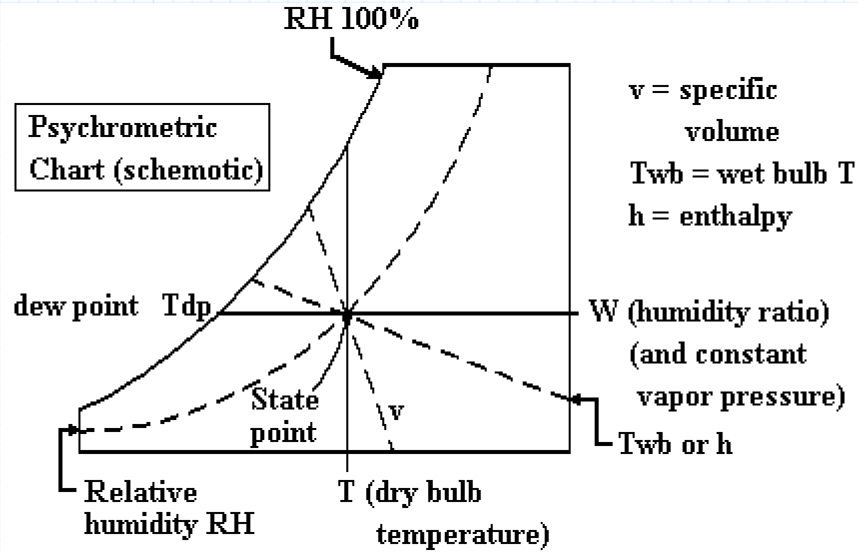
## CHAPTER 8 PSYCHROMETRY AND THERMAL COMFORT

# 8.2 Properties of Moist Air - Three Cases

### Psychrometric Chart

Moist air properties such as enthalpy and specific volume may be determined with the equations given in **Section 8.1** or using a psychrometric chart. The main limitation of the chart is, of course, accuracy and that it is drawn for a specific pressure.




Usually, the problem in determining moist air properties can be reduced to three cases. In all cases, there are two known parameters: the pressure (total) and the dry bulb temperature. A third parameter is required to define the state of moist air. This third parameter is relative humidity, or dew-point or wet-bulb temperature. We consider below examples for the three cases.

**Case 1:** Given T, p, RH (relative humidity)

$kJ := 1000\ J$

Determine the various properties of moist air given the following data:

$p := 1\ atm$ $\quad T := 20$ deg C $\quad RH := 0.45$

First calculate the saturated vapor pressure:

$T := 20 + 273.15$

$a := -5.8002206 \cdot 10^{3}$ $\quad b := 1.3914993$

$c := -4.8640239 \cdot 10^{-2}$ $\quad d := 4.1764768 \cdot 10^{-5}$

$e := -1.4452093 \cdot 10^{-8}$ $f := 6.5459673$

$$p_s := \exp\left(\frac{a}{T} + b + c \cdot T + d \cdot T^2 + e \cdot T^3 + f \cdot \ln(T)\right) Pa = 2338.8037\ Pa$$

Water vapor pressure:

$$p_v := RH \cdot p_s = 1052.4617\ Pa$$

$p_a := p - p_v = 100273\ Pa$ $p = 101325\ Pa$

Specific humidity:

$$W := 0.622 \cdot \frac{p_v}{p_a} = 0.0065$$

$R_a := 287.08 \cdot \frac{J}{kg \cdot K}$ $R_v := 461.38 \cdot \frac{J}{kg \cdot K}$

Density of air:

$$\rho_a := \frac{p_a}{R_a \cdot T \cdot K} = 1.1915\ \frac{kg}{m^3}$$

Density of vapor:

$$\rho_v := \frac{p_v}{R_v \cdot T \cdot K} = 0.0078\ \frac{kg}{m^3}$$

Density of mixture:

$$\rho := \rho_a + \rho_v = 1.1993\ \frac{kg}{m^3}$$

Specific volume:

$v := \frac{1}{\rho_a} = 0.8393\ \frac{m^3}{kg}$ (for kg of dry air)

Enthalpy:

$T := 20$ degrees C

$$h := (T + W \cdot (2501 + 1.805 \cdot T))\ \frac{kJ}{kg}$$

$h = 36.5635 \ \frac{kJ}{kg}$ (note that T should be degC in the calculation of h)

Wet-bulb temperature (requires solution of a system of equations):

Guess Values

$T_{wb} := 14$

$T_2 := 20 + 273.15$

$p_{s2} := 2000 \ Pa$

Constraints

$T_2 = T_{wb} + 273.15$

$p_{s2} = e^{\frac{a}{T_2} + b + c \cdot T_2 + d \cdot T_2^2 + e \cdot T_2^3 + f \cdot \ln(T_2)} \cdot Pa$

$0.622 \cdot \frac{p_v}{p_a} = \frac{(2501 - 2.381 \cdot T_{wb}) \cdot \frac{0.622 \cdot p_{s2}}{p - p_{s2}} - (T - T_{wb})}{2501 + 1.805 \cdot T - 4.186 \cdot T_{wb}}$

Solver

$sol := \mathbf{Find}(T_{wb}, T_2, p_{s2}) = \begin{bmatrix} 13.0616 \\ 286.2116 \\ 1503.8625 \ \frac{kg}{m \cdot s^2} \end{bmatrix}$

Therefore

$T_{wb} := sol_0 \ °C$

$p' := \frac{p_v}{1000 \cdot Pa}$ pressure in kPa but without units for calculation below

$T_{dp} := 6.54 + 14.526 \cdot \ln(p') + 0.7389 \cdot \ln(p')^2 + .09486 \cdot \ln(p')^3 + 0.4569 \cdot p'^{.1984}$

$T_{dp} = 7.7462$

**Case 2:** Given T, p, Tdp (dew-point temperature)

Determine the various properties of moist air given the following data:

$p := 1\ atm$ $\quad T := 20$ (deg C)

$T_{dp} := 7.746$

First calculate the saturated vapor pressures at the two temperatures:

$T_1 := T + 273.15$ $\quad T_2 := T_{dp} + 273.15$

$$p_{s1} := \exp\left(\frac{a}{T_1} + b + c \cdot T_1 + d \cdot T_1^{\ 2} + e \cdot T_1^{\ 3} + f \cdot \ln\left(T_1\right)\right)\ Pa$$

$$p_{s2} := \exp\left(\frac{a}{T_2} + b + c \cdot T_2 + d \cdot T_2^{\ 2} + e \cdot T_2^{\ 3} + f \cdot \ln\left(T_2\right)\right)\ Pa$$

$p_{s1} = 2338.8037\ Pa$ $\quad p_{s2} = 1054.43\ Pa$

Note that

$p_v := p_{s2}$ (vapor pressure equals saturation vapor pressure at dew-point temperature)

$W := \frac{0.622 \cdot p_v}{p - p_v}$ $\quad W = 0.0065$ humidity ratio

$W_s := \frac{0.622 \cdot p_{s1}}{p - p_{s1}}$ $\quad W_s = 0.0147$ saturation humidity ratio at dry bulb temperature

$RH := \frac{p_v}{p_{s1}}$ $\quad RH = 0.4508$ relative humidity

$$h := (T + W \cdot (2501 + 1.805 \cdot T))\ \frac{kJ}{kg} = 36.5948\ \frac{kJ}{kg}$$

$v := \frac{R_a \cdot T_1}{p - p_v} \cdot K = 0.8393\ \frac{m^3}{kg}$ specific volume per kg dry air

Wet-bulb temperature:

Guess Values

$T_{wb} := 14 \qquad T_2 := 273.15 + T \qquad p_{s2} := 2000\ Pa$

Constraints

$$T_2 = T_{wb} + 273.15$$

$$p_{s2} = \exp\left(\frac{a}{T_2} + b + c \cdot T_2 + d \cdot T_2^{\ 2} + e \cdot T_2^{\ 3} + f \cdot \ln\left(T_2\right)\right) \cdot Pa$$

$$\frac{0.622 \cdot p_v}{p - p_v} = \frac{\left(2501 - 2.381 \cdot T_{wb}\right) \cdot \dfrac{0.622 \cdot p_{s2}}{p - p_{s2}} - \left(T - T_{wb}\right)}{2501 + 1.805 \cdot T - 4.186 \cdot T_{wb}}$$

Solver

$$sol := \mathbf{Find}\left(T_{wb}, T_2, p_{s2}\right) = \begin{bmatrix} 13.0737 \\ 286.2237 \\ 1505.0478\ \dfrac{kg}{m \cdot s^2} \end{bmatrix}$$

Therefore

$$T_{wb} := sol_0\ °C = 13.0737\ °C$$

**Case 3:** Given T, p, Twb (wet-bulb temperature)

Determine the various properties of moist air given the following data:

$p := 1\ atm \qquad T := 20\ \Delta°C \qquad T_{wb} := 13.06\ \Delta°C \qquad \frac{T}{\mathrm{UnitsOf}(T)} = 20$

First calculate the saturated vapor pressures at the two temperatures:

$T_1 := \frac{T}{\Delta°C} + 273.15 \qquad T_2 := \frac{T_{wb}}{\Delta°C} + 273.15$

$$p_{s1} := \exp\left(\frac{a}{T_1} + b + c \cdot T_1 + d \cdot {T_1}^2 + e \cdot {T_1}^3 + f \cdot \ln(T_1)\right) Pa$$

$$p_{s2} := \exp\left(\frac{a}{T_2} + b + c \cdot T_2 + d \cdot {T_2}^2 + e \cdot {T_2}^3 + f \cdot \ln(T_2)\right) Pa$$

$p_{s1} = 2338.8037\ Pa \qquad p_{s2} = 1503.7006\ Pa$

$W_{s2} := \frac{0.622 \cdot p_{s2}}{p - p_{s2}} \qquad W_{s2} = 0.0094$ saturation humidity ratio at wet-bulb temperature

$W_s := \frac{0.622 \cdot p_{s1}}{p - p_{s1}} \qquad W_s = 0.0147$ saturation humidity ratio at dry bulb temperature

$$W := \frac{\left(2501 - 2.381 \cdot \frac{T_{wb}}{\Delta°C}\right) \cdot W_{s2} - (T - T_{wb}) \cdot \frac{1}{\Delta°C}}{2501 + 1.805 \cdot \frac{T}{\Delta°C} - 4.186 \cdot \frac{T_{wb}}{\Delta°C}}$$

$$p_v := p \cdot \frac{W}{0.622 + W}$$

$W = 0.0065$

$p_v = 1052.1928\ Pa$

humidity ratio and partial pressure of vapor in moist air mixture considered

$RH := \frac{p_v}{p_{s1}}$ $\quad RH = 0.4499$ $\quad$ relative humidity

$h := \left(\frac{T}{\Delta°C} + W \cdot \left(2501 + 1.805 \cdot \frac{T}{\Delta°C}\right)\right) \frac{kJ}{kg} = 36.5592 \ \frac{kJ}{kg}$

$v := \frac{R_a \cdot T_1}{p - p_v} K = 0.8393 \ \frac{m^3}{kg}$ $\quad$ specific volume per kg dry air

$p' := \frac{p_v}{1000 \ Pa}$ $\quad$ pressure in kPa but without units for calculation below

$T_{dp} := 6.54 + 14.526 \cdot \ln(p') + .7389 \cdot \ln(p')^2 + .09486 \cdot \ln(p')^3 + .4569 \cdot p'^{.1984} = 7.7425$