CHAPTER 3 HEAT CONDUCTION IN BUILDINGS WITH THE FINITE DIFFERENCE METHOD

3.3 Transient One-Dimensional Finite Difference Wall Model

Transient thermal analysis of walls or rooms may be performed with the following objectives:

1. Peak heating/cooling load calculations
2. Calculation of dynamic temperature variation within walls including solar effects, room temperature swings and condensation on wall interior surfaces.

In the transient finite difference method, we represent each wall layer by one or more sub-layers (regions). Each region is represented by a node with a thermal capacitance C connected to two thermal resistances, each equal to half the R-value of the layer, forming a T-section as shown in the figure below for the concrete layer.

For a multilayered wall, an energy balance is applied at each node at regular time intervals to obtain the temperature of the nodes as a function of time. These equations may be solved with the implicit method as a set of simultaneous equations or with the explicit method in which we march forward in time from a set of initial conditions.

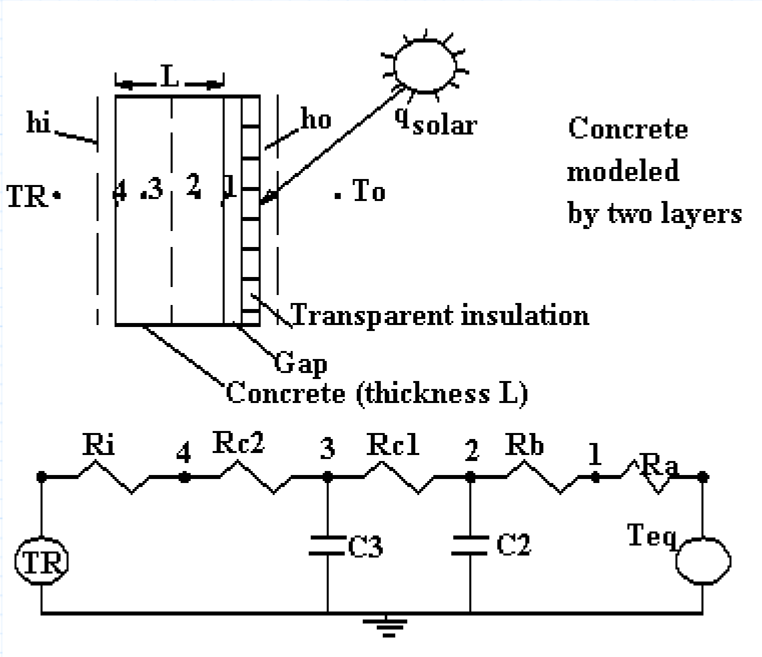
The general form of the explicit finite difference formulation corresponding to node i and time interval p is

$$T(i,p+1)=\left(\frac{\Delta t}{C_i}\right)\cdot\left(q_i+\sum_j \frac{T(j,p)-T(i,p)}{R(i,j)}\right)+T(i,p)$$

Critical time step:

$$\Delta t_{critical}=min\left(\frac{C_i}{\sum_j \frac{1}{R_{i,j}}}\right)$$ for all nodes i.

Example: A wall employs the new concept of transparent insulation. It consists of a transparent exterior layer, an air gap and a concrete thermal storage layer.



$A := 1\ m^2$ $\qquad L := 0.12\ m$

Concrete properties:

$c := 800\ \dfrac{J}{kg \cdot \Delta°C}$ specific heat

$k := 1.7\ \dfrac{W}{m \cdot \Delta°C}$ conductivity

$\rho := 2200\ \dfrac{kg}{m^3}$ density

$h_i := 10\ \dfrac{W}{m^2 \cdot \Delta°C}$

$h_o := 20\ \dfrac{W}{m^2 \cdot \Delta°C}$

film coefficients

$\tau\alpha := 0.7$ effective transmittance-absorptance

$R_{ins} := 0.3\ m^2 \cdot \dfrac{\Delta°C}{W}$ $\qquad R_{gap} := 0.3\ m^2 \cdot \dfrac{\Delta°C}{W}$

$$R_a := \frac{R_{ins} + R_{gap} + \frac{1}{h_o}}{A} = 0.65\ \frac{\Delta°C}{W}$$

$$R_c := \frac{L}{k \cdot A}$$

$$R_b := \frac{R_c}{4} = 0.018\ \frac{\Delta°C}{W}$$

$$R_{c1} := \frac{R_c}{2} = 0.035\ \frac{\Delta°C}{W}$$

$$R_{c2} := \frac{R_c}{4}$$

$$R_i := \frac{1}{A \cdot h_i} = 0.1\ \frac{\Delta°C}{W}$$

$$C2 := \rho \cdot c \cdot \frac{L}{2} \cdot A = \left(1.056 \cdot 10^{5}\right)\ \frac{J}{\Delta°C}$$

$C3 := C2$ thermal capacitances

Stability Test

The time step Δt should be lower than the minimum of the two values in the vector TS.

$$TS := \left[\frac{C2}{\frac{1}{R_a + R_b} + \frac{1}{R_{c1}}} \quad \frac{C3}{\frac{1}{R_{c1}} + \frac{1}{R_{c2} + R_i}} \right]$$

$\Delta t_{critical} := min(TS)$ $\Delta t_{critical} = \left(2.867 \cdot 10^{3}\right)\ s$

Choose Δt:

$\Delta t := 1800 \cdot s$

$Steps := 96$ number of time steps

$t := 0 \cdot s, \Delta t .. Steps \cdot \Delta t$ $i := 0 .. Steps$

Assume

$$w := 2 \cdot \frac{\pi}{86400} \frac{rad}{s}$$

frequency based on period of one day

$$To(t) := \left(5 \cdot \cos\left(w \cdot t + 3 \cdot \frac{\pi}{4}\right) - 5\right) \Delta°C$$

outside temperature

$$f(t) := 500 \cdot \cos(w \cdot (t - 43200 \cdot sec))\ W$$

$$q_{solar}(t) := \mathbf{if}(f(t) > 0\ W, f(t), 0\ W)$$

incident solar radiation modeled as half-sinusoid

$$Teq(t) := To(t) + q_{solar}(t) \cdot \tau\alpha \cdot R_a$$

equivalent "sol-air" temperature

$$TR := 22\ \Delta°C$$

room temperature

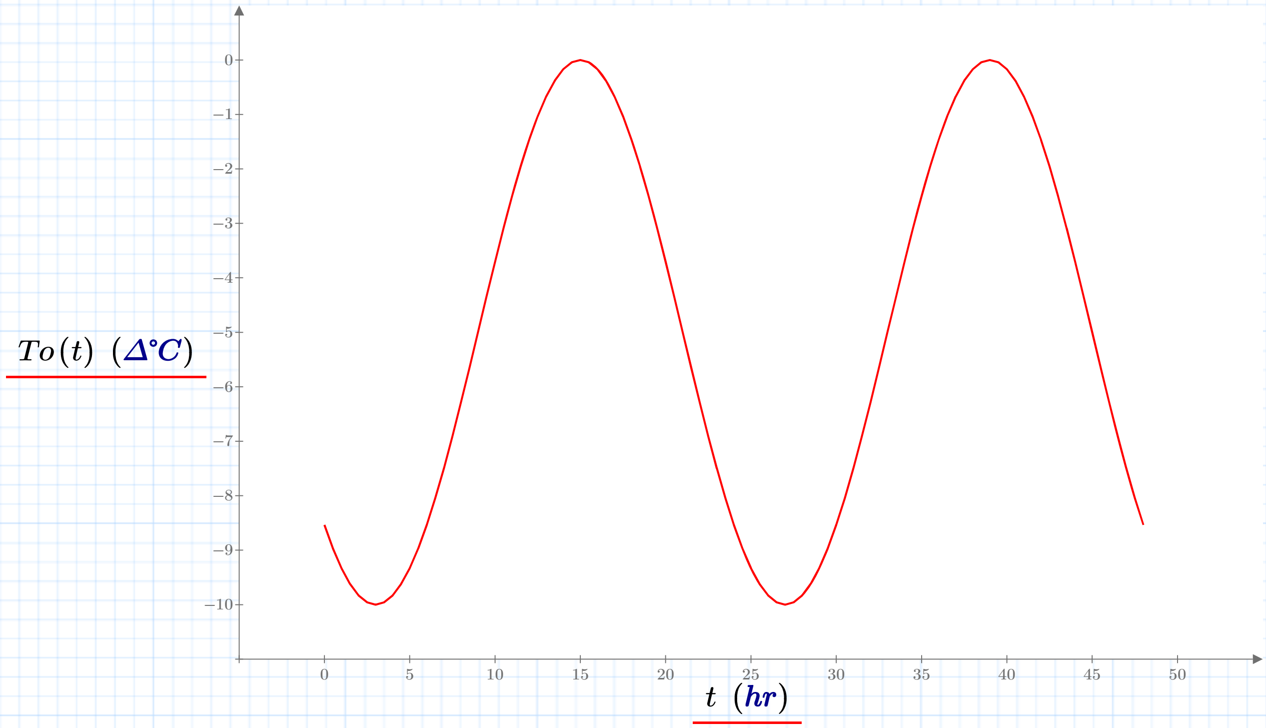
Initial estimates of temperatures:

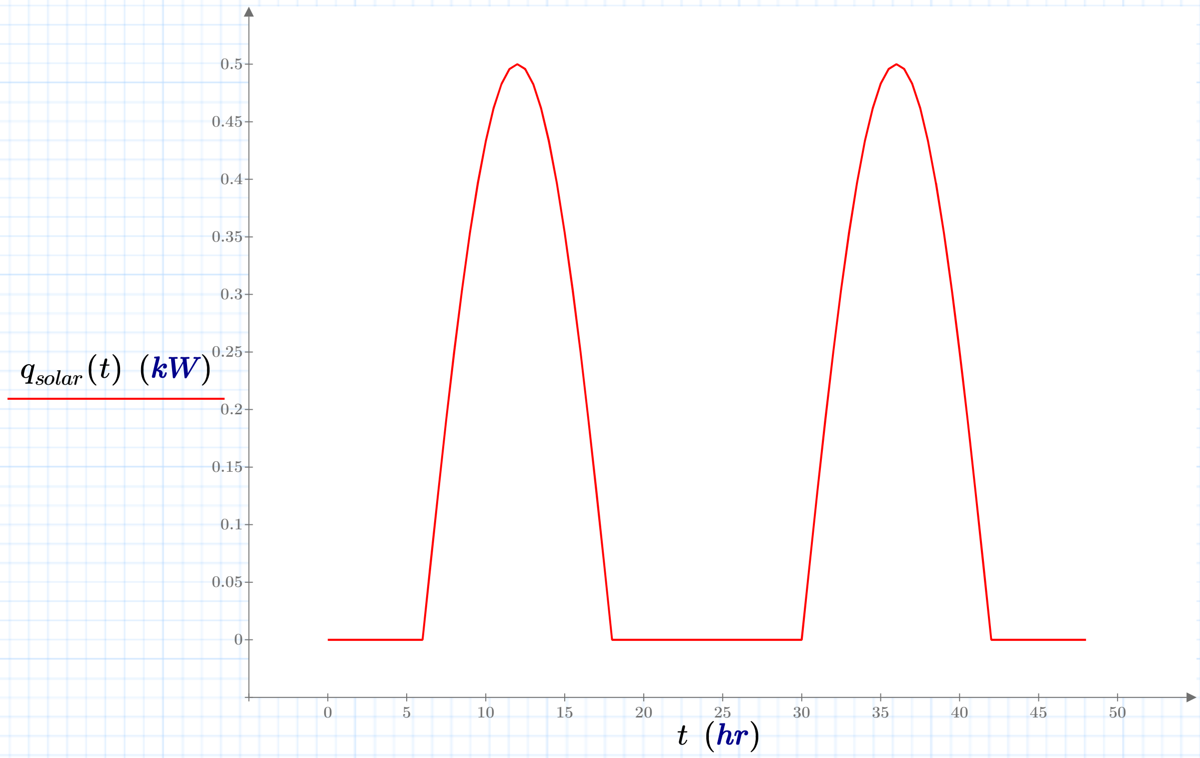
$$\begin{bmatrix} T2_0 \\ T3_1 \end{bmatrix} := \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Delta°C$$

$$\begin{bmatrix} T2_{i+1} \\ T3_{i+1} \end{bmatrix} := \begin{bmatrix} \frac{\Delta t}{C2} \cdot \left(\frac{Teq(i \cdot \Delta t) - T2_i}{R_a + R_b} + \frac{T3_i - T2_i}{R_{c1}}\right) + T2_i \\ \frac{\Delta t}{C3} \cdot \left(\frac{T2_i - T3_i}{R_{c1}} + \frac{TR - T3_i}{R_i + R_{c2}}\right) + T3_i \end{bmatrix}$$

$$T4_i := TR + R_i \cdot \frac{T3_i - TR}{R_{c2} + R_i}$$

$$T1_i := T2_i + R_b \cdot \frac{Teq(i \cdot \Delta t) - T2_i}{R_b + R_a}$$


$To(t)$ $(\Delta°C)$
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
0 5 10 15 20 25 30 35 40 45 50
t (hr)


$q_{solar}(t)$ (kW)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 5 10 15 20 25 30 35 40 45 50
t (hr)

Variation of nodal temperatures for two days (T4 is room-side wall surface temperature, T1 is the concrete gap-facing temperature).