CHAPTER 9 HEATING AND COOLING LOAD CALCULATIONS

# 9.2 Detailed Steady-Periodic Zone Model and Heating Load Calculations

Methods for calculation of heat transfer through walls have already been introduced.

The explicit finite difference method (**Section 3.3**) requires discretization in both space and time.

The (frequency domain) admittance method (**Sections 4.1-4.3**) permits both lumped and distributed parameter elements and requires Fourier series models for all heat sources and specified temperatures.

In addition, a simple periodic model of this method was introduced in **Section 9.1**.

The main advantage of the admittance method is that it is convenient for steady-periodic analysis (with about three harmonics). However, it cannot accommodate nonlinearities. The explicit finite difference method may accommodate nonlinear heat transfer coefficients and control strategies. Here, we will consider a detailed model based on the admittance method. The following inputs are required.

**1. Weather inputs:**

**a.** Outside temperature

**b.** Solar radiation (approximate model)

Note that, since heating equipment should be sized based on extreme weather conditions, solar radiation may be excluded from this analysis. However, if a passive solar analysis is to be performed, the solar gains should be considered. The model employed for heating load calculation may be considered a special case of the cooling load model presented in the next section.

**2. Building data:**

NS: number of surfaces contributing to the zone energy balance.

$NS_e$: number of exterior surfaces (walls and roof).

$A_i$ : area of exterior surface i.

Nw: number of windows (= NSe  normally)

$Aw_i$: area of window i

**3. Window type:**

U value or thermal resistance, single or double glazing and kL value (extinction coefficient x thickness)

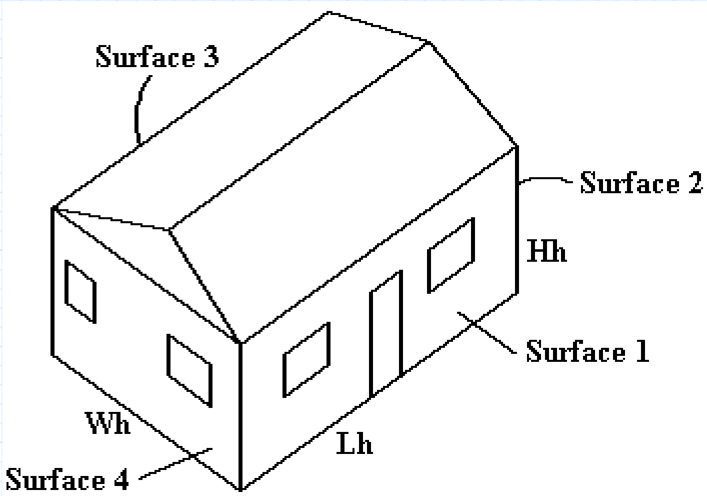
$A_{door}$: external door area        $R_{door}$: external  door R-value

**4. Wall construction:**
Wall layer properties. For the interior layer, properties for transient analysis are required.

ach: infiltration - air changes per hour
$h_i$: inside combined surface heat transfer (film) coefficient for surface i

**5. Internal gains:**
$Q_{intr}$: radiative internal gains   $Q_{intc}$: convective internal gains

**Example:** Consider a house which consists of a basement and a ground level floor, with a pitched roof. The basement heating load may be determined with the techniques of **Section 3.2**. Here we consider the ground level zone.




$Hh := 2.7\ m$ $\quad Lh := 14\ m$ $\quad Wh := 12\ m$

Surfaces contributing to energy balance:

$NS := 6$

$i := 1, 2 .. NS$ — 1-4 walls, 5-ceiling, 6-floor

$se := 1 .. 5$ — exterior surfaces

$Nw := 4$ $\quad iw := 1, 2 .. Nw$

(assume four windows — sum the window areas on each house side)

Window and door areas:

$Aw_1 := 12\ m^2$ $\quad Aw_2 := 3\ m^2$

$Aw_3 := 2\ m^2$ $\quad Aw_4 := 3\ m^2$

$Ad_1 := 2\ m^2$ $\quad Ad_2 := 2\ m^2$

$Ad_3 := 2\ m^2$ $\quad Ad_4 := 2\ m^2$

Wall net areas:

$A_1 := Lh \cdot Hh - Aw_1 - Ad_1$

$A_2 := Wh \cdot Hh - Aw_2 - Ad_2$

$A_3 := Lh \cdot Hh - Aw_3 - Ad_3$

$A_4 := Wh \cdot Hh - Aw_4 - Ad_4$

$A_5 := Wh \cdot Lh$ $\quad A_6 := A_5$

$Hi := 2.4\ m$ internal height

$Vol := A_5 \cdot Hi$

$Rd := 1\ \dfrac{m^2 \cdot \Delta°C}{W}$ door thermal resistance

$R_w := 0.34\ \dfrac{m^2 \cdot \Delta°C}{W}$ window resistance (double-glazed)

see **Section 6.3.1**

$h_1 := 8.3\ \dfrac{W}{m^2 \cdot \Delta°C}$ $\quad h_2 := h_1$

$h_3 := h_1$ $\quad h_4 := h_1$

$h_5 := 9.3\ \dfrac{W}{m^2 \cdot \Delta°C}$ film coefficients

$h_6 := 6.1\ \dfrac{W}{m^2 \cdot \Delta°C}$ (cold floor)

$ach := 0.5$ ach= air changes /hour

Calculation of infiltration conductance:

$c_{pair} := 1000 \ \frac{J}{kg \cdot \Delta°C}$ $\rho_{air} := 1.2 \ \frac{kg}{m^3}$ specific heat and density of air

$U_{inf} := \frac{ach \cdot Vol}{3600 \cdot s} \cdot \rho_{air} \cdot c_{pair}$ $U_{inf} = 67.2 \ \frac{W}{\Delta°C}$

## Thermal Resistance of Walls (including air films)

### Vertical Walls

**1.** gypsum board

$L_1 := 0.013 \ m$ thickness

$\rho_1 := 800 \ \frac{kg}{m^3}$ density

$k_1 := 0.16 \ \frac{W}{m \cdot \Delta°C}$ conductivity

$c_1 := 750 \ \frac{J}{kg \cdot \Delta°C}$ specific heat

**2.** insulation

$R_{ins} := 2.2 \ \frac{m^2 \cdot \Delta°C}{W}$

**3.** siding and sheathing

$R_{sid} := 0.37 \ \frac{m^2 \cdot \Delta°C}{W}$

**4.** exterior film

$h_o := 22 \ \frac{W}{m^2 \cdot \Delta°C}$ (**Section 5.3**)

15% of area is framing $f_f := 0.15$ fraction of area which is framing

2-by-4 wood stud with R value:

$$R_f := 0.77 \frac{m^2 \cdot \Delta°C}{W}$$

$$R_1 := \frac{1}{\dfrac{1-f_f}{\dfrac{L_1}{k_1}+R_{ins}+R_{sid}+\dfrac{1}{h_o}+\dfrac{1}{h_1}}+\dfrac{f_f}{\dfrac{L_1}{k_1}+R_f+R_{sid}+\dfrac{1}{h_o}+\dfrac{1}{h_1}}}$$

$$R_1 = 2.44 \frac{\Delta°C \cdot m^2}{W}$$

Calculation of wall conductance excluding interior layer and film (to be used for admittance calculations):

$$u_1 := \frac{1}{R_1 - \dfrac{L_1}{k_1} - \dfrac{1}{h_1}}$$

Assume that all exterior walls are of the same construction:

$ii := 1, 2 .. 4$

$L_{ii} := L_1$ $R_{ii} := R_1$ $u_{ii} := u_1$

$k_{ii} := k_1$ $\rho_{ii} := \rho_1$ $c_{ii} := c_1$

## Calculation of Roof-Ceiling Thermal Resistance
(see **Section 1.6**)

### Ceiling

**1.** gypsum board

$L_5 := L_1$ $\qquad k_5 := k_1$

$c_5 := c_1$ $\qquad \rho_5 := \rho_1$

**2.** insulation

$$R_{insc} := 2.8 \; \frac{m^2 \cdot \Delta°C}{W}$$

**3.** air-film (attic)

$$h_a := 12 \cdot \frac{W}{m^2 \cdot \Delta°C}$$

$$R_c := \frac{1}{\dfrac{1 - f_f}{\dfrac{L_5}{k_5} + R_{insc} + \dfrac{1}{h_a} + \dfrac{1}{h_5}} + \dfrac{f_f}{\dfrac{L_5}{k_5} + R_f + \dfrac{1}{h_a} + \dfrac{1}{h_5}}}$$

$$R_c = 2.377 \; \frac{\Delta°C \cdot m^2}{W}$$

## Roof

**1.** exterior air film

$$h_o := 20 \frac{W}{m^2 \cdot \Delta°C}$$

**2.** shingle backer board

$$R_b := 0.19 \frac{\Delta°C \cdot m^2}{W}$$

**3.** wood shingles

$$R_{sh} := 0.17 \frac{\Delta°C \cdot m^2}{W}$$

$$R_r := \frac{1}{\dfrac{1-f_f}{R_b + R_{sh} + \dfrac{1}{h_o} + \dfrac{1}{h_a}} + \dfrac{f_f}{R_f + R_b + R_{sh} + \dfrac{1}{h_o} + \dfrac{1}{h_a}}}$$

$$R_r = 0.543 \frac{\Delta°C \cdot m^2}{W}$$

Assuming a 30 degree slope for the roof, we calculate the ceiling-roof combined resistance per unit ceiling area (assuming no ventilation in the attic -- see **Section 1.6** for ventilated attic) as follows:

$$A_r := \frac{A_5}{\cos(30\ deg)} \qquad R_5 := \left(\frac{R_c}{A_5} + \frac{R_r}{A_r}\right) \cdot A_5$$

$$R_5 = 2.848 \frac{\Delta°C \cdot m^2}{W} \qquad u_5 := \frac{1}{R_5 - \dfrac{L_5}{k_5} - \dfrac{1}{h_5}}$$

for admittance calculation

## Floor

**1.** Carpet and underpad

$L_6 := 0.02\ m$

$k_6 := 0.06\ \frac{W}{m \cdot \Delta°C}$

$\rho_6 := 800\ \frac{kg}{m^3}$

**2.** Insulation and plywood

$R_{ins} := 1.0\ \frac{m^2 \cdot \Delta°C}{W}$

$c_6 := 1400\ \frac{J}{kg \cdot \Delta°C}$

**3.** Air film (horizontal heat flow downward)

$h_o := 6.13 \cdot \frac{W}{m^2 \cdot \Delta°C}$

$R_6 := R_{ins} + \frac{L_6}{k_6} + \frac{1}{h_o} + \frac{1}{h_6}$

$R_6 = 1.66\ \frac{m^2 \cdot \Delta°C}{W}$

$u_6 := \frac{1}{R_6 - \frac{L_6}{k_6} - \frac{1}{h_6}}$

## Calculation of Wall Admittances

The self-admittance and the transfer admittance will be calculated for each wall, considering the thermal capacity of the room interior layer. Note that the steady-state value of the admittance is equal to the wall conductance. We will calculate admittances to the interior surface and to the room air point. The analysis will be performed for the mean term and three harmonics of the weather inputs and heat sources.

## Admittances

Steady state admittance to interior surface is equal to wall U value (excluding interior film); first subscript indicates frequency, second subscript indicates surface number.

$$Ys_{0,i} := \frac{A_i}{R_i - \frac{1}{h_i}} \qquad Yt_{0,i} := Ys_{0,i}$$

$$Y_{0,i} := \frac{A_i}{R_i} \qquad Yta_{0,i} := Y_{0,i}$$

admittances from outside to room air (steady state)

$$n := 1, 2 .. 3 \qquad j := \sqrt{-1}$$

$$\gamma_{n,i} := \sqrt{j \cdot \frac{2 \cdot \pi \cdot n}{\frac{k_i}{\rho_i \cdot c_i} \cdot 24\ hr}}$$

$$Ui_i := A_i \cdot h_i$$

$$Uo_i := h_o \cdot A_i$$

interior and exterior surface conductances

$$iw := 1, 2 .. 4$$

$$Uw_{iw} := \frac{Aw_{iw}}{R_w} + \frac{Ad_{iw}}{Rd}$$

conductance of double-glazed windows and doors

$$Ys_{n,i} := A_i \cdot \frac{u_i + k_i \cdot \gamma_{n,i} \cdot \tanh\left(\gamma_{n,i} \cdot L_i\right)}{\left(\frac{u_i}{k_i \cdot \gamma_{n,i}} \cdot \tanh\left(\gamma_{n,i} \cdot L_i\right)\right) + 1}$$

$$Yt_{n,i} := \frac{A_i}{\frac{\cosh\left(\gamma_{n,i} \cdot L_i\right)}{u_i} + \frac{\sinh\left(\gamma_{n,i} \cdot L_i\right)}{k_i \cdot \gamma_{n,i}}}$$

$$Y_{n,i} := \frac{Ys_{n,i} \cdot Ui_i}{Ys_{n,i} + Ui_i}$$

$$Yta_{n,i} := Yt_{n,i} \cdot \frac{Ui_i}{Ys_{n,i} + Ui_i}$$ wall admittances from outside to inside air

**Zone Admittance Yz** (from room temperature node)

$n := 0,1..3$ $\qquad$ $Yz_n := U_{inf} + \sum_{iw} Uw_{iw} + \sum_i Y_{n,i}$

$$Yz = \begin{bmatrix} 340.267 \\ 403 + 298.766j \\ 539.342 + 519.654j \\ 683.104 + 670.462j \end{bmatrix} \frac{W}{\Delta °C}$$

Note that $Yz_0$ is simply the total U value of the house.

**Energy Balance**

The energy balance may be written in a similar form to that for the simple model of **Section 9.1**. All walls are represented by their self-admittances, together with the equivalent heat source at their inside surface, equal to the transfer admittance times the outside or other equivalent known temperature. Radiation and convection in the room are represented by combined film coefficients. Thus, an analytical solution for room temperature or auxiliary heating may be obtained. A more detailed model with separate radiation exchanges requires an energy balance at the interior surfaces
(Athienitis et al, 1990).

Energy balance at room air node (in admittance form):

[Y] [T] = [Q]

and since surface nodes may be eliminated (**Section 4.3**):

$$Yz \cdot T_R = \sum Q_{eq} + q_{aux}$$

$q_{aux}$= auxiliary heating (or cooling - if negative).

All heat gains/losses are expressed as equivalent heat sources (negative if losses) $Q_{eq}$ at the room temperature node based on a reference temperature of 0 degC as follows:

Internal gains plus heat source due to infiltration:

$$Qintc_n + \left(U_{inf} + \sum_{iw} Uw_{iw}\right) \cdot To_n \qquad \text{(a)}$$

Equivalent heat sources at wall interior surfaces due to sol-air temperatures or specified temperatures in adjacent zones. These can be added to radiant gains Qintr at the interior surfaces:

$$\left(Yt_{n,i} \cdot Teq_{n,i} + Qintr_{n,i}\right) \qquad \text{(b1)}$$

Source (b1) must be converted to a heat gain at the room air point as follows:

$$\left(Yt_{n,i} \cdot Teq_{n,i} + Qintr_{n,i}\right) \cdot \frac{Ui_i}{Ys_{n,i} + Ui_i} \qquad \text{(b)}$$

The energy balance may then be written as

$$q_{aux_n} = \left( TR_n - \frac{Qintc_n + \left(U_{inf} + \sum_{iw} Uw_{iw}\right) \cdot To_n + \sum_i \left( \left(Yt_{n,i} \cdot Teq_{n,i} + Qintr_{n,i}\right) \cdot \frac{Ui_i}{Ys_{n,i} + Ui_i} \right)}{Yz_n} \right) \cdot Yz_n$$

For heating load calculations, we will consider a minimal level of solar radiation equivalent to a cloudy day. The outside temperature will be modeled by a three harmonic Fourier series based on its value at eight different times. Alternatively, we may employ the one-harmonic model given in **Section 4.3**.

## Outside Temperature

The outside temperature is modeled by a Fourier series based on NTo+1 values that are an input to the array below. If more detail is required, NTo may be increased.

$NTo := 7$ $\qquad it := 0, 1 .. NTo$ $\qquad$ time index

$t_{it} := it \cdot 3\ hr$ $\qquad$ time

$n := 0, 1 .. 3$ $\qquad$ harmonics

$$w_n := 2 \cdot \pi \cdot \frac{n}{24\ hr} \qquad j := \sqrt{-1}$$

$$To_{it} := \begin{bmatrix} -1 \\ -4 \\ 0 \\ 3 \\ 5 \\ 6 \\ 5 \\ 1 \end{bmatrix} K \qquad Ton_n := \left( \sum_{it} \left( To_{it} \cdot \frac{\exp\left(-1j \cdot w_n \cdot t_{it}\right)}{NTo + 1} \right) \right)$$

$$Ton_0 = \begin{bmatrix} -1 \\ -4 \\ 0 \\ 3 \\ 5 \\ 6 \\ 5 \\ 1 \end{bmatrix} K$$

mean daily temperature

**Solar Radiation**

Solar radiation will be modeled by a half-sinusoid from sunrise to sunset time $t_s$.

Redefine time array: $it := 0, 1 .. 23$ $\qquad t_{it} := it \cdot hr$

Let

$t_s := 5\ hr$ time from solar noon to sunset

$S_{max} := 100\ \frac{W}{m^2}$ peak solar radiation (at noon) (assume minimal level for cloudy day)

$$f_{it} := S_{max} \cdot \cos\left(\pi \cdot \frac{t_{it} - 12 \cdot hr}{2 \cdot t_s}\right)$$

$$S_{it} := \mathbf{if}\left(f_{it} > 0.0\ \frac{W}{m^2}, f_{it}, 0.0\ \frac{W}{m^2}\right)$$

S(t) may be modeled with a discrete Fourier series as follows:

$$Sn_n := \left(\sum_{it} \left(S_{it} \cdot \frac{\exp\left(-j \cdot w_n \cdot t_{it}\right)}{24}\right)\right) \cdot \sum_{iw} Aw_{iw}$$

We multiply by total window area to determine total solar radiation.

We will assume that the fraction of this radiation absorbed by each interior surface is proportional to its area:

$$Atot := \sum_i A_i \qquad Qr_{n,i} := Sn_n \cdot \frac{A_i}{Atot}$$

We may similarly model internal gains with Fourier series. In this example, we will ignore internal gains. Also, we will ignore solar radiation absorbed on wall exterior surfaces since our objective is to determine the peak heating load.

Room-air temperature TR (assume specified):

$$T_{R_0} := 20\ \Delta°C \qquad n1 := 1, 2 .. 3 \qquad T_{R_{n1}} := 0\ \Delta°C$$

A variable room temperature may be employed, e.g. with a night setback\setup. In such a case, TR should be modeled by a Fourier series like To.

Equivalent temperatures for exterior surfaces:

$$Teq_{n,se} := Ton_n$$

For basement:

$$T_b := 20\ \Delta°C \qquad Teq_{0,6} := T_b \qquad Teq_{n1,6} := 0\ \Delta°C$$

First determine the mean auxiliary load:

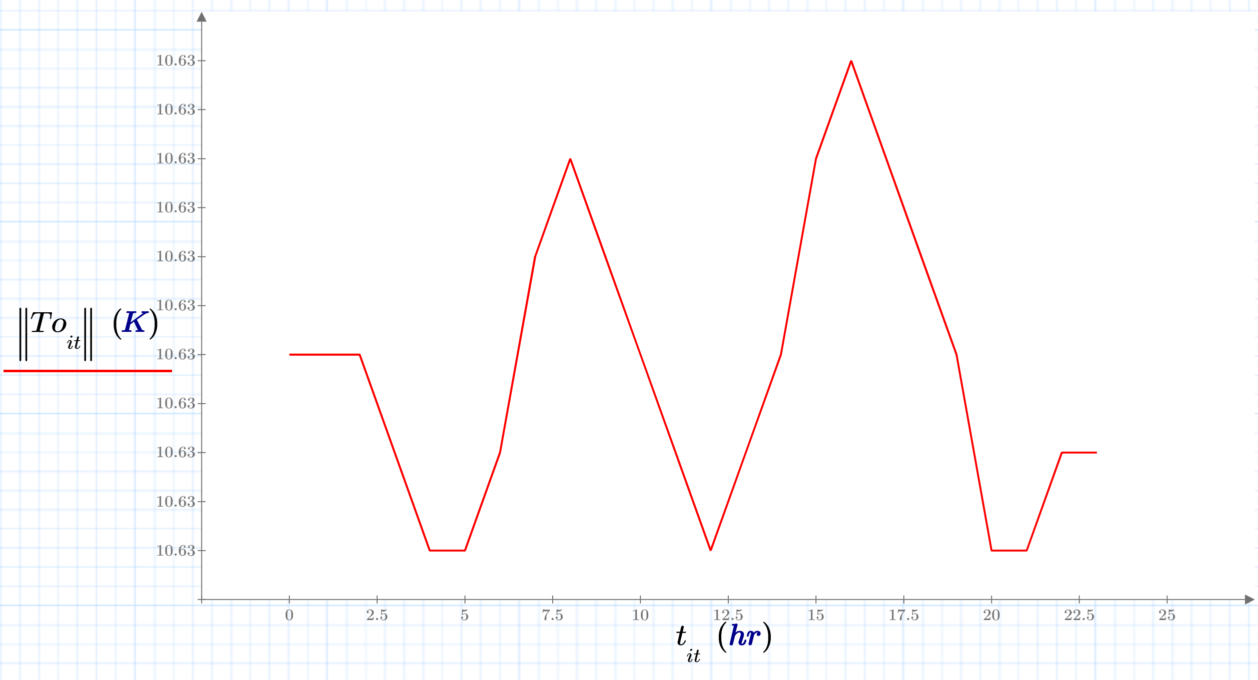
$$q_{aux_0} := \left(T_{R_0} - \frac{\left(U_{inf} + \sum_{iw} Uw_{iw}\right) \cdot Ton_0 + \sum_i \left(\left(Yt_{0,i} \cdot Teq_{0,i} + Qr_{0,i}\right) \cdot \frac{Ui_i}{Ys_{0,i} + Ui_i}\right)}{Yz_0}\right) \cdot Yz_0$$

$$q_{aux_0}=\begin{bmatrix}4.528\cdot 10^3\\5.245\cdot 10^3\\4.289\cdot 10^3\\3.572\cdot 10^3\\3.094\cdot 10^3\\2.854\cdot 10^3\\3.094\cdot 10^3\\4.05\cdot 10^3\end{bmatrix} W$$

$$q_{aux_{n1}}:=\left(T_{R_{n1}}-\frac{\left(U_{inf}+\sum_{iw} Uw_{iw}\right)\cdot Ton_{n1}+\sum_{i}\left(\left(Yt_{n1,i}\cdot Teq_{n1,i}+Qr_{n1,i}\right)\cdot\frac{Ui_i}{Ys_{n1,i}+Ui_i}\right)}{Yz_{n1}}\right)\cdot Yz_{n1}$$

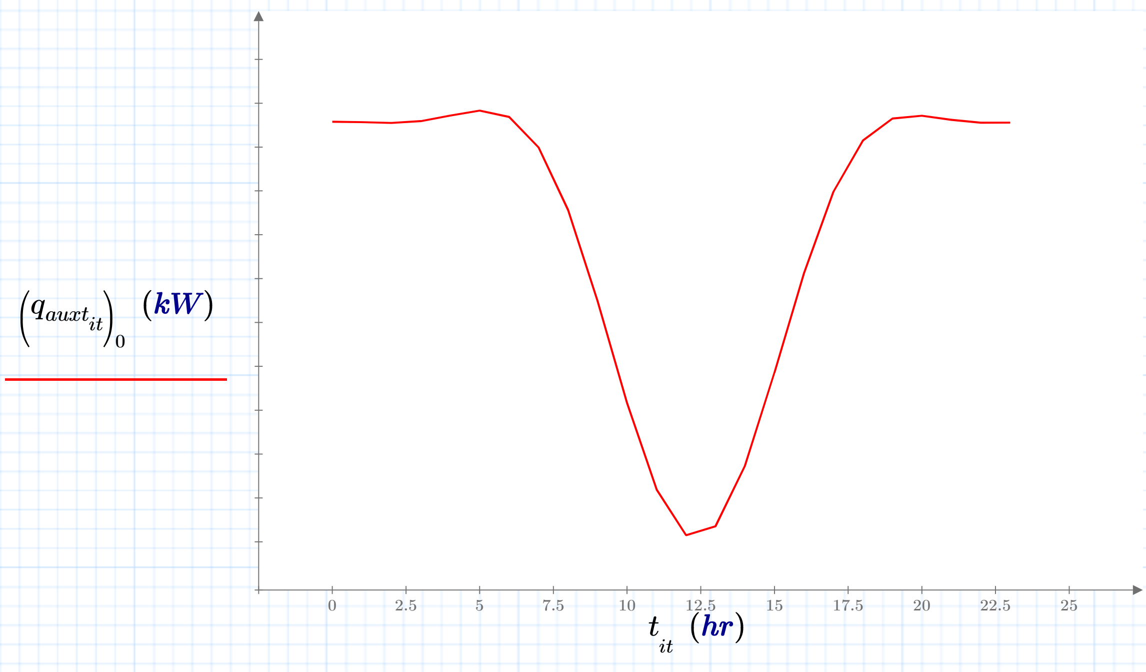
$$To_{it}:=Ton_0+2\cdot\sum_{n1}\mathrm{Re}\left(\left(Ton_{n1}\right)\cdot\exp\left(j\cdot w_{n1}\cdot t_{it}\right)\right)$$

Ambient temperature



$$q_{auxt_{it}} := q_{aux_0} + 2 \cdot \sum_{n1} \text{Re}\left(q_{aux_{n1}} \cdot \exp\left(j \cdot w_{n1} \cdot t_{it}\right)\right)$$

Auxiliary power




Peak heating load (used to size heating system):

$$max\left(q_{auxt}\right) = ?\ W$$

Heating energy consumption Qh obtained by numerical integration of the positive part of the auxiliary power curve:

$q_{auxt_{24}} := q_{auxt_0}$ $\quad t_{24} := 24\ hr$ $\quad MJ := 10^6\ J$

$$Qh := \sum_{it} \left( \frac{q_{auxt_{it}} + \left|q_{auxt_{it}}\right| + q_{auxt_{it+1}} + \left|q_{auxt_{it+1}}\right|}{4} \cdot \left(t_{it+1} - t_{it}\right) \right)$$

$$Qh = \begin{bmatrix} 674.808 \\ 705.793 \\ 664.479 \\ 633.493 \\ 612.836 \\ 602.508 \\ 612.836 \\ 654.15 \end{bmatrix} MJ \qquad q_{aux_0} \cdot 24 \cdot hr = \begin{bmatrix} 391.227 \\ 453.198 \\ 370.57 \\ 308.599 \\ 267.285 \\ 246.628 \\ 267.285 \\ 349.913 \end{bmatrix} MJ$$

Note that in this case, because there is no cooling, Qh may be obtained from the mean power.

A variable room temperature may be employed, e.g. with a night setback. In such a case, TR should be modeled as a Fourier series.
Note that combined radiative-convective coefficients were employed in the above model in order to permit a simple analytical solution. A more detailed model is described by Athienitis et al (1990).

The method is similar in concept to the CLTD and transfer functions methods of ASHRAE (1989). However, all building transfer functions (admittances) are directly calculated.

**References**

Athienitis, A. K., M. Stylianou and J. Shou. 1990. "A Methodology for Building Thermal Dynamics Studies and Control Applications," *ASHRAE Transactions*, Vol. 96, Pt. 2, pp. 839-48.

Athienitis, A. K., H. F. Sullivan and K. G. T. Hollands. 1986. "Analytical Model, Sensitivity Analysis, and Temperature Swings in Direct Gain Rooms," *Solar Energy*, Vol. 36, pp. 303-12.

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