CHAPTER 8 PSYCHROMETRY AND THERMAL COMFORT

8.3 Thermal Comfort Calculation

Thermal Indices

A person's feeling of thermal comfort is a function of air temperature T_{ai}, mean radiant T_{mr} at the location of interest, air movement and humidity, and also personal factors such as clothing and activity.

Various environmental indices are often used as a measure of thermal comfort, including equivalent temperature, effective temperature, humid operative temperature, globe temperature and (in the UK) the resultant temperature. Several environmental parameters that affect thermal comfort can be measured directly:

1. Air temperature T_{air}
2. Wet-bulb temperature T_{wb}
3. Dew-point temperature T_{dp}
4. Water-vapor pressure p_v
5. Total atmospheric pressure p
6. Relative humidity RH
7. Specific humidity W
8. Air Velocity V
9. Mean radiant temperature T_{mr}. The temperatures of individual surfaces are usually combined into T_{mr}

Also, globe temperature, which can be measured directly, is close to the operative temperature T_{op}.

The operative temperature is the average of the mean radiant and air temperatures weighted by their respective heat transfer coefficients.

$$T_{op} = \frac{h_c \cdot T_{ai} + h_r \cdot T_{mr}}{h_c + h_r}$$

The mean radiant temperature is a very important variable in all thermal comfort calculations. It is the uniform temperature of an imaginary black enclosure in which radiant heat transfer from a person equals the radiant heat transfer in the actual enclosure. T_{mr} may be estimated from measurements of globe temperature, air temperature and air velocity. T_{mr} can also be calculated from measured temperatures of each room interior surface. Since building surfaces have a high longwave emittance, the following blackbody approximation for Tmr is satisfactory:

$$T_{mr}^{\;4} = \sum_i \left(\left(T_i \right)^4 \cdot F_{p-i} \right)$$

where i = 1.. . . N (N surfaces in room)
T_i = temperature of surface i, k
$F_{p\text{-}i}$ = view factor between a person and surface i

If small temperature differences exist between the room surfaces, then the above equation can be expressed by its linear form:

$$T_{mr} = \sum_i \left(T_i \cdot F_{p-i}\right)$$

In a rectangular room, one possible approximation in calculating the view-factors is to model a person as a sphere for ease of calculation (this would be a reasonable approximation for a seated person). The mean radiant temperature may also be calculated from the plane radiant temperature in six directions (ASHRAE 1989).

PMV (Predicted Mean Vote) Model

A steady-state model developed by Fanger (ASHRAE 1989) assumes that the body is in thermal equilibrium with negligible heat storage.

Rate of heat generation = rate of heat loss

$$M - W = Q_{sk} + Q_{res}$$

$$M - W = \left(C + R + E_{sk}\right) + Q_{res} \qquad (1)$$

where

M = rate of metabolic energy production, Watt/m^2
W = rate of mechanical work, Watt/m^2
Q_{res} = total rate of respiration heat loss
Q_{sk} = total rate of heat loss from skin
C = rate of convective heat loss
R = radiation heat loss from skin
E_{sk} = rate of total evaporative heat loss from the skin

Fanger developed the PMV model for thermal comfort by correlating collected comfort data to physiological variables. At a given level of metabolic activity M, when the body is not far from thermal neutrality, the mean skin temperature T_{sk} and sweat rate E_{rsw} are the only physiological parameters affecting a steady-state heat balance. Fanger developed the following correlation of environmental and personal variables that produce a neutral sensation (cl indicates clothing).

$$(M - W) = \left(3.96 \cdot 10^{-8} \cdot f_{cl}\right) \cdot \left(\left(T_{cl} + 273\right)^4 - \left(T_{mr} + 273\right)^4\right) + f_{cl} \cdot h_c \left(T_{cl} - T_{ai}\right) + C1 + C2 \qquad (2)$$

where

$$C1 = \left(3.05 \cdot \left(5.73 - 0.007 \cdot (M - W) - p_v\right)\right)$$

$$C2 = \left(\left(0.42 \cdot ((M - W) - 58.15) + 0.0173 \cdot M \cdot \left(5.87 - p_v\right)\right) + \left(0.0014 \cdot M \cdot \left(34 - T_{ai}\right)\right)\right)$$

$$T_{cl} = 35.7 - 0.0275 \cdot (M - W) - 0.155 \cdot I_{cl} \cdot ((M - W) - C1 - C2)$$

$$h_c = max\left(\begin{bmatrix} \left(2.38 \cdot \left(T_{cl} - T_{ai}\right)\right)^{0.25} \\ 12.1 \cdot \sqrt{V} \end{bmatrix}\right)$$ convective heat transfer coefficient

$$f_{cl} = 1.0 + 0.2 \cdot I_{cl} \quad \text{if} \quad I_{cl} < 0.5 \quad clo$$

$$f_{cl} = 1.05 + 0.1 \cdot I_{cl} \quad \text{if} \quad I_{cl} > 0.5 \quad clo$$

clothing area factor

Equation (2) is expanded to include a range of thermal sensations by using a predicted mean vote (PMV) index. The PMV index predicts the mean response of a large group of people according to the following ASHRAE thermal sensation scale:

+3 hot
+2 warm
+1 slightly warm
 0 neutral
-1 slightly cool
-2 cool
-3 cold

Fanger related PMV to the imbalance between the actual heat flow from the body in a given environment and the heat flow required for optimum comfort at the specified activity by the equation:

$$PMV = (0.303 \cdot exp(-0.036 \cdot M) + 0.028) \cdot L \qquad (3)$$

where L is the thermal load on the body equal to the difference between the left- and the right-hand sides of equation (1) or (2):

$$L = M - W - \left(3.96 \cdot 10^{-8} \cdot f_{cl} \cdot \left(\left(T_{cl} + 273\right)^4 - \left(T_{mr} + 273\right)^4\right) + \left(f_{cl} \cdot h_c \cdot \left(T_{cl} - T_{ai}\right) + C1 + C2\right)\right)$$

Example:
Determine the thermal comfort level in an office based on the PMV model given the following data:

$M := 58.2$

$W := 0$ (for normal office activity)

$T_{mr} := 23 \cdot \Delta°C$ $\qquad p_v := 1.419$ kPa (water vapor pressure)

$T_{ai} := 23 \cdot \Delta°C$ $\quad I_{cl} := 1.0$ (Units are clo)

clothing resistance: fitted trousers and long-sleeve shirt

$V := 0.1$ velocity m/s

Calculate C1 and C2 (equation 2):

$$C1 := (3.05 \cdot (5.73 - 0.007 \cdot (M - W) - p_v))$$

$$C2 := \left(0.42 \cdot ((M - W) - 58.15) + \left(0.0173 \cdot M \cdot (5.87 - p_v) + \left(0.0014 \cdot M \cdot \left(34 - \frac{T_{ai}}{K}\right)\right)\right)\right)$$

$$T_{cl} := (35.7 - 0.0275 \cdot (M - W) - 0.155 \cdot I_{cl} \cdot ((M - W) - C1 - C2))$$

$T_{cl} = 27.761$ cloth (body) surface temperature

$$h_c := \max\left(\begin{bmatrix} \left(2.38 \cdot \left(T_{cl} - \frac{T_{ai}}{K}\right)\right)^{0.25} \\ 12.1 \cdot \sqrt{V} \end{bmatrix}\right)$$

$h_c = 3.826$ convective heat transfer coefficient

$f_{cl} := 1.05 + 0.1 \cdot I_{cl}$ since $I_{cl} > 0.5$ (Units are clo)

clothing area factor

$$L := M - W - \left(3.96 \cdot 10^{-8} \cdot f_{cl} \cdot \left((T_{cl} + 273)^4 - \left(\frac{T_{mr}}{K} + 273\right)^4\right) + \left(f_{cl} \cdot h_c \cdot \left(T_{cl} - \frac{T_{ai}}{K}\right) + C1 + C2\right)\right)$$

$$PMV := (0.303 \cdot \exp(-0.036 \cdot M) + 0.028) \cdot L$$

$PMV = -0.202$ (comfortable)

References

ASHRAE. 1989. *ASHRAE Handbook of Fundamentals.* Atlanta, GA.