CHAPTER 9 HEATING AND COOLING LOAD CALCULATIONS

9.3 Steady-Periodic Zone Model and Cooling Load Calculations

The model employed for cooling load calculations is the same as the model used for heating load calculations in **Section 9.2**. The main difference here is that solar radiation transmitted through windows or absorbed by outside surfaces is calculated in detail. Also, latent cooling load is calculated.

Each wall is modeled by a self-admittance and a transfer admittance as described in **Sections 4.2-4.3**. Analysis is performed for a design day with major objective of determining peak heating load.

1. Weather inputs:

a. Outside temperature.
b. Solar radiation transmitted by each window and solar radiation absorbed by each exterior surface.

2. Building data:

NS: number of surfaces contributing to the zone energy balance.

NSe: number of exterior surfaces (walls and roof).

Ai: area of exterior surface i.

Nw: number of windows $\qquad$ A_{wi}: area of window i

ψ_{se} se = 0,1. . . NSe exterior surface azimuth angle.

α_{Sse} se = 0,1. . . Nse solar absorptance of exterior surfaces.

3. Window type:
U value or thermal resistance, single or double glazing and kL value (extinction coefficient x thickness)

A_{door}: external door area $\qquad$ R_{door}: external door R-value

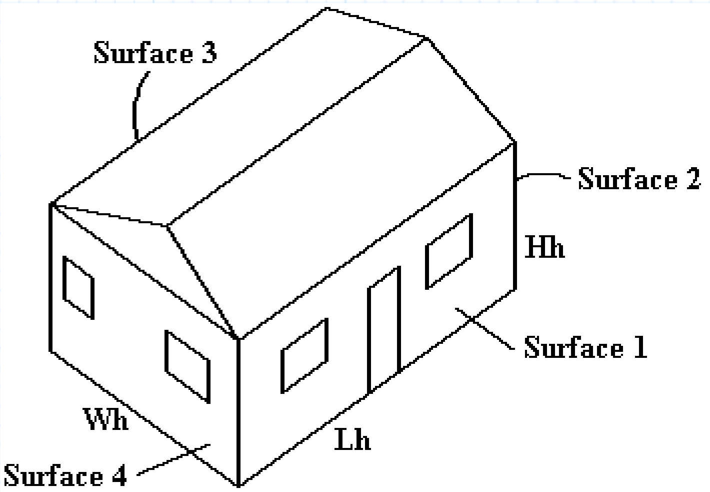
4. Wall construction:
Wall layer properties. Interior layer properties for transient analysis are also required.

ach: infiltration - air changes per hour
h_i: inside surface heat transfer coefficient for surface i
(combined radiative - convective coefficients are assumed)

5. Internal gains:

Qintr: radiative internal gains   Qintc: convective internal gains

Example: Consider a house which consists of a basement and a ground level floor with a pitched roof. The basement load may be determined with the techniques of **Section 3.2**. Here we consider the ground level zone. In cases with significantly different solar exposure and separate thermal control of rooms, a number of separate zones should be considered.



$Hh := 2.7\ m$ $Lh := 14\ m$ $Wh := 12\ m$

Surfaces contributing to energy balance:

$NS := 6$

$i := 1, 2..NS$ 1-4 walls, 5-ceiling, 6-floor

$se := 1..5$ exterior surfaces

$Nw := 4$ $iw := 1, 2..Nw$

(assume four windows — sum the window areas on each house side)

Window and door areas:

$Aw_1 := 12\ m^2$ $Aw_2 := 3\ m^2$

$Aw_3 := 2\ m^2$ $Aw_4 := 3\ m^2$

$Ad_1 := 2\ m^2$ $\quad$ $Ad_2 := 2\ m^2$

$Ad_3 := 2\ m^2$ $\quad$ $Ad_4 := 2\ m^2$

Wall net areas:

$A_1 := Lh \cdot Hh - Aw_1 - Ad_1$

$A_2 := Wh \cdot Hh - Aw_2 - Ad_2$

$A_3 := Lh \cdot Hh - Aw_3 - Ad_3$

$A_4 := Wh \cdot Hh - Aw_4 - Ad_4$

$A_5 := Wh \cdot Lh$ $\quad$ $A_6 := A_5$

$Hi := 2.4\ m$ internal height

$Vol := A_5 \cdot Hi$

$Rd := 1\ \dfrac{m^2\ \Delta°C}{W}$ door thermal resistance

$R_w := 0.32\ \dfrac{m^2\ \Delta°C}{W}$ window resistance (double-glazed)

see **Section 6.3.1**

Wall/window azimuth angles:

$\psi_1 := -45\ deg$ $\quad$ $\psi_2 := -135\ deg$

$\psi_3 := 135\ deg$ $\quad$ $\psi_4 := 45\ deg$

Wall absorptances:

$\alpha s_{iw} := 0.9$

$h_1 := 8.3\ \dfrac{W}{m^2\ \Delta°C}$ $\quad$ $h_2 := h_1$ $\quad$ film coefficients (combined)

$h_4 := h_1$

$h_3 := h_1$

$h_5 := 6.1 \frac{W}{m^2 \ \Delta°C}$

$h_6 := 9.3 \frac{W}{m^2 \ \Delta°C}$ (hot floor)

$ach := 0.5$ ach= air changes /hour

Calculation of infiltration conductance:

$c_{pair} := 1000 \frac{J}{kg \cdot \Delta°C}$ $\rho_{air} := 1.2 \frac{kg}{m^3}$ specific heat and density of air

$U_{inf} := \frac{ach \cdot Vol}{3600 \ s} \cdot \rho_{air} \cdot c_{pair}$ $U_{inf} = 67.2 \frac{W}{\Delta°C}$

Thermal Resistance of Walls (including air films)

Vertical Walls

1. gypsum board

$L_1 := 0.013 \ m$ thickness

$\rho_1 := 800 \frac{kg}{m^3}$ density

$k_1 := 0.16 \frac{W}{m \cdot \Delta°C}$ conductivity

$c_1 := 750 \frac{J}{kg \cdot \Delta°C}$ specific heat

2. insulation

$R_{ins} := 2.2 \frac{m^2 \cdot \Delta°C}{W}$

3. siding and sheathing

$$R_{sid} := 0.37 \frac{m^2 \cdot \Delta°C}{W}$$

4. exterior film

$$h_o := 22 \frac{W}{m^2\ \Delta°C}$$ (**Section 5.3**)

15% of area is framing $f_f := 0.15$ fraction of area which is framing

2-by-4 wood stud with R value: $R_f := 0.77 \frac{m^2 \cdot \Delta°C}{W}$

$$R_1 := \frac{1}{\dfrac{1-f_f}{\dfrac{L_1}{k_1}+R_{ins}+R_{sid}+\dfrac{1}{h_o}+\dfrac{1}{h_1}}+\dfrac{f_f}{\dfrac{L_1}{k_1}+R_f+R_{sid}+\dfrac{1}{h_o}+\dfrac{1}{h_1}}}$$

$$R_1 = 2.44 \frac{\Delta°C \cdot m^2}{W}$$

Calculation of wall conductance excluding interior layer and film (to be used for admittance calculations):

$$u_1 := \frac{1}{R_1 - \dfrac{L_1}{k_1} - \dfrac{1}{h_1}}$$

Assume that all exterior walls are of the same construction:

$ii := 1, 2 .. 4$ vertical walls

$L_{ii} := L_1$ $R_{ii} := R_1$ $u_{ii} := u_1$

$k_{ii} := k_1$ $\rho_{ii} := \rho_1$ $c_{ii} := c_1$

Calculation of Roof-Ceiling Thermal Resistance

(see **Section 1.6**)

Ceiling

1. gypsum board

$L_5 := L_1$ $\quad k_5 := k_1$

$c_5 := c_1$ $\quad \rho_5 := \rho_1$

2. insulation

$R_{insc} := 2.8 \, \frac{m^2 \, \Delta°C}{W}$

3. air-film (attic)

$h_a := 12 \, \frac{W}{m^2 \, \Delta°C}$

$$R_c := \frac{1}{\dfrac{1-f_f}{\dfrac{L_5}{k_5} + R_{insc} + \dfrac{1}{h_a} + \dfrac{1}{h_5}} + \dfrac{f_f}{\dfrac{L_5}{k_5} + R_f + \dfrac{1}{h_a} + \dfrac{1}{h_5}}}$$

$R_c = 2.45 \, \frac{m^2 \, \Delta°C}{W}$

Roof

1. exterior air film

$$h_o := 20\ \frac{W}{m^2\ \Delta°C}$$

2. shingle backer board

$$R_b := 0.19\ \frac{m^2 \cdot \Delta°C}{W}$$

3. wood shingles

$$R_{sh} := 0.17\ \frac{m^2\ \Delta°C}{W}$$

$$R_r := \frac{1}{\dfrac{1-f_f}{R_b+R_{sh}+\dfrac{1}{h_o}+\dfrac{1}{h_a}}+\dfrac{f_f}{R_f+R_b+R_{sh}+\dfrac{1}{h_o}+\dfrac{1}{h_a}}}$$

$$R_r = 0.543\ \frac{m^2\ \Delta°C}{W}$$

Assuming a 30 degree slope for the roof, we calculate the ceiling-roof combined resistance per unit ceiling area (assuming no ventilation in the attic — see **Section 1.6** for ventilated attic) as follows:

$$A_r := \frac{A_5}{\cos(30\ deg)} \qquad R_5 := \left(\frac{R_c}{A_5}+\frac{R_r}{A_r}\right)\cdot A_5$$

$$R_5 = 2.92\ \frac{m^2\ \Delta°C}{W} \qquad u_5 := \frac{1}{R_5-\dfrac{L_5}{k_5}-\dfrac{1}{h_5}}$$

for admittance calculation

Floor

1. Carpet and underpad

$L_6 := 0.02\ m$ $\qquad k_6 := 0.06\ \frac{W}{m \cdot \Delta°C}$

$\rho_6 := 800\ \frac{kg}{m^3}$

2. Insulation and plywood

$R_{ins} := 1.0\ \frac{m^2\ \Delta°C}{W}$ $\qquad c_6 := 1400 \cdot \frac{J}{kg \cdot \Delta°C}$

3. Air film (horizontal heat flow downward)

$h_o := 6.13\ \frac{W}{m^2\ \Delta°C}$

$$R_6 := R_{ins} + \frac{L_6}{k_6} + \frac{1}{h_o} + \frac{1}{h_6}$$

$R_6 = 1.604\ \frac{m^2\ \Delta°C}{W}$ $\qquad u_6 := \frac{1}{R_6 - \frac{L_6}{k_6} - \frac{1}{h_6}}$

Calculation of Wall Admittances

The self-admittance and the transfer admittance will be calculated for each wall, considering the thermal capacity of the room interior layer. Note that the steady-state value of the admittance is equal to the wall conductance. We will calculate admittances to the interior surface and to the room air point. The analysis will be performed for the mean term and three harmonics of the weather inputs.

Admittances

Steady state admittance is equal to wall U value (excluding interior film); first subscript indicates frequency, second subscript indicates surface number.

$$Ys_{0,i} := \frac{A_i}{R_i - \frac{1}{h_i}} \qquad Yt_{0,i} := Ys_{0,i}$$

$$Y_{0,i} := \frac{A_i}{R_i} \qquad Yta_{0,i} := Y_{0,i}$$

admittances from outside to room air (steady state)

$$n := 1, 2 .. 3 \qquad j := \sqrt{-1}$$

$$\gamma_{n,i} := \sqrt{j \cdot \frac{2 \cdot \pi \cdot n}{\frac{k_i}{\rho_i \cdot c_i} \cdot 86400\ s}}$$

$$Ui_i := A_i \cdot h_i$$

$$Uo_i := h_o \cdot A_i$$

interior and exterior surface conductances

$$iw := 1, 2 .. 4$$

$$Uw_{iw} := \frac{Aw_{iw}}{R_w} + \frac{Ad_{iw}}{Rd}$$

conductance of double-glazed windows and doors

$$Ys_{n,i} := A_i \cdot \frac{u_i + k_i \cdot \gamma_{n,i} \cdot \tanh\left(\gamma_{n,i} \cdot L_i\right)}{\left(\frac{u_i}{k_i \cdot \gamma_{n,i}} \cdot \tanh\left(\gamma_{n,i} \cdot L_i\right)\right) + 1}$$

$$Yt_{n,i} := \frac{A_i}{\frac{\cosh\left(\gamma_{n,i} \cdot L_i\right)}{u_i} + \frac{\sinh\left(\gamma_{n,i} \cdot L_i\right)}{k_i \cdot \gamma_{n,i}}}$$

$$Y_{n,i} := \frac{Ys_{n,i} \cdot Ui_i}{Ys_{n,i} + Ui_i}$$

$$Yta_{n,i} := Yt_{n,i} \cdot \frac{Ui_i}{Ys_{n,i} + Ui_i}$$

wall admittances from outside to inside air.

Zone Admittance Yz (from air point)

$$n := 0, 1..3 \qquad Yz_n := U_{inf} + \sum_{iw} Uw_{iw} + \sum_i Y_{n,i}$$

$$Yz = \begin{bmatrix} 346.045 \\ 405.779 + 312.28j \\ 547.992 + 556.606j \\ 713.747 + 725.151j \end{bmatrix} \frac{W}{\Delta°C}$$

Note that Yz_0 is simply the total U value of the house.

Energy Balance

The energy balance may then be written as (see **Section 9.2**):

$$q_{aux_n} = \left(TR_n - \frac{Qintc_n + \left(U_{inf} + \sum_{iw} Uw_{iw} \right) \cdot To_n + \sum_i \left(\left(Yt_{n,i} \cdot Teq_{n,i} + Qintr_{n,i} \right) \cdot \frac{Ui_i}{Ys_{n,i} + Ui_i} \right)}{Yz_n} \right) \cdot Yz_n$$

Outside Temperature

The outside temperature is modeled by a Fourier series based on NTo+1 values that are an input to the array below. If more detail is required, NTo may be increased.

$NTo := 7$ $\quad it := 0, 1 .. NTo$ $\quad$ time index

$t_{it} := it \cdot 3\ hr$ $\quad$ time

$n := 0, 1 .. 3$ $\quad$ harmonics

$w_n := 2 \cdot \pi \cdot \dfrac{n}{24\ hr}$ $\quad j := \sqrt{-1}$

$$To_{it} := \begin{bmatrix} 25 \\ 26 \\ 27 \\ 28 \\ 29 \\ 32 \\ 28 \\ 24 \end{bmatrix} \Delta°C \qquad Ton_n := \left(\sum_{it} \left(To_{it} \cdot \frac{\exp\left(-j \cdot w_n \cdot t_{it}\right)}{NTo + 1} \right) \right)$$

$$Ton = \begin{bmatrix} [8 \times 1] \\ [8 \times 1] \\ [8 \times 1] \\ [8 \times 1] \end{bmatrix} \Delta°C$$

$$Ton_0 = \begin{bmatrix} 25 \\ 26 \\ 27 \\ 28 \\ 29 \\ 32 \\ 28 \\ 24 \end{bmatrix} \Delta°C$$

mean daily temperature

Solar Radiation

Solar radiation transmitted through the windows will be modeled as absorbed 70% at the floor surface and the remainder by the other surfaces in proportion to their areas. First, we use the technique of **Section 7.3** to determine solar radiation absorbed by exterior surfaces and the quantities transmitted through windows.

Location data:

$L := 35\ deg$ $\quad$ latitude

$\beta := 90\ deg$ $\quad$ tilt angle

$n_d := 185$ $\quad$ day number

$\rho_g := 0.2$ $\quad$ ground reflectance

First perform solar geometry calculations:

Declination angle:

$$\delta := 23.45\ deg \cdot \sin\left(360 \cdot \frac{284 + n_d}{365}\ deg\right) = 22.887\ deg$$

Sunset time:

$$t_s := (\mathrm{acos}(-\tan(L) \cdot \tan(\delta)))\ \frac{hr}{15\ deg} = 7.146\ hr$$

Time array:

$$it := 0, 1 .. 23$$

$t_{it} := (it - 11.99)\ hr$ solar time for solar radiation calculations

$ha_{it} := 15\ \frac{deg}{hr} \cdot t_{it}$ hour angle (0 at solar noon)

$ha_s := t_s \cdot 15\ \frac{deg}{hr}$ sunset hour angle

Solar altitude:

$$\alpha_{it} := \mathrm{asin}\left(\cos(L) \cdot \cos(\delta) \cdot \cos\left(ha_{it}\right) + \sin(L) \cdot \sin(\delta)\right) \cdot \left(\left|t_{it}\right| < \left|t_s\right|\right)$$

Solar azimuth:

$$\phi_{it} := \mathrm{acos}\left(\frac{\sin\left(\alpha_{it}\right) \cdot \sin(L) - \sin(\delta)}{\cos\left(\alpha_{it}\right) \cdot \cos(L)}\right) \cdot \frac{ha_{it}}{\left|ha_{it}\right|}$$

Angle of incidence:

$$cos\theta_{it,iw} := \cos\left(\alpha_{it}\right) \cdot \cos\left(\left|\phi_{it} - \psi_{iw}\right|\right) \cdot \sin(\beta) + \sin\left(\alpha_{it}\right) \cdot \cos(\beta)$$

$$\theta_{it,iw} := \operatorname{acos}\left(\frac{cos\theta_{it,iw} + \left|cos\theta_{it,iw}\right|}{2}\right)$$

Calculate transmittance of atmosphere and glazing:

Beam atmospheric transmittance calculations:

$Al := 0.5$ altitude (km)

$$a_o := 1.03 \cdot \left(0.4237 - 0.00821 \cdot (6 - Al)^2\right)$$

$$a_1 := 1.01 \cdot \left(0.5055 + (0.00595 \cdot (6.5 - Al))^2\right)$$

$$k := 1.0 \cdot \left(0.2711 + (0.01858 \cdot (2.5 - Al))^2\right)$$

$$\tau_{b_{it}} := \text{if}\left(\left(\left|t_{it}\right| < \left|t_s\right|\right), a_o + a_1 \cdot \exp\left(\frac{-k}{\sin\left(\alpha_{it}\right)}\right), 0\right)$$

Determine now the glazing properties as a function of time interval j:

Glass properties:

$kL := 0.1$ extinction coeff. × glazing thickness

$n_g := 1.53$ refractive index

Angle of refraction and component reflectivity:

$$\theta'_{it,iw} := \operatorname{asin}\left(\frac{\sin\left(\theta_{it,iw}\right)}{n_g}\right)$$

$$r_{it,iw} := \frac{1}{2} \cdot \left(\left(\frac{\sin\left(\theta_{it,iw} - \theta'_{it,iw}\right)}{\sin\left(\theta_{it,iw} + \theta'_{it,iw}\right)}\right)^2 + \left(\frac{\tan\left(\theta_{it,iw} - \theta'_{it,iw}\right)}{\tan\left(\theta_{it,iw} + \theta'_{it,iw}\right)}\right)^2\right)$$

Beam transmittance, τ, reflectance, ρo, and absorptance, α, of glazing:

$$a_{it,iw} := \exp\left(-\frac{kL}{\sqrt{1-\left(\frac{\sin\left(\theta_{it,iw}\right)}{n_g}\right)^2}}\right)$$

$$\tau_{it,iw} := \frac{\left(1-r_{it,iw}\right)^2 \cdot a_{it,iw}}{1-\left(r_{it,iw}\right)^2 \cdot \left(a_{it,iw}\right)^2}$$

$$\rho o_{it,iw} := r_{it,iw} + \frac{r_{it,iw} \cdot \left(1-r_{it,iw}\right)^2 \cdot \left(a_{it,iw}\right)^2}{1-\left(r_{it,iw}\right)^2 \cdot \left(a_{it,iw}\right)^2}$$

$$\alpha_{s_{it,iw}} := 1 - \rho o_{it,iw} - \tau_{it,iw}$$

For double glazed windows:

$$\tau_{e_{it,iw}} := \frac{\left(\tau_{it,iw}\right)^2}{1-\left(\rho o_{it,iw}\right)^2}$$

$$\alpha_{i_{it,iw}} := \alpha_{s_{it,iw}} \cdot \frac{\tau_{it,iw}}{1-\left(\rho o_{it,iw}\right)^2}$$

$$\alpha_{o_{it,iw}} := \alpha_{s_{it,iw}} + \alpha_{s_{it,iw}} \cdot \frac{\tau_{it,iw} \cdot \rho o_{it,iw}}{1-\left(\rho o_{it,iw}\right)^2}$$

Determine the solar radiation incident on exterior walls and transmitted by windows:

Extraterrestrial normal solar radiation:

$$I_{on} := 1353 \, \frac{W}{m^2} \cdot \left(1 + 0.033 \cdot \cos\left(360 \cdot \frac{n_d}{365} \, deg\right)\right)$$

Determine beam solar radiation:

$$I_{b_{it,iw}} := \left(I_{on} \cdot \tau_{b_{it}} \cdot \cos\left(\theta_{it,iw}\right)\right)$$ incident beam radiation

$$G_{b_{it,iw}} := I_{b_{it,iw}} \cdot \tau_{e_{it,iw}}$$ transmitted beam radiation

$$\tau_{ed_{iw}} := \tau_{e_{10,1}}$$ approximate value for diffuse transmittance (equal for all windows)

$$\theta_{10,1} = 64.973 \, deg$$

$$I_{ds_{it}} := I_{on} \cdot \sin\left(\alpha_{it}\right) \cdot \left(0.2710 - 0.2939 \cdot \tau_{b_{it}}\right) \cdot \frac{1 + \cos(\beta)}{2}$$ incident instantaneous sky diffuse radiation

$$I_{dg_{it}} := \left(I_{on} \cdot \sin\left(\alpha_{it}\right) \cdot \left(0.2710 - 0.2939 \cdot \tau_{b_{it}} + \tau_{b_{it}}\right)\right) \cdot \rho_g \cdot \frac{1 - \cos(\beta)}{2}$$ ground reflected

$$G_{d_{it,iw}} := \tau_{ed_{iw}} \cdot \left(I_{ds_{it}} + I_{dg_{it}}\right)$$ transmitted diffuse irradiation (instantaneous)

$$G_{b_{it,iw}} := I_{b_{it,iw}} \cdot \tau_{e_{it,iw}}$$ beam transmitted solar radiation; this equation may be modified to include the effect of an overhang based on the equations of **Section 7.4**

Total instantaneous solar irradiation incident on exterior surfaces:

$$I_{it,iw} := I_{b_{it,iw}} + I_{ds_{it}} + I_{dg_{it}}$$ on vertical walls

$$I_{h_{it}} := I_{on} \cdot \sin\left(\alpha_{it}\right) \cdot \left(0.2710 - 0.2939 \cdot \tau_{b_{it}} + \tau_{b_{it}}\right)$$ on roof

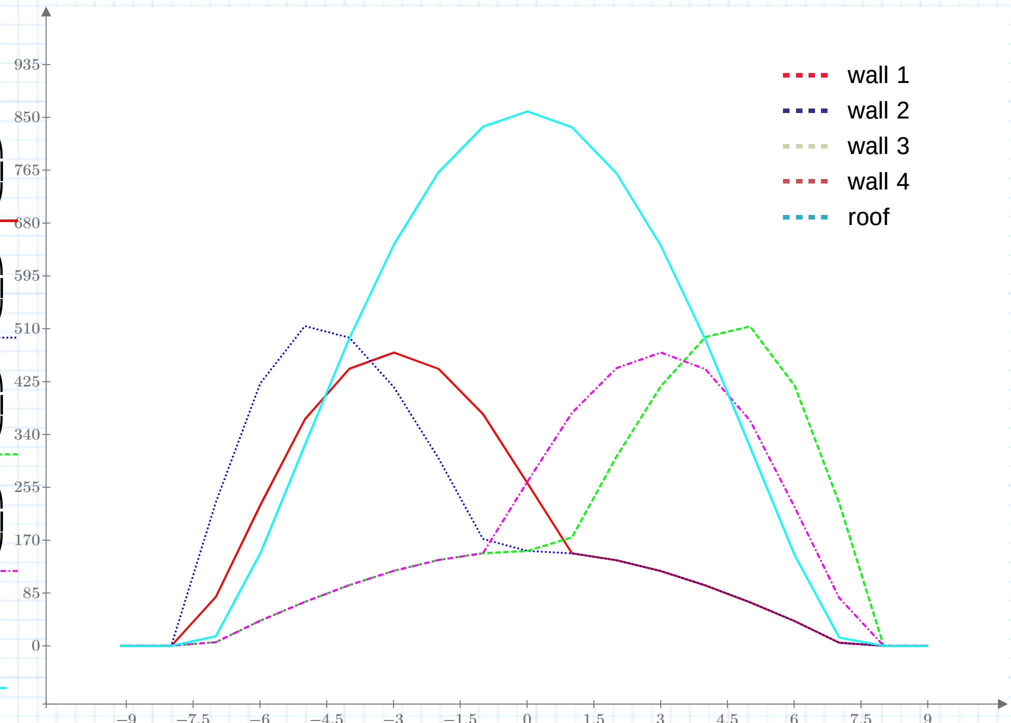
Total instantaneous solar radiation transmitted by windows:

$$G_{it,iw} := G_{b_{it,iw}} + G_{d_{it,iw}}$$

$$G_{ao_{it,iw}} := \alpha_{o_{it,iw}} \cdot I_{b_{it,iw}} + \alpha_{o_{10,iw}} \cdot \left(I_{ds_{it}} + I_{dg_{it}}\right)$$ radiation absorbed in outer glazings

$$G_{ai_{it,iw}} := \alpha_{i_{it,iw}} \cdot I_{b_{it,iw}} + \alpha_{i_{10,iw}} \cdot \left(I_{ds_{it}} + I_{dg_{it}}\right)$$ radiation absorbed in inner glazings

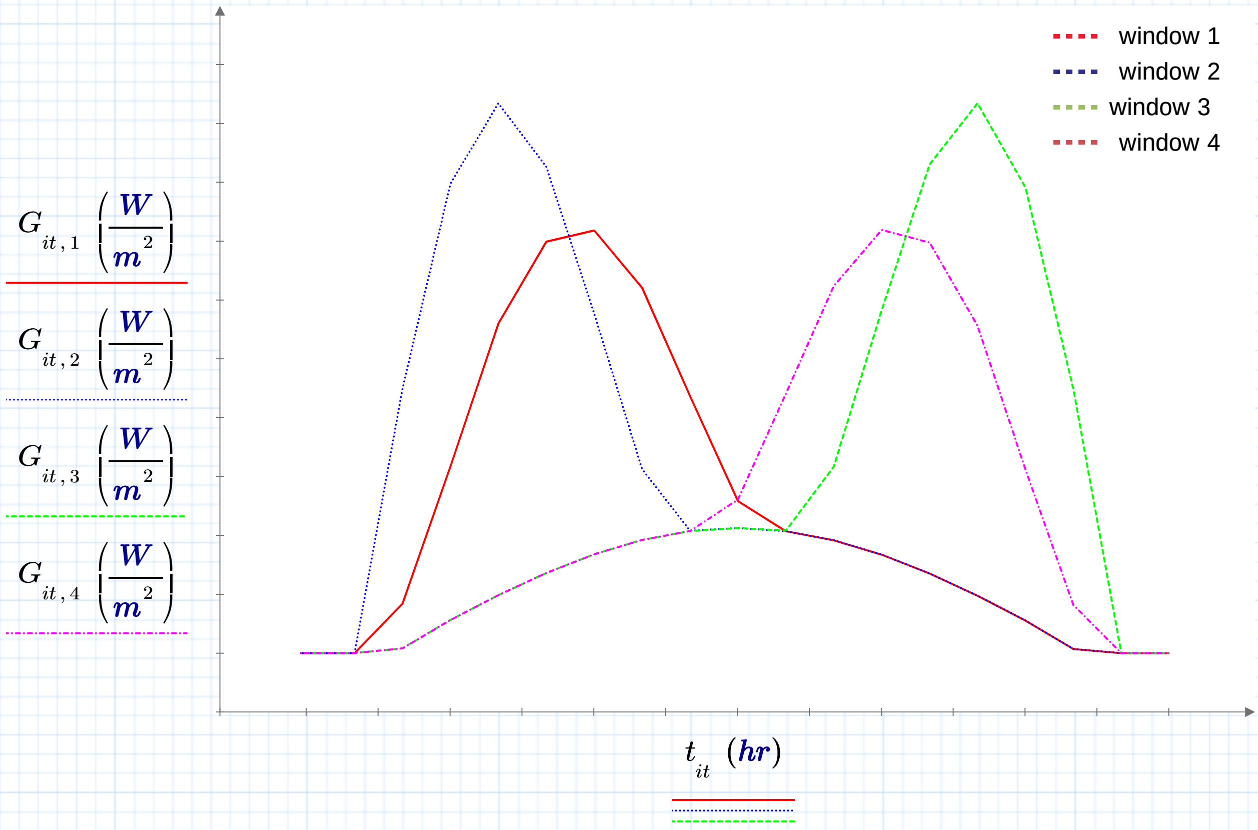
Incident solar irradiation flux (watts/m²):



$I_{it,1} \left(\frac{W}{m^2}\right)$, $I_{it,2} \left(\frac{W}{m^2}\right)$, $I_{it,3} \left(\frac{W}{m^2}\right)$, $I_{it,4} \left(\frac{W}{m^2}\right)$, $I_{h_{it}} \left(\frac{W}{m^2}\right)$

$t_{it}\ (hr)$

Transmitted solar radiation flux:



Before performing Fourier series calculations, change the time origin from solar noon back to midnight:

$$t_{it} := t_{it} + 11.99\ hr$$

Solar component of sol-air temperature for exterior surfaces:

$$Ts_{it,iw} := I_{it,iw} \cdot \frac{\alpha s_{iw}}{h_{iw}}$$ vertical surfaces

$$Ts_{it,5} := I_{h_{it}} \cdot \frac{\alpha s_4}{h_4}$$ roof

Represent with Fourier series:

$$n := 0, 1..3 \qquad Tsn_{n,se} := \left(\sum_{it} \left(Ts_{it,se} \cdot \frac{\exp\left(-j \cdot w_n \cdot t_{it}\right)}{24} \right) \right)$$

Equivalent or sol-air temperatures:

$$Teq_{n,se} := Ton_n + Tsn_{n,se}$$

Total instantaneous solar radiation transmitted through all windows:

$$Gt_{it} := \sum_{iw} \left(G_{it,iw} \cdot Aw_{iw} \right)$$

Solar radiation absorbed by room surfaces:

$$S_{it,i} := 0.3 \cdot Gt_{it} \cdot \frac{A_i}{\sum_{iw} A_i}$$

Floor:

$$S_{it,6} := 0.7 \cdot Gt_{it}$$

Represent with Fourier series:

$$Sn_{n,i} := \left(\sum_{it} \left(S_{it,i} \cdot \frac{\exp\left(-j \cdot w_n \cdot t_{it}\right)}{24} \right) \right)$$

Solar radiation absorbed in glazings and released to room air:

$$Qg_{it} := \sum_{iw} \left(\left(\left(\frac{1}{R_w \cdot h_o} \cdot \left({\alpha_o}_{it,iw} \cdot I_{it,iw} \right) + \left(\frac{1}{R_w} \cdot \left(R_w - \frac{1}{h_6} \right) \cdot \left({\alpha_i}_{it,iw} \cdot I_{it,iw} \right) \right) \right) \right) \cdot Aw_{iw} \right)$$

$$Qgn_n := \left(\sum_{it} \left(Qg_{it} \cdot \frac{\exp\left(-j \cdot w_n \cdot t_{it}\right)}{24} \right) \right)$$

$$Qgn_0 = 257.483 \; W$$

We may similarly model internal gains with Fourier series. In this example, we will not consider internal gains.

Room-air temperature TR (assume specified):

$T_{R_0} := 23\ \Delta°C$ $\quad n1 := 1,2..3$ $\quad T_{R_{n1}} := 0\ \Delta°C$

A variable room temperature may be employed, e.g. with a night setback/setup. In such a case, TR should be modeled by a Fourier series like To.

For basement:

$T_b := 23\ \Delta°C$ $\quad Teq_{0,6} := T_b$ $\quad Teq_{n1,6} := 0\ \Delta°C$

Sensible Cooling Load

First determine the mean auxiliary load:

$$q_{aux_0} := \left(T_{R_0} - \frac{\left(U_{inf} + \sum_{iw} Uw_{iw}\right) \cdot Ton_0 + Qgn_0 + \sum_i \left(\left(Yt_{0,i} \cdot Teq_{0,i} + Sn_{0,i}\right) \cdot \frac{Ui_i}{Ys_{0,i} + Ui_i} \right)}{Yz_0} \right) \cdot Yz_0$$

$$q_{aux_0} = \begin{bmatrix} -4.711 \cdot 10^3 \\ -4.953 \cdot 10^3 \\ -5.194 \cdot 10^3 \\ -5.435 \cdot 10^3 \\ -5.677 \cdot 10^3 \\ -6.401 \cdot 10^3 \\ -5.435 \cdot 10^3 \\ -4.47 \cdot 10^3 \end{bmatrix} W$$

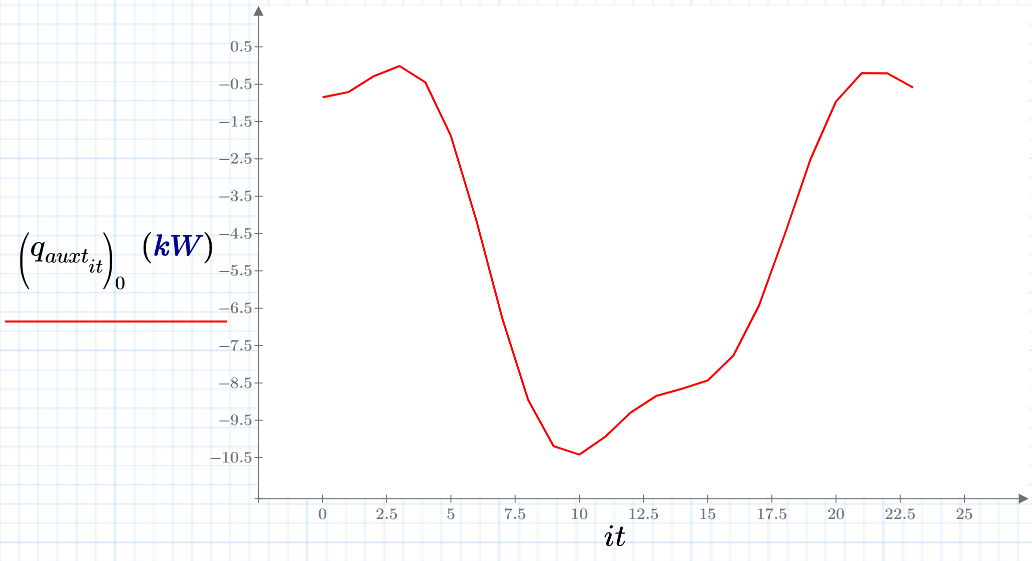
mean auxiliary load

$$q_{aux_{n1}} := \left(T_{R_{n1}} - \frac{\left(U_{inf} + \sum_{iw} Uw_{iw}\right) \cdot Ton_{n1} + Qgn_{n1} + \sum_i \left(\left(Yt_{n1,i} \cdot Teq_{n1,i} + Sn_{n1,i}\right) \cdot \frac{Ui_i}{Ys_{n1,i} + Ui_i} \right)}{Yz_{n1}} \right) \cdot Yz_{n1}$$

$$q_{auxt_{it}} := q_{aux_0} + 2 \cdot \sum_{n1} \mathrm{Re}\left(q_{aux_{n1}} \cdot \exp\left(j \cdot w_{n1} \cdot t_{it} \right) \right)$$

Peak Cooling Load

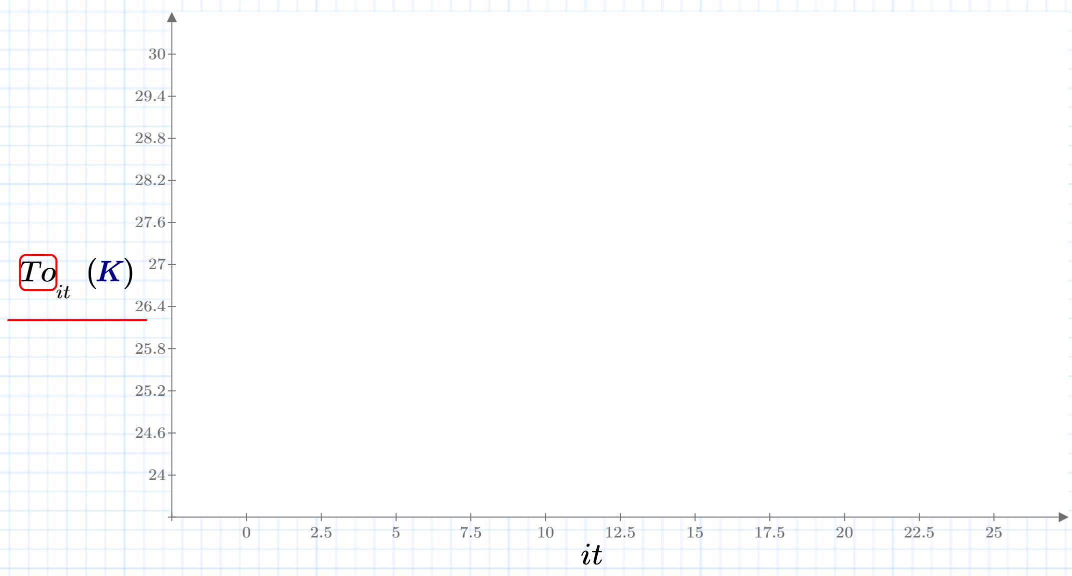
(Used to size cooling system)



$\min(q_{auxt}) = ?$

$$To_{it} := Ton_0 + 2 \cdot \sum_{n1} \mathrm{Re}\left(\left(Ton_{n1}\right) \cdot \exp\left(j \cdot w_{n1} \cdot t_{it}\right)\right)$$

Ambient temperature:



Note that the peak cooling load in this case is due to gains from the large southeast facing window and the outside temperature.

Cooling Energy Consumption

(Based only on load) Qc is obtained by numerical integration of the negative part of the auxiliary power curve:

$q_{auxt_{24}} := q_{auxt_0}$ $\quad t_{24} := 24\ hr$ $\quad MJ := 10^6 \cdot joule$

$$Qc := \sum_{it} \left(\frac{\left(-q_{auxt}\right)_{it} + \left|\left(-q_{auxt}\right)_{it}\right| + \left(-q_{auxt}\right)_{it+1} + \left|\left(-q_{auxt}\right)_{it+1}\right|}{4} \cdot \left(t_{it+1} - t_{it}\right) \right)$$

$$Qc = \begin{bmatrix} 858.1 \\ 868.5 \\ 879 \\ 889.4 \\ 899.8 \\ 931.1 \\ 889.4 \\ 847.7 \end{bmatrix} MJ \qquad q_{aux_0} \cdot 24 \cdot hr = \begin{bmatrix} -407.1 \\ -427.9 \\ -448.8 \\ -469.6 \\ -490.5 \\ -553 \\ -469.6 \\ -386.2 \end{bmatrix} MJ$$

Note that in this case, because there is no heating, Qc may be obtained from the mean power.

Combined radiative-convective coefficients were employed in the above model in order to obtain a simple analytical solution. A more detailed model is described by Athienitis et al (1990).

The method is similar in concept to the CLTD method of ASHRAE (1989), but all factors and wall dynamic parameters are directly evaluated.

Latent Cooling Load

The latent cooling load is due to exchange of inside air and outside air with different moisture contents and due to generation of moisture by occupants, equipment etc.

If moisture must be removed from the outdoor air to reduce the humidity level to the comfort range, the energy required (assume indoor relative humidity 50% and outdoor humidity 80%) is determined as follows:

$h_{fg} := 2465\ \frac{J \cdot 1000}{kg}$ latent heat

$\rho_{air} := 1.2\ \frac{kg}{m^3}$ density

Determine the humidity (kg_vapor / kg_dry air) from the psychrometric chart or the equations given in **Section 8.2.**

$W_i := 0.0073$

$W_o := 0.0117$

interior and exterior humidity ratios (from **Section 8.2**)

$$q_{lat} := \frac{ach}{3600 \cdot s} \cdot Vol \cdot \rho_{air} \cdot \left(W_i - W_o\right) \cdot h_{fg}$$

$$q_{lat} = -728.851\ W$$

Note that there is also a small, usually negligible additional load (sensible) due to the difference in temperature between the room air moisture and the outside air moisture.

Other heat gains that may be included: An internal heat gain of approximately 67 watts per person (for residences) may also be included, as well as a heat gain of 470 watts from appliances. These heat gains may be assumed to be approximately 50% convective and 50% radiative (absorbed by each wall proportionally to its area).

References

ASHRAE. 1989. *ASHRAE Handbook of Fundamentals*. Atlanta, GA.

Athienitis, A. K., M. Stylianou and J. Shou. 1990. "A Methodology for Building Thermal Dynamics Studies and Control Applications," *ASHRAE Transactions*, Vol. 96, Pt. 2, pp. 839-48.

Athienitis, A. K., H. F. Sullivan and K. G. T. Hollands. 1986. "Analytical Model, Sensitivity Analysis, and Temperature Swings in Direct Gain Rooms," *Solar Energy*,Vol. 36, pp. 303-12.