$$

$$T=\begin{bmatrix} 6.923 \\ -8.983 \\ 3.897 \\ 17.081 \\ 14.211 \\ 0.013 \\ -8.752 \\ 1.815 \\ -1.04 \\ -3.899 \\ -9.453 \\ -9.958 \\ -9.997 \\ 1.298 \\ 15.456 \end{bmatrix} \Delta°C \qquad Q=\begin{bmatrix} 54 \\ -45 \\ 0 \\ 37.241 \\ 37.241 \\ 0 \\ -45 \\ 0 \\ 0 \\ 0 \\ -63.75 \\ -63.75 \\ -70.373 \\ 0 \\ 35.308 \end{bmatrix} \frac{W}{m}$$

$$U=\begin{bmatrix} 7.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 5.346 & -0.346 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0 & \\ 0 & -0.346 & 0.691 & -0.34 & 0 & -0.005 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & -0.34 & 2.535 & -0.333 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & -0.333 & 3.092 & 0 & 0 & -0.557 & 0 & 0 & 0 & \\ 0 & 0 & -0.005 & 0 & 0 & 0.701 & 0 & 0 & -0.01 & 0 & 0 & \\ 0 & -0.5 & 0 & 0 & 0 & 0 & 5.904 & 0 & 0 & -0.557 & 0 & \\ 0 & 0 & 0 & 0 & -0.557 & 0 & 0 & 3.674 & -2.267 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & -0.01 & 0 & -2.267 & 4.543 & -2.267 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.557 & 0 & -2.267 & 3.504 & -0.68 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.68 & 7.622 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.567 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.85 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \end{bmatrix} \frac{W}{\Delta°C \cdot m}$$