CHAPTER 8 PSYCHROMETRY AND THERMAL COMFORT

8.1 Psychrometry and Thermodynamic Properties of Moist Air

Psychrometry is the study and measurement of the properties of moist air. Moist air is a mixture of dry air and water vapor. The amount of water vapor that air may contain depends on temperature and pressure; this amount is maximum at **saturation,** a state of equilibrium between moist air and the condensed water phase (liquid, or ice).

Moist air, up to about 3 atmospheres, obeys the perfect gas law with sufficient accuracy for engineering calculations. Thus, the Gibbs-Dalton law for a mixture of perfect gases is obeyed and the total atmospheric pressure, p, is equal to the sum of the partial pressures of the components of air:

$$p = p_a + p_v \tag{1}$$

where p_a = pressure of dry air and p_v = pressure of water vapor.

Definitions

Humidity ratio W (also called specific humidity) is the ratio of mass of water vapor m_v to dry air mass m_a

$$W = \frac{m_v}{m_a} \qquad \text{or} \qquad W = \frac{v_a}{v_v} \tag{2}$$

where v is specific volume.

Relative humidity RH is the ratio of the mole fraction of water vapor x_v in a mixture to the mole fraction x_s of the vapor in a saturated mixture at the same temperature T and pressure p.

$$RH = \left(\frac{x_v}{x_s}\right)_{T,p} \qquad \text{and} \qquad x_v = \frac{p_v}{p} \qquad x_s = \frac{p_s}{p} \tag{3}$$

Therefore $\qquad RH = \frac{p_v}{p_s} \cdot \text{"}100\%\text{"} \tag{4}$

Degree of saturation μ is equal to the air humidity ratio divided by the humidity ratio of a saturated mixture at the same temperature and pressure.

$$\mu = \left(\frac{W}{W_s}\right)_{T,P} \qquad\qquad (5)$$

Dew point T_dp is the temperature to which unsaturated air must be cooled at constant pressure for it to become saturated (or for condensation to begin). Note that throughout this process the humidity ratio is constant.

$$W_s(p, T_{dp}) = W(p, T) \qquad\qquad (6)$$

The dew-point temperature may be determined as a function of the partial water vapor pressure with the following correlations (ASHRAE 1989), in which the vapor pressure should be in kPa.

For dew-points in the range 0 to 93 degC:

$$T_{dp} = 6.54 + 14.526 \cdot ln\,(p_v) + .7389 \cdot ln\,(p_v)^2 + \left(\left(.09486 \cdot ln\,(p_v)^3\right) + .4569 \cdot p_v^{.1984}\right) \qquad (6a)$$

For dew-points below 0 degC:

$$T_{dp} = 6.09 + 12.608 \cdot ln\,(p_v) + 0.4959 \cdot ln\,(p_v)^2 \qquad\qquad (6b)$$

Relation Between Relative Humidity (RH) and Specific Humidity W:

From the perfect gas law

$$m_v = \frac{p_v \cdot V}{R_v \cdot T} = \frac{p_v \cdot V \cdot M_v}{R \cdot T} \qquad \text{and} \qquad m_a = \frac{p_a \cdot V \cdot M_a}{R \cdot T} \qquad (7)$$

where

$$R := 8.314 \cdot \frac{joule}{mole \cdot K} \qquad \text{universal gas constant}$$

V = volume
T = temperature, K

$$M_a := 0.02896 \cdot \frac{kg}{mole}$$ 　　　molecular masses

$$M_v := 0.01802 \cdot \frac{kg}{mole}$$

$$\frac{M_v}{M_a} = 0.62224$$ 　　　　$$R_a := \frac{R}{M_a}$$

$$R_v := \frac{R}{M_v}$$ 　　　　$$R_a = 287.08564 \; \frac{joule}{kg \cdot K}$$

Therefore, since

$$W = \frac{m_v}{m_a}$$ 　　　　$$W = 0.622 \cdot \frac{p_v}{p_a}$$ 　　　(8)

and since

$$p_a = p - p_v$$ 　　　　$$W = 0.622 \cdot \frac{p_v}{p - p_v}$$ 　　　(9)

Substitute pv from (8) in

$$RH = \frac{p_v}{p_s}$$

to obtain

$$RH = \frac{W \cdot p_a}{0.622 \cdot p_s}$$

The density of the mixture of dry air and vapor is given by the sum of their respective densities. The specific volume, v, of the air-vapor mixture is expressed for unit mass of dry air:

$$v = \frac{R_a \cdot T}{p_a} = \frac{R_a \cdot T}{p - p_v}$$ 　　　(10)

Enthalpy of moist air is the sum of the enthalpies of dry air and vapor and is referred to unit mass of dry air because the mass of water vapor may vary:

$$h = h_a + W \cdot h_v \tag{11}$$

$$kJ := joule \cdot 1000$$

Assuming perfect gas behavior:

$$h_a = c_{pa} \cdot (T - T_{ref}) + h_a(T_{ref})$$
$$\tag{12}$$
$$h_v = h_g + c_{pv} \cdot (T - T_{ref})$$

where

$$c_{pa} := 1 \cdot \frac{kJ}{kg \cdot \Delta°C}$$ specific heat capacity of air at constant pressure

$$c_{pv} := 1.805 \cdot \frac{kJ}{kg \cdot \Delta°C}$$ specific heat capacity of vapor at constant pressure

$$T_{ref} := 0 \cdot \Delta°C$$ reference temperature (note that storage of energy is always relative to a reference temperature)

$$h_g := 2501 \cdot \frac{kJ}{kg}$$ saturated vapor enthalpy at T_{ref}

Therefore, substituting equation (12) in equation (11), we obtain

$$h = (T + W \cdot (2501 + 1.805 \cdot T)) \cdot \frac{kJ}{kg} \tag{13}$$

T= temperature, kg is dry air

Thermodynamic Wet-Bulb Temperature T$_{wb}$

The temperature at which water, by evaporating into the air, can bring the air to saturation adiabatically at the same temperature and constant pressure. During this process, the humidity ratio is increased from an initial value W to the value W's corresponding to the wet-bulb temperature; the enthalpy is increased from the initial value h to h$_s$. The mass of water added per kg dry air is (W's-W), which adds energy to the moist air equal to (W's-W)·h'$_w$. Therefore, an energy balance yields

$$h + \left(W'_s\left(T_{wb}\right) - W\right) \cdot h'_w\left(T_{wb}\right) = h_s\left(T_{wb}\right) \qquad\qquad (14)$$

The above equation defines thermodynamic wet-bulb temperature. A psychrometer consists of two thermometers, one covered with a wet wick. When the wet-bulb is exposed to an air stream, water evaporates and then reaches an equilibrium temperature called the wet-bulb temperature, which is close to the T$_{wb}$. Substituting equation (13) in (14) and

$$h'_w = 4.186 \cdot T \cdot \frac{kJ}{kg}$$
approximate relationship for specific enthalpy of water

and solving for W we obtain

$$W = \frac{\left(2501 - 2.381 \cdot T_{wb}\right) \cdot W'_s - \left(T_{db} - T_{wb}\right)}{2501 + 1.805 \cdot T_{db} - 4.186 \cdot T_{wb}}$$

This equation is used in an iterative manner to calculate wet-bulb temperature.

Water Vapor Saturation Pressure p$_s$

To determine moist air properties, the water vapor saturation pressure is required. This may be obtained from Tables or more easily mathematically as follows (ASHRAE 1989):

Let

$$T := (20 + 273.15)$$
Temperature transformed from degC to Kelvin

Water vapor saturation pressure over water (0 to 200 degC):

$$a := -5.8002206 \cdot 10^3 \qquad\qquad b := 1.3914993$$

$$c := -4.8640239 \cdot 10^{-2} \qquad\qquad d := 4.1764768 \cdot 10^{-5}$$

$$e := -1.4452093 \cdot 10^{-8} \qquad\qquad f := 6.5459673$$

$$p_s := \exp\left(\frac{a}{T} + b + c \cdot T + d \cdot T^2 + e \cdot T^3 + f \cdot \ln(T)\right) \cdot Pa$$

$p_s = 2338.8037 \; Pa$ compare with 2338.9 Pa from Tables

The saturation pressure over ice for the temperature range -100 to 0 degC is calculated as follows:

E.g., for

$$T := (273.15 - 10)$$

Equation constants:

$$g := -5.6745359 \cdot 10^3$$

$h := 6.3925247$ $k := -9.677843 \cdot 10^{-3}$

$l := 6.22115701 \cdot 10^{-7}$ $ll := 2.0747825 \cdot 10^{-9}$

$n := -9.484024 \cdot 10^{-13}$ $p := 4.1635019$

$$p_{sice} := \exp\left(\frac{g}{T} + h + k \cdot T + l \cdot T^2 + ll \cdot T^3 + n \cdot T^4 + p \cdot \ln(T)\right) \cdot Pa$$

$p_{sice} = 259.90212 \; Pa$ compare with 259.91 Pa from Tables

References

ASHRAE. 1989. *ASHRAE Handbook of Fundamentals*. Atlanta, GA.