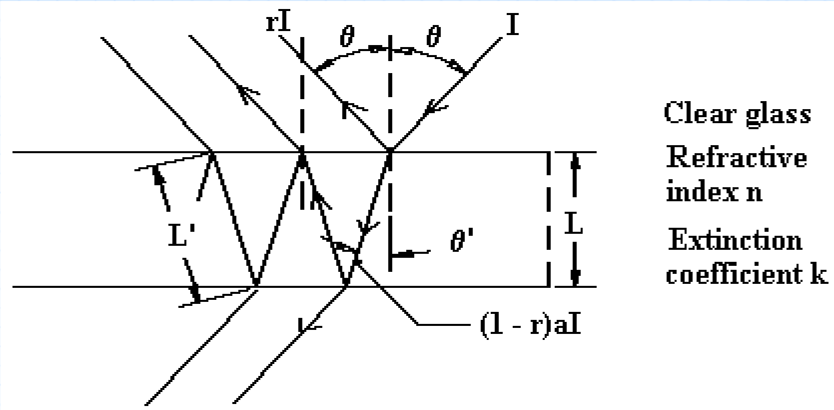
CHAPTER 7 SOLAR RADIATION

7.2 Solar Properties of Windows

The solar transmittance, reflectance and absorptance of windows or other transparent building components such as solar collectors need to be determined in order to calculate how much solar radiation they transmit.

First we determine the solar properties of a layer of glass with thickness L, refractive index n and extinction coefficient k. Consider the figure below:



Snell's Law

$$\sin(\theta') = \frac{\sin(\theta)}{n}$$

θ = incidence angle

Component reflectivity (r) is the fraction of each ray component reflected. It is determined from the Fresnel relations of electromagnetic theory:

$$r = \frac{1}{2} \cdot \left(\left(\frac{\sin(\theta - \theta')}{\sin(\theta + \theta')} \right)^2 + \left(\frac{\tan(\theta - \theta')}{\tan(\theta + \theta')} \right)^2 \right)$$

$$L' = \frac{L}{\sqrt{1 - \left(\frac{\sin(\theta)}{n} \right)^2}}$$

Refractive index of glass:

n = 1.51 - 1.53

Extinction coefficients for different types of glass:

1. Double strength, A quality k = 7.76/m

2. Clear plate glass k = 6.96/m

3. Heat absorbing glass k = 132/m

The intensity I(x) of radiation after it has traveled distance x in a material is an exponential decay function of its intensity at x=0, I_o and the extinction coefficient k:

$$I(x) = I_o \cdot exp(-k \cdot x)$$

Therefore, after traveling a distance L' as shown in the figure, the intensity is equal to

$I(L') = I_o \cdot a$ where $a = exp(-k \cdot L')$

The quantity a (=exp(k L')) is the fraction of radiation available after each reflection in the glazing. Through simple ray-tracing techniques, one may show that

$$\tau = (1-r)^2 \cdot a + r^2 \cdot (1-r)^2 \cdot a^3 + r^4 \cdot (1-r) \cdot a^5 + ..$$

This is a convergent geometric series. Therefore, the effective transmittance of the glazing is

$$\tau = \frac{(1-r)^2 \cdot a}{1-r^2 \cdot a^2}$$

Similarly, the effective reflectance of the glazing is

$$\rho = r + \frac{r \cdot (1-r)^2 \cdot a^2}{1-r^2 \cdot a^2}$$

The absorptance is determined from the conservation relationship:

$$\alpha = 1 - \rho - \tau$$

Note that the above properties are a function of incidence angle, and wavelength. They are typically reported averaged over two wavelength ranges:

1. solar: 0.3 - 3 micrometers
2. longwave: 3 - 30 micrometers

The averaged properties are determined as a weighted average of spectral values:

$$\alpha_s = \frac{\int_{0.3 \cdot \mu m}^{3 \cdot \mu m} \alpha_\lambda \cdot I_\lambda \, d\lambda}{\int_{0.3 \cdot \mu m}^{3 \cdot \mu m} I_\lambda \, d\lambda}$$

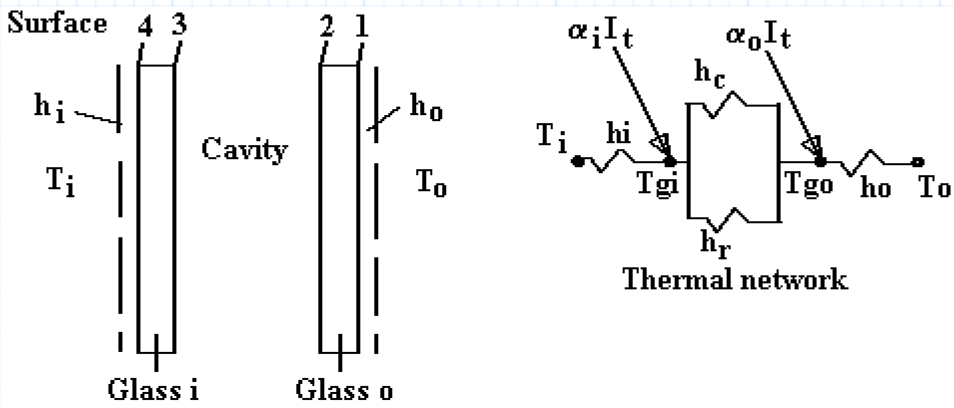
(See also **Section 6.2**.)

For conversion of solar energy to heat, a high solar absorptance and a low longwave emissivity are required. Selective surface coatings may be used to achieve such properties.

Double-Glazed Windows

For a double-glazed window, the effective transmittance t_e is usually required given τ_i, ρ_2, τ_o, and ρ_3 for the two glazings. The effective transmittance is given by

$$\tau_e = \frac{\tau_i \cdot \tau_o}{1 - \rho_2 \cdot \rho_3}$$



In heat transmission calculations, the effective absorptance of each glazing is also required:

$$\alpha_o = \alpha_1 + \alpha_2 \cdot \frac{\tau_o \cdot \rho_3}{1 - \rho_2 \cdot \rho_3} \qquad \alpha_i = \alpha_3 \cdot \frac{\tau_o}{1 - \rho_2 \cdot \rho_3}$$

α_o absorptance of outer glass

α_i absorptance of inner glass

α_2 absorptance of outer glass for solar radiation incident on indoor surface (2)

α_3 absorptance of glass i for solar radiation incident on surface (3)

ρ_j reflectance of surface j

Total solar radiation absorbed in the outer glass is:

$$I_{ao} = \alpha_o \cdot I_t$$

Total solar radiation absorbed in the inner glass is:

$$I_{ai} = \alpha_i \cdot I_t$$

Note that for uncoated surfaces the directional reflectance (and absorptance) of the glazing surfaces are equal (Note: the diffuse component of I should be multiplied by the diffuse absorptance for a more accurate calculation).

Part of the solar radiation absorbed in the glazings is transmitted by convection or as longwave radiation heat into the room. An energy balance at the two glazings will show that the total amount of solar radiation that flows as heat into the room (after absorption in the glazings) is:

$$q_{sa} = \frac{U}{h_o} \cdot \left(\alpha_o \cdot I_t\right) + U \cdot \left(\frac{1}{h_o} + \frac{1}{h_g}\right) \cdot \left(\alpha_i \cdot I_t\right)$$

where

$$U = \frac{1}{\frac{1}{h_o} + \frac{1}{h_g} + \frac{1}{h_i}}$$ window U value

h_o = outside film coefficient $\qquad$ $h_g = h_c + h_r$

h_g = cavity heat transfer coefficient $\qquad$ h_i = inside film coefficient

The first term in q_{sa} is for a single (outer) glazing. The total amount of solar radiation that enters a room is equal to the transmitted portion plus q_{sa}. The *ASHRAE Handbook of Fundamentals* (ASHRAE, 1989) defines a related parameter called **solar heat gain factor (SHGF)** which is equal to the solar radiation transmitted plus the inward flow of absorbed solar radiation for double strength (DSA) clear glass. Double-glazed and other types of windows are related to this amount through the use of a shading coefficient (SC) defined as

SC = SHGF (of window) / SHGF (of DSA glass)

* SHGF values may be directly determined here using the equations given above, and a procedure similar to that of **Section 7.3**.

Example-single glazing: Determine the transmittance, reflectance, and absorptance as a function of angle of incidence and kL for a single layer of glass.

$i := 1, 2 .. 15$ $\qquad$ $n := 1.53$

$kL_i := 0.01 + 0.01 \cdot i$ $\qquad$ kL products

$j := 0, 1 .. 17$

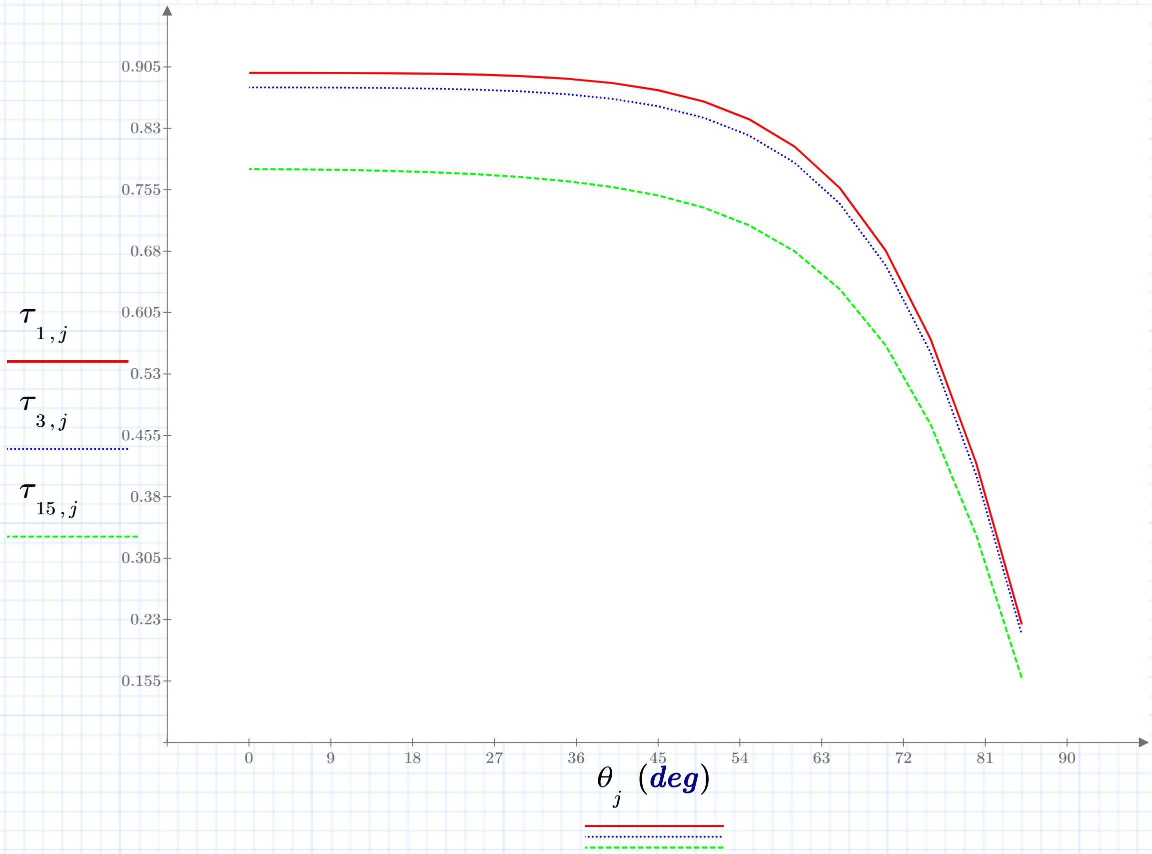
$\theta_j := (5 \cdot j + 0.01)\ deg$ $\qquad$ angle of incidence

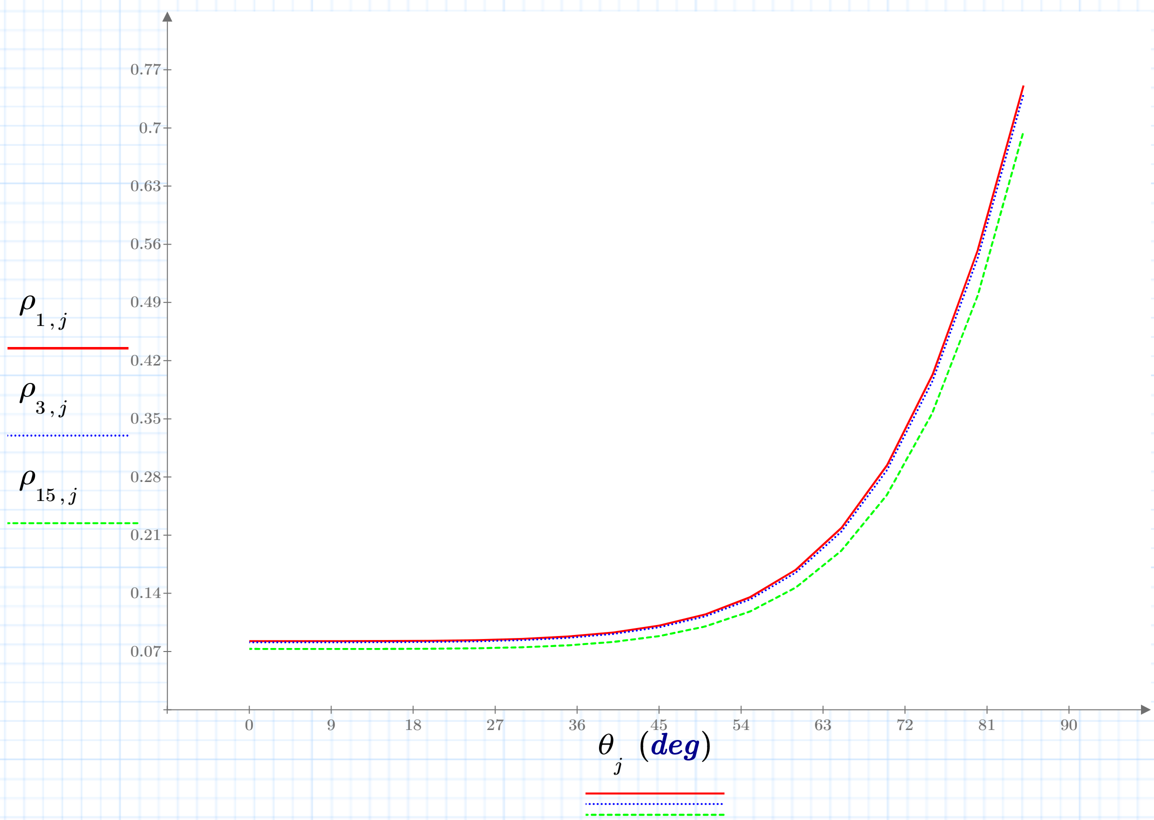
$$\theta'_j := \operatorname{asin}\left(\frac{\sin\left(\theta_j\right)}{n}\right)$$

$$r_j := \frac{1}{2} \cdot \left(\left(\frac{\sin\left(\theta_j - \theta'_j\right)}{\sin\left(\theta_j + \theta'_j\right)}\right)^2 + \left(\frac{\tan\left(\theta_j - \theta'_j\right)}{\tan\left(\theta_j + \theta'_j\right)}\right)^2\right)$$

$$a_{i,j} := \exp\left(-\frac{kL_i}{\sqrt{1 - \left(\frac{\sin\left(\theta_j\right)}{n}\right)^2}}\right) \qquad \tau_{i,j} := \frac{\left(1 - r_j\right)^2 \cdot a_{i,j}}{1 - \left(r_j\right)^2 \cdot \left(a_{i,j}\right)^2}$$

$$\rho_{i,j} := r_j + \frac{r_j \cdot \left(1 - r_j\right)^2 \cdot \left(a_{i,j}\right)^2}{1 - \left(r_j\right)^2 \cdot \left(a_{i,j}\right)^2} \qquad \alpha_{i,j} := 1 - \rho_{i,j} - \tau_{i,j}$$


0.905
0.83
0.755
0.68
0.605
0.53
0.455
0.38
0.305
0.23
0.155
$\tau_{1,j}$
$\tau_{3,j}$
$\tau_{15,j}$
0
9
18
27
36
45
54
63
72
81
90
θ_j (deg)

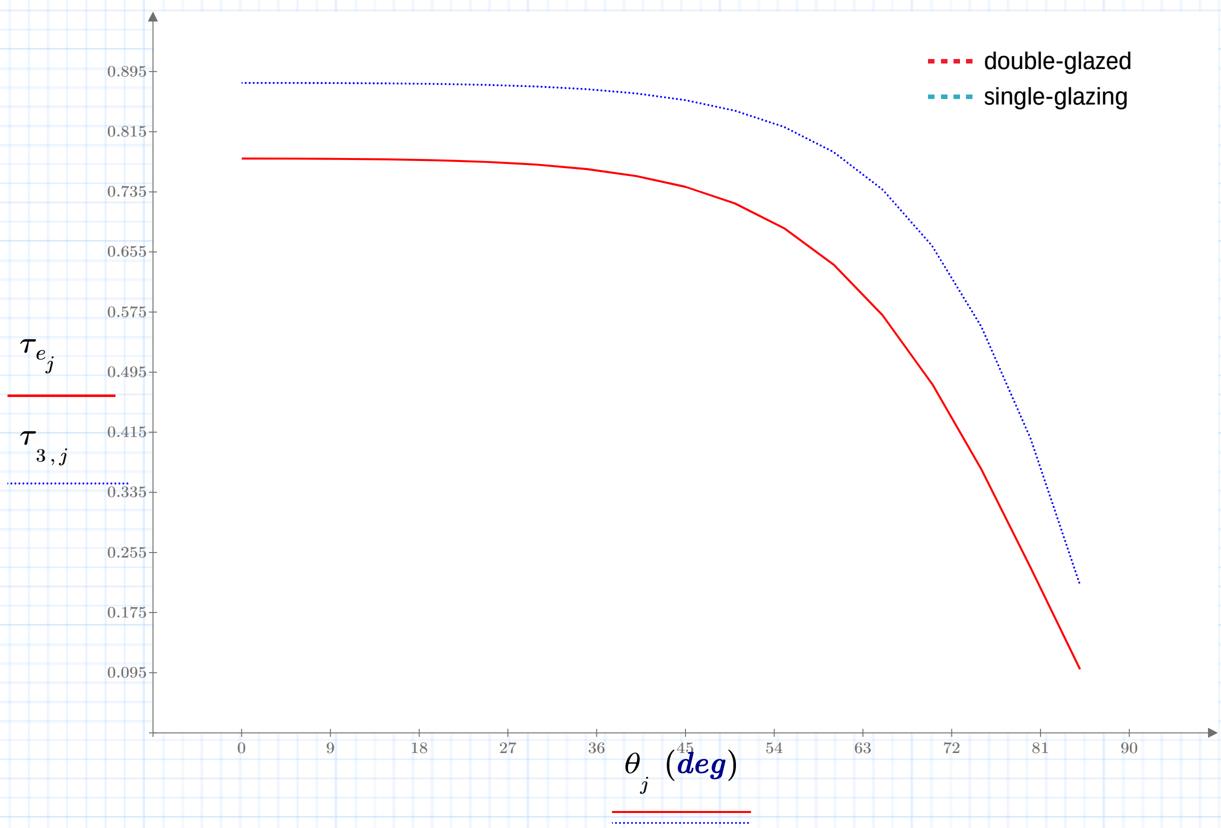

0.77
0.7
0.63
0.56
0.49
0.42
0.35
0.28
0.21
0.14
0.07
$\rho_{1,j}$
$\rho_{3,j}$
$\rho_{15,j}$
0
9
18
27
36
45
54
63
72
81
90
θ_j (deg)

Double-glazed window-example: Consider a window consisting of two 3 mm thick glazings with kL = 0.040 (corresponds to i = 3 in the above example). Determine the effective transmittance of the window and the effective absorptance of the indoor and outdoor glazings.

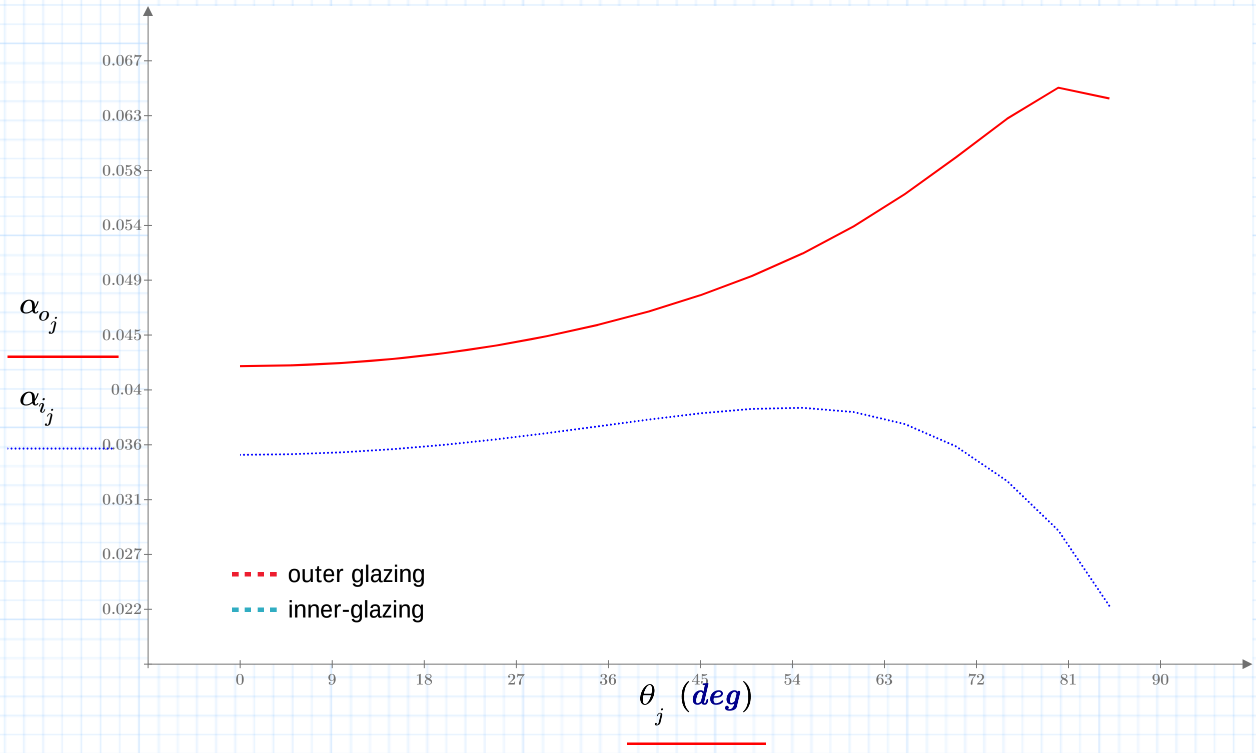
$$\tau_{e_j} := \frac{\left(\tau_{3,j}\right)^2}{1-\left(\rho_{3,j}\right)^2} \qquad \alpha_{i_j} := \alpha_{3,j} \cdot \frac{\tau_{3,j}}{1-\left(\rho_{3,j}\right)^2}$$

$$\alpha_{o_j} := \alpha_{3,j} + \alpha_{3,j} \cdot \frac{\tau_{3,j} \cdot \rho_{3,j}}{1-\left(\rho_{3,j}\right)^2}$$

Transmittance as a function of angle



Absorptance as a function of angle



References

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