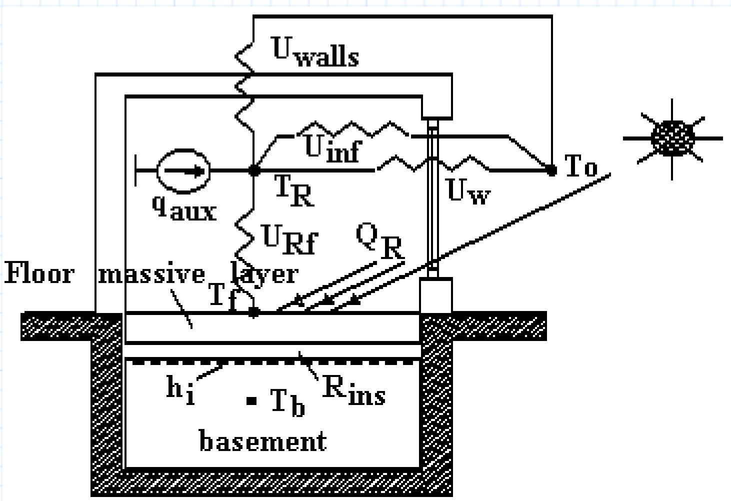
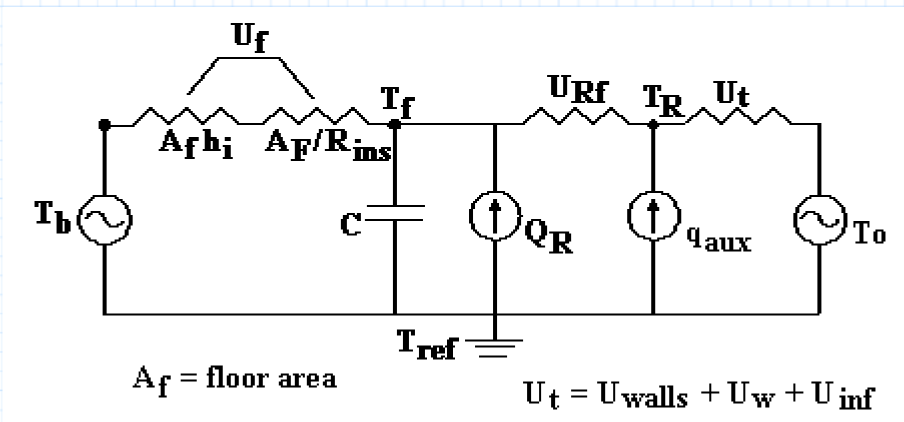
CHAPTER 9 HEATING AND COOLING LOAD CALCULATIONS

9.1 First Order Room Model

Building thermal network models are commonly used for heating/cooling load analysis, thermal comfort calculations and for building enclosure heat transfer studies. Room thermal models consisting of resistances representing convection, conduction and radiation, as well as capacitances representing thermal storage effects can have various degrees of modeling detail. Generally, detailed models represent thermal storage in each wall with separate capacitances or distributed elements (**Section 4.2**). An energy balance is performed for all capacitances, leading to a set of coupled first order differential equations. A first order model for a room represents its thermal storage capacity with only one thermal capacitance; this "effective" room thermal capacitance is usually lumped at the room temperature node or at a surface.

Consider a zone over a basement. Assume that only the floor has significant thermal capacitance and that it consists of a massive interior layer (room side) and layers without significant thermal capacity under it with insulation value R_{ins}. The room schematic together with an approximate network are shown:





U is for total area

Infiltration heat transfer (sensible):

$$q_s = U_{inf}\left(T_R - To\right)$$

$$c_{pair} := 1000 \cdot \frac{joule}{kg \cdot \Delta°C}$$ specific heat of air

$$\rho_{air} := 1.2 \cdot \frac{kg}{m^3}$$ density of air

Infiltration conductance:

$$U_{inf} = \frac{ach \cdot Vol}{3600 \cdot s} \cdot \rho_{air} \cdot c_{pair}$$ Vol= zone volume

$$U_{inf} = \frac{ach \cdot Vol}{3}$$ ach= air changes/hour

The total conductance between the basement temperature Tb and the top room temperature TR is given by

$$U_f = \frac{1}{\left(\frac{1}{A_f \cdot hi}\right) + R_{ins}}$$

Now performing an energy balance at the two nodes (R - room air and f- floor surface), we obtain

R: $$U_{Rf} \cdot \left(T_f - T_R\right) + U_t \cdot \left(T_o - T_R\right) + q_{aux} = 0$$

f: $$-C \cdot \frac{dT_f}{dt} + U_{Rf} \cdot \left(T_R - T_f\right) + U_f \cdot \left(T_b - T_f\right) + Q_R = 0$$

(1)

$-C \cdot \frac{dT_f}{dt}$ is $-Cs \cdot T_f$ in the Laplace domain

i.e. equations (1) in the Laplace domain (see **Section 10.1**) become

[Y] [T] = [Q]

$$\begin{bmatrix} U_{Rf} + U_t & -U_{Rf} \\ -U_{Rf} & sC + U_f + U_{Rf} \end{bmatrix} \cdot \begin{bmatrix} T_R \\ T_f \end{bmatrix} = \begin{bmatrix} q_{aux} + U_t \cdot T_o \\ Q_R + U_f \cdot T_b \end{bmatrix}$$

Therefore, the solution (in the Laplace domain) is

[T] = [Y^{-1}] [Q] or

$$\begin{bmatrix} T_R \\ T_f \end{bmatrix} = \frac{1}{D} \cdot \begin{bmatrix} sC + U_f + U_{Rf} & U_{Rf} \\ U_{Rf} & U_{Rf} + U_t \end{bmatrix} \cdot \begin{bmatrix} q_{aux} + U_T \cdot T_o \\ Q_R + U_f \cdot Tb \end{bmatrix}$$

where the determinant D is given by

$$D = \left(U_{Rf} + U_t\right) \cdot \left(sC + U_f + U_{Rf}\right) - U_{Rf}^{\,2}$$

Therefore, the room temperature is

$$T_R = \frac{sC + U_f + U_{Rf}}{D} \cdot \left(q_{aux} + U_t \cdot T_o\right) + \left(\frac{U_{Rf}}{D} \cdot \left(Q_R + U_f \cdot Tb\right)\right) \qquad (2)$$

Note that T_o, q_{aux}, T_b, and T_R are in the Laplace domain. Equation (2) may also be expressed as

$$T_R = Z_{11} \cdot \left(q_{aux} + U_t \cdot T_o\right) + Z_{12} \cdot \left(Q_R + U_f \cdot T_b\right) \qquad (3)$$

where

$$Z_{11}(s) = \frac{s \cdot C + U_f + U_{Rf}}{\left(U_{Rf} + U_t\right) \cdot \left(s \cdot C + U_f + U_{Rf}\right) - U_{Rf}^{\,2}}$$

$$Z_{12}(s) = \frac{U_{Rf}}{\left(U_{Rf} + U_t\right) \cdot \left(s \cdot C + U_f + U_{Rf}\right) - U_{Rf}^{\,2}}$$

(4)

Equations 3 and 4 relate T_R (s) to inputs (forcing functions) q_{aux}(s), T_o(s), Q_R(s) and T_b(s). Note that if T_R is specified (known) the auxiliary heating/cooling may be determined (by rearranging (3)) as

$$q_{aux} = \frac{T_R - Z_{12} \cdot \left(Q_R + U_f \cdot T_b\right) - Z_{11} \cdot U_t \cdot T_o}{Z_{11}} \qquad (5)$$

For steady state calculations the capacitance term sC is set to zero in equation (4), i.e. Z_{12} and Z_{11} become effectively resistances. We can determine the periodic variation of $q_{aux}(t)$ and $T_R(t)$ by representing the variation of the inputs T_o and Q_R by sinusoids as demonstrated in chapter 4. Then we have

Total response = mean term + harmonic variation

Frequency response analysis and input-output analysis may be performed more easily by using complex numbers.

Z_{11} and Z_{12} are impedance transfer functions (analogous to impedances in a.c. electric circuits) and their phase and magnitude may be evaluated by substituting s = jω where j = (-1) to power 0.5, and ω is the frequency of interest (one cycle per day). In general, given a transfer function Z which relates the effect of an input Q on a temperature T we have the following:

for

$$Q = A \cdot cos(\omega \cdot t + \theta)$$

we obtain

$$T(t) = A \cdot |Z(j \cdot w)| \cdot cos(w \cdot t + \theta + \phi_z) \qquad (6)$$

where

$|Z(j \cdot w)|$ = magnitude of Z and $\phi_z = arg(Z(j \cdot w))$

Note that the phase angle of the transfer function ϕ_z determines the time lag between cause (Q) and effect (T). Because of the superposition principle, we can consider each input (e.g. absorbed solar radiation Q_R) alone or all together. Simple models for the sources are employed as follows:

Outside temperature:

$$T_o = T_{om} + \frac{\Delta T_o}{2} \cdot cos(w \cdot t + \theta_1) \qquad (6a)$$

Solar radiation absorbed on floor:

$$Q_R = Q_{Rm} + \Delta Q_R \cdot cos(w \cdot t + \theta_2) \qquad (6b)$$

Basement temperature:

$$T_b = T_{bm} = constant \qquad (6c)$$

Room temperature:

$$T_R = constant \qquad (6d)$$

Let Z_{11m} = value of Z_{11} for w = 0 and Z_{12m} = value of Z_{12} for w = 0 (steady-state). The general solution for qaux (from equation 5) is then given by

$$q_{mean} = \frac{T_R}{Z_{11m}} - \frac{Z_{12m}}{Z_{11m}} \cdot (Q_{Rm} + U_f \cdot Tb) - U_t \cdot T_{om}$$ steady state term

$$q_{To} = -U_t \cdot \frac{\Delta T_o}{2} \cdot cos(w \cdot t + \theta_2)$$ variation due to T_o

$$q_{QR}(t) = -\left(\frac{|Z_{12}(j \cdot w)|}{|Z_{11}(j \cdot w)|} \cdot \Delta Q_R \cdot cos(w \cdot t + \theta_1 + \phi_{z12} - \phi_{z11}) \right)$$ variation due to Q_R

$$q_{aux}(t) = q_{mean} + q_{QR} + q_{To}$$

The variation of $q_{aux}(t)$ due to the solar gains is particularly interesting. There is a phase lag of $\phi_{z12} - \phi_{z11}$, that is a time lag of $(\phi_{z12} - \phi_{z11})/w$ in $q_{aux}(t)$ relative to QR(t).

Example: We will consider a simple zone with the following data.

Dimensions:

$L := 5 \cdot m$ $\qquad W := 5 \cdot m$ $\qquad H := 3 \cdot m$

$A_f := L \cdot W$ $\qquad A_w := 4 \cdot m^2$ $\qquad$ floor and window areas (window on wall WxH)

$R_{wall} := 2.1\ m^2 \dfrac{\Delta °C}{watt}$ $\qquad R_{roof} := 2.5\ m^2 \dfrac{\Delta °C}{watt}$

$R_w := 0.34\ m^2 \dfrac{\Delta °C}{watt}$ $\qquad R_{ins} := 1\ m^2 \dfrac{\Delta °C}{watt}$

$hi := 9 \ \frac{watt}{m^2 \cdot \Delta°C}$ film coefficient

$ach := 1$ air changes per hour

$A_{wall} := (2 \cdot L \cdot H + W \cdot H \cdot 2) - A_w$

$A_{roof} := L \cdot W$ $Vol := L \cdot W \cdot H$

Floor cover layer -tiles with properties:

$k := 1.0 \ \frac{watt}{m \cdot \Delta°C}$ conductivity

$\rho := 1200 \ \frac{kg}{m^3}$ density

$c := 700 \ \frac{joule}{kg \cdot \Delta°C}$ specific heat

$x := 4 \ cm$ thickness

Calculate conductances:

$U_{inf} := \frac{ach \cdot Vol}{3600 \cdot sec} \cdot \rho_{air} \cdot c_{pair}$ $U_{inf} = 25 \ \frac{watt}{\Delta°C}$

$U_t := U_{inf} + \frac{A_w}{R_w} + \frac{A_{wall}}{R_{wall}} + \frac{A_{roof}}{R_{roof}}$

$U_t = 73.43137 \ \frac{watt}{\Delta°C}$

$U_f := \frac{A_f}{\frac{1}{hi} + R_{ins}}$ $U_{Rf} := A_f \cdot hi$

Thermal capacitance:

$C := c \cdot \rho \cdot A_f \cdot x$ $\qquad C = \left(8.4 \cdot 10^5\right) \dfrac{joule}{\Delta °C}$

$w := 2 \cdot \dfrac{\pi}{86400 \cdot s}$ $\qquad$ frequency (1 cycle/day) $\qquad j := \sqrt{-1}$

Room transfer functions:

$$Z_{11}(s) := \frac{s \cdot C + U_f + U_{Rf}}{\left(U_{Rf} + U_t\right) \cdot \left(s \cdot C + U_f + U_{Rf}\right) - U_{Rf}^{\ 2}}$$

$$Z_{12}(s) := \frac{U_{Rf}}{\left(U_{Rf} + U_t\right) \cdot \left(s \cdot C + U_f + U_{Rf}\right) - U_{Rf}^{\ 2}}$$

$$Z_{11}\left(\frac{0}{s}\right) = 0.01065 \ \frac{\Delta °C}{watt}$$

$$Z_{12}\left(\frac{0}{s}\right) = 0.00968 \ \frac{\Delta °C}{watt}$$

$Z_{11m} := Z_{11}\left(\dfrac{0}{s}\right)$ $\qquad Z_{12m} := Z_{12}\left(\dfrac{0}{s}\right)$

$$Z_{11}(j \cdot w) = (0.00787 - 0.00355\mathrm{j}) \ \frac{\Delta °C}{watt}$$

$$Z_{12}(j \cdot w) = (0.00599 - 0.0047\mathrm{j}) \ \frac{\Delta °C}{watt}$$

Specified temperature and solar source:

$T_R := 20 \ \Delta °C$ $\qquad T_b := 16 \ \Delta °C$

$T_{om} := 0 \ \Delta °C$ $\qquad \Delta T_o := 10 \ \Delta °C$

$Q_{Rm} := A_w \cdot 200 \cdot \dfrac{watt}{m^2}$ $\qquad \Delta Q_R := Q_{Rm}$

$t := 1 \cdot hr, 2 \cdot hr .. 24 \cdot hr$ $\qquad \theta_1 := -5 \cdot \dfrac{\pi}{4}$ $\qquad \theta_2 := -\pi$

Load calculation:

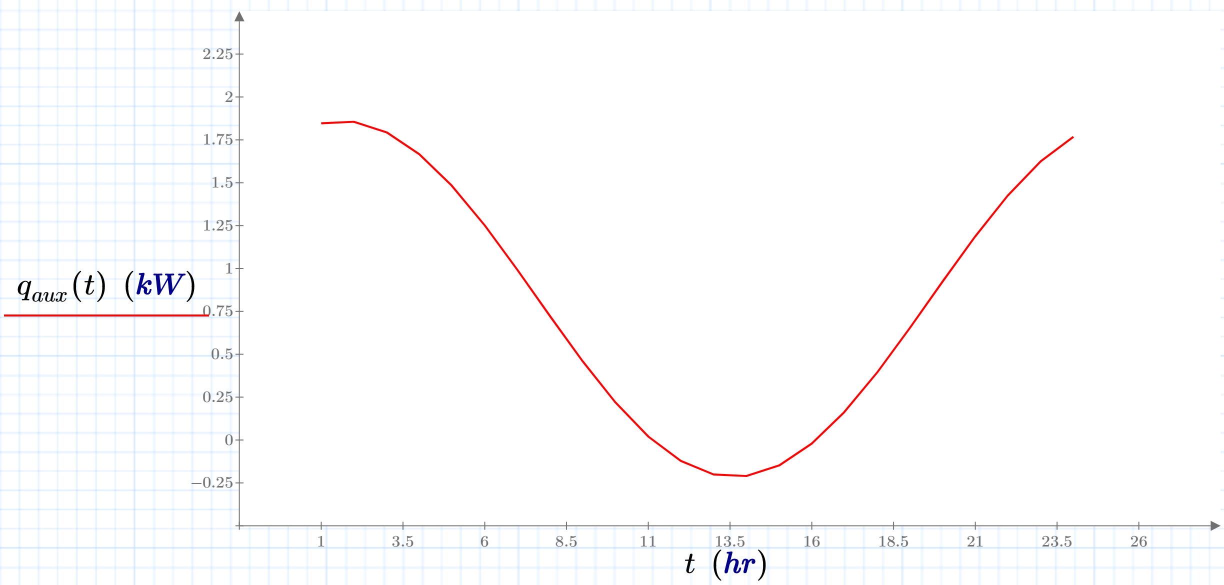
$$q_{mean} := \frac{T_R}{Z_{11m}} - \frac{Z_{12m}}{Z_{11m}} \cdot \left(Q_{Rm} + U_f \cdot T_b\right) - U_t \cdot T_{om}$$

$$q_{mean} = 823\ watt$$

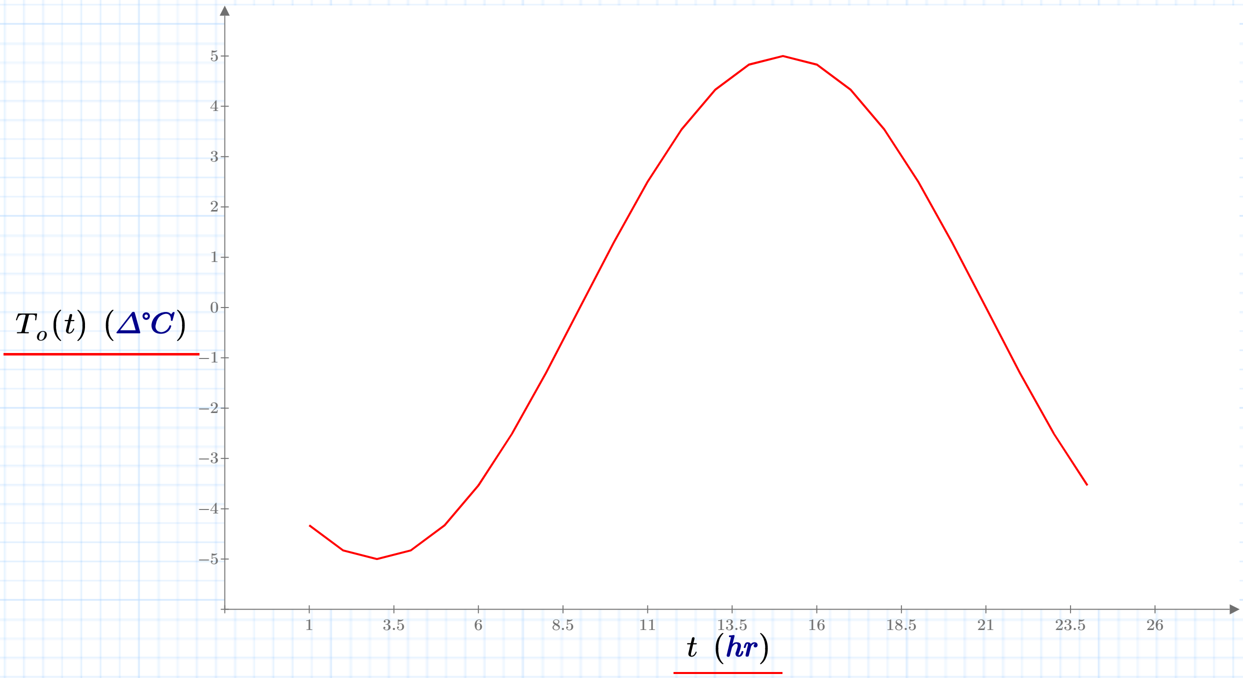
$$q_{To}(t) := -U_t \cdot \frac{\Delta T_o}{2} \cdot \cos\left(w \cdot t + \theta_1\right)$$

$$q_{QR}(t) := -\left(\frac{\left|Z_{12}(j \cdot w)\right|}{\left|Z_{11}(j \cdot w)\right|} \cdot \Delta Q_R \cdot \cos\left(\left(w \cdot t + \theta_2\right) - \arg\left(Z_{11}(j \cdot w)\right) + \arg\left(Z_{12}(j \cdot w)\right)\right)\right)$$

$$q_{aux}(t) := q_{mean} + q_{QR}(t) + q_{To}(t)$$



$$T_o(t) := T_{om} + \frac{\Delta T_o}{2} \cdot \cos\left(w \cdot t + \theta_1\right)$$



The above results indicate a peak heating load of 2.1 kW based on this simple room model and the approximate solar radiation model. More detailed and accurate models are employed in the next sections, including complete solar radiation calculations. This model may be employed for fast analysis of simple cases and to understand the basic concepts employed in the next two sections.

References

Athienitis, A. K., H. F. Sullivan and K. G. T. Hollands. 1986. "Analytical Model, Sensitivity Analysis, and Temperature Swings in Direct Gain Rooms," *Solar Energy,* vol.36, pp. 303-12.