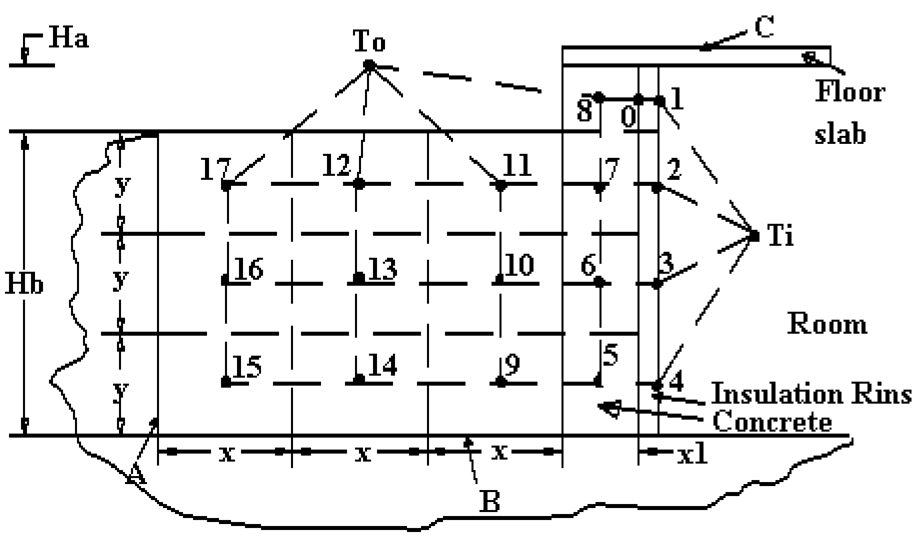
CHAPTER 3 HEAT CONDUCTION IN BUILDINGS WITH THE FINITE DIFFERENCE METHOD

3.2 Heat Flow in Basements

Heat flow through insulated or uninsulated basement walls, partly above grade and partly below grade can be determined with a two-dimensional thermal network or a finite difference grid. Usually, the ground temperature is approximately constant within each month and follows a sinusoidal variation with the seasons throughout the year.

Consider the insulated basement wall shown below with height Ha above grade and Hb below grade, made up of a layer of concrete of thickness x1 and insulated on the interior side with insulation of thermal resistance Rins per unit area.



Lines connecting nodes 1-17 and Ti & To indicate conductances

Note: the insulation is often applied up to a certain depth on the ground side.

$h_i := 9 \, \frac{W}{m^2 \cdot \Delta°C}$ $\qquad h_o := 20 \, \frac{W}{m^2 \cdot \Delta°C}$

$R_{ins} := 2 \, m^2 \cdot \frac{\Delta°C}{W}$ insulation thermal resistance (and cover layer)

$k_c := 1.7 \, \frac{W}{m \cdot \Delta°C}$ concrete thermal conductivity

$k := 0.8 \dfrac{W}{m \cdot \Delta°C}$ soil thermal conductivity

Consider unit wall width:

$H_a := 0.6\ m$ $H_b := 1.8\ m$ $L := 1\ m$

$Ti := 20\ \Delta°C$ $To := -15\ \Delta°C$ inside and outside temperatures

Grid size: $x := 0.6\ m$ $x1 := 0.3\ m$ $y := 0.6\ m$

Assumptions:

1. Steady-state two-dimensional heat transfer.
2. Surfaces A, B, C are zero-heat loss surfaces (adiabatic).
3. No heat-transfer between nodes 1, 2, 3 and 4.

A grid is set up as shown in the figure with each node located in the center of an element, except nodes 1-4 that are located on the surface of the room side. Each conductance Uij, connecting nodes i and j, is determined as follows based on the governing heat flow equation:

$$q_{ij} = Uij \cdot \left(T_i - T_j\right)$$

Calculate the conductances between all nodes for unit width (L equal to 1m):

$U08 := \dfrac{k_c \cdot H_a \cdot 2}{x1}$ $U01 := \dfrac{H_a}{R_{ins}}$ $U1i := H_a \cdot h_i$

$U2i := y \cdot h_i$ $U3i := U1i$ $U4i := U1i$

$$U27 := \dfrac{1}{\dfrac{R_{ins}}{y} + \dfrac{x1}{2 \cdot y \cdot k_c}}$$

$U36 := U27$

$U45 := U27$

$U67 := \dfrac{x1 \cdot k_c}{y}$ $U78 := \dfrac{2 \cdot x1 \cdot k_c}{y + H_a}$ $U56 := U67$

$$U7\_11 := \frac{1}{\frac{x1}{2 \cdot y \cdot k_c} + \frac{x}{2 \cdot y \cdot k}}$$

$U59 := U7\_11$ $\qquad U6\_10 := U7\_11$

$U10\_11 := \frac{x \cdot k}{y}$ $\qquad U9\_10 := U10\_11$

$U12\_13 := U9\_10$ $\qquad U13\_14 := U9\_10$

$U15\_16 := U9\_10$ $\qquad U16\_17 := U9\_10$

$U9\_14 := \frac{y \cdot k}{x}$ $\qquad U10\_13 := U9\_14$

$U11\_12 := U9\_14$ $\qquad U14\_15 := U9\_14$

$U13\_16 := U9\_14$ $\qquad U12\_17 := U9\_14$

$$U17o := \frac{1}{\frac{1}{x \cdot h_o} + \frac{y}{2 \cdot k \cdot x}}$$

$U12o := U17o$ $\qquad U8o := \frac{1}{\frac{1}{H_a \cdot h_o} + \frac{1}{U08}}$

$U11o := U17o$

The energy balance at all nodes may be written as follows: Sum of heat flows into node i = 0. For example, for node 1 we have

$$U01 \cdot (T1 - To) + U1i \cdot (T1 - Ti) = 0$$

Because Ti is known, we may rewrite this equation in the form

$$(U01 + U1i) \cdot T1 - U01 \cdot To = U1i \cdot Ti$$

As we can see, we can similarly write equations for all nodes in the form

[U] [T] = [Q],

where the matrix elements are determined as follows:

1. Diagonal element U(i, i) is equal to the sum of conductances connected to node i.

2. Off-diagonal element U(i, j) is equal to the total conductance between nodes i and j times -1.

3. The source vector element Q(i) is equal to the sum of the heat sources into node i plus equivalent heat sources due to specified temperatures (e.g. U1i * Ti in the above example).

Initialize the elements of U:

$i := 0, 1 .. 17$ $j := 0, 1 .. 17$

$U_{i,j} := 0 \frac{W}{\Delta°C \cdot m}$

$U_{0,0} := U01 + U08$ $U_{1,1} := U01 + U1i$

$U_{2,2} := U2i + U27$ $U_{3,3} := U3i + U36$

$U_{4,4} := U4i + U45$ $U_{5,5} := U45 + U56 + U59$

$U_{6,6} := U36 + U67 + U56 + U6\_10$

$U_{7,7} := U27 + U78 + U67 + U7\_11$

$U_{8,8} := U8o + U08 + U78$

$U_{9,9} := U9\_10 + U59 + U9\_14$

$U_{10,10} := U9\_10 + U6\_10 + U10\_13 + U10\_11$

$U_{12,12} := U11\_12 + U12\_13 + U12\_17 + U12o$

$U_{13,13} := U10\_13 + U12\_13 + U13\_14 + U13\_16$

$U_{11,11} := U11o + U11\_12 + U7\_11 + U10\_11$

$U_{14,14} := U9\_14 + U14\_15 + U13\_14$

$U_{15,15} := U14\_15 + U15\_16$

$U_{16,16} := U13\_16 + U15\_16 + U16\_17$

$U_{17,17} := U17o + U16\_17 + U12\_17$

$U_{0,8} := -U08$ $U_{0,1} := -U01$ $U_{2,7} := -U27$

$U_{3,6} := -U36$ $U_{4,5} := -U45$ $U_{7,8} := -U78$

$U_{6,7} := -U67$ $U_{5,6} := -U56$ $U_{7,11} := -U7\_11$

$U_{6,10} := -U6\_10$ $U_{5,9} := -U59$ $U_{10,11} := -U10\_11$

$U_{9,10} := -U9\_10$ $U_{12,13} := -U12\_13$

$U_{13,14} := -U13\_14$ $U_{15,16} := -U15\_16$

$U_{16,17} := -U16\_17$ $U_{9,14} := -U9\_14$

$U_{10,13} := -U10\_13$ $U_{11,12} := -U11\_12$

$U_{14,15} := -U14\_15$ $U_{13,16} := -U13\_16$

$U_{12,17} := -U12\_17$

Since the conductance matrix U is symmetric, it can be shown that

$U_{i,j} := \mathbf{if}\left(i > j, U_{j,i}, U_{i,j}\right)$

Initialize source vector elements:

$Q_j := 0\ \frac{W}{m}$

$Q_1 := U1i \cdot Ti$ $Q_2 := U2i \cdot Ti$

$Q_3 := U3i \cdot Ti$ $Q_4 := U4i \cdot Ti$

$Q_8 := U8o \cdot To$ $Q_{11} := U11o \cdot To$

$Q_{12} := U12o \cdot To$ $Q_{17} := U17o \cdot To$

$T := U^{-1} \cdot Q$

$$Q=\begin{bmatrix}0\\108\\108\\108\\108\\0\\0\\0\\-65.106\\0\\0\\-21.176\\-21.176\\0\\0\\0\\0\\-21.176\end{bmatrix}\frac{W}{m} \qquad T=\begin{bmatrix}-10.573\\18.391\\18.687\\18.874\\18.961\\-0.567\\-2.291\\-5.99\\-11.85\\-3.768\\-5.689\\-10.045\\-12.221\\-8.758\\-7.029\\-8.561\\-10.092\\-12.958\end{bmatrix}\Delta°C$$

Heat loss

$$q_{aux}:=\left(U1i\cdot\left(Ti-T_{1}\right)+U2i\cdot\left(Ti-T_{2}\right)+U3i\cdot\left(Ti-T_{3}\right)+U4i\cdot\left(Ti-T_{4}\right)\right)\cdot L$$

$$q_{aux}=27.471\ W$$

$$\text{submatrix}(U,0,17,0,8)=\begin{bmatrix}7.1&-0.3&0&0&0&0&0&0&-6.8\\-0.3&5.7&0&0&0&0&0&0&0\\0&0&5.687&0&0&0&0&-0.287&0\\0&0&0&5.687&0&0&-0.287&0&0\\0&0&0&0&5.687&-0.287&0&0&0\\0&0&0&0&-0.287&2.433&-0.85&0&0\\0&0&0&-0.287&0&-0.85&3.283&-0.85&0\\0&0&-0.287&0&0&0&-0.85&3.283&-0.85\\-6.8&0&0&0&0&0&0&-0.85&11.99\\0&0&0&0&0&-1.295&0&0&0\\0&0&0&0&0&0&-1.295&0&0\\0&0&0&0&0&0&0&-1.295&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\end{bmatrix}\frac{W}{m\cdot\Delta°C}$$

$$\text{submatrix}(U,0,17,9,17)=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1.295 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.295 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.295 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2.895 & -0.8 & 0 & 0 & 0 & -0.8 & 0 & 0 & 0 \\ -0.8 & 3.695 & -0.8 & 0 & -0.8 & 0 & 0 & 0 & 0 \\ 0 & -0.8 & 4.307 & -0.8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.8 & 3.812 & -0.8 & 0 & 0 & 0 & -0.8 \\ 0 & -0.8 & 0 & -0.8 & 3.2 & -0.8 & 0 & -0.8 & 0 \\ -0.8 & 0 & 0 & 0 & -0.8 & 2.4 & -0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.8 & 1.6 & -0.8 & 0 \\ 0 & 0 & 0 & 0 & -0.8 & 0 & -0.8 & 2.4 & -0.8 \\ 0 & 0 & 0 & -0.8 & 0 & 0 & 0 & -0.8 & 3.012 \end{bmatrix} \frac{W}{m \cdot \Delta °C}$$

Heat loss through basement floors is generally less than heat loss through the perimeter walls and is usually between 0.12 and 0.18 watts per square meter per degC.