CHAPTER 1: Analysis of Beams

1.1 Simple Span Beams

Description

This application computes the reactions and the maximum bending moment for simple span beams loaded with any practical number of uniformly distributed and concentrated loads. This application is limited to beams with all applied loads in the same direction. The user must divide the beam into segments with each segment supporting a single uniformly distributed load over its length and/or a concentrated load at the right end of each segment.

The user must enter the length of each segment, the uniformly distributed loads on each segment, and the concentrated loads at the right end of each segment.

Four sample problems are shown below to demonstrate use of the application. Each of the problems demonstrates a different loading condition and alternate ways of entering input. The first problem shows the most flexible method of entering input which may be used for any combination of uniform and concentrated loads.

Sketches and text labels are included in this document to provide additional information about the application to the user. A user can save the application to another file and use that file as the customized working document. Note that any practical number of beams may be entered for analysis. To do this the user would simply add to the **Input** and **Summary** sections.

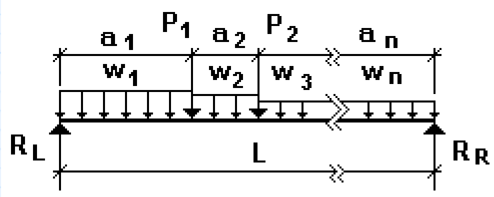
A summary of input and calculated values is shown on page 10.

The user should be familiar with subscript notation, entering numbers as vectors, and using the **transpose** and **vectorize** operators on the pallete.

Input

Notation

n is the last segment



Input Variables

a   segment lengths

w   uniformly distributed loads on segments

P   concentrated loads at right end of segments

Computed Variables

The following variables are calculated in this document:

L   span length

R_L beam reaction at left end

R_R beam reaction at right end

X_L distance from the left reaction to the point of maximum moment

M_{max} maximum bending moment

Defined Units

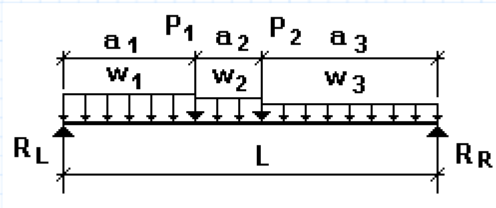
$$plf := \frac{lbf}{ft}$$

ORIGIN set equal to 1 to agree with customary usage. **ORIGIN** is a PTC Mathcad variable, the index number of the first element of a vector or matrix.

$$\text{ORIGIN} := 1$$

Sample 1

This sample problem shows a series of uniform and concentrated loads with differing load magnitudes and segment lengths, with the values of **a**, **w** and **P** entered as transposed vectors. This is the most flexible format since any beam with uniformly distributed and concentrated loads may be entered using this format.



Enter beam number, segment lengths, uniformly distributed loads, and concentrated loads starting from the left reaction:

Beam number: $b := 1$

Segment lengths: $a := [3.5 \;\; 10 \;\; 13.5]^{T} \; ft$

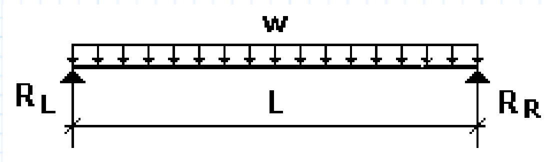
Span length: $L_b := \sum a^{\langle b \rangle}$ $\qquad L_b = 27 \; ft$

Uniformly distributed loads: $w := [2.6 \;\; 1.8 \;\; 0.5]^{T} \; \frac{kip}{ft}$

Concentrated loads at the right end of segments: $P := [7.8 \;\; 10.3]^{T} \; kip$

Sample 2

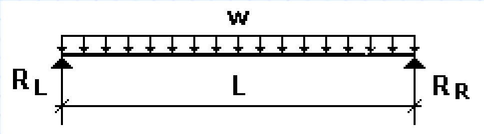
This sample problem shows a series of four equal concentrated loads at uniform spacing with a single uniformly distributed load over the length of the beam.



Beam number:	$b := 2$
Number of segments:	$n := 5$
Range variable i for segments:	$i := 1 .. n$
Segment lengths:	$a_{i,b} := 6\ ft$
Span length:	$L_b := \sum a^{\langle b \rangle}$ $L_b = 30\ ft$
Uniformly distributed loads:	$w_{i,b} := 60.0\ plf$
Range variable i1, for concentrated loads:	$i1 := 1 .. n - 1$
Concentrated loads:	$P_{i1,b} := 7.5\ kip$

Sample 3

This sample problem shows a single uniform load over the length of the beam.



Beam number: $b := 3$

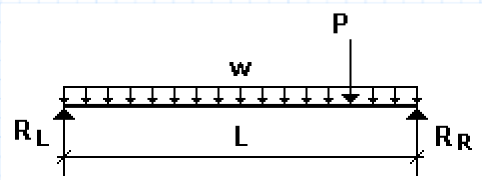
Segment lengths: $a_{1,b} := 27\ ft$

Span length: $L_b := \sum a^{\langle b \rangle}$ $L_b = 27\ ft$

Uniformly distributed loads: $w_{1,b} := 1.45\ \dfrac{kip}{ft}$

Sample 4

This sample problem shows a single uniform load over the length of the beam with a single concentrated load not at midspan. Since the load is not symmetrical, the "template" for Sample 1 is used with the single concentrated load entered as a subscripted variable.



Beam number: $b := 4$

Segment lengths: $a^{\langle b \rangle} := [15\ \ 5]^{T}\ ft$

Span length: $L_b := \sum a^{\langle b \rangle}$ $L_b = 20\ ft$

Uniformly distributed loads: $w^{\langle b \rangle} := [250\ \ 250]^{T}\ plf$

Concentrated loads at right end of segments: $P_{1,b} := 52.5\ kip$

Calculations

Beam reactions, location of the point of zero shear, and maximum bending moment are computed within this section.

Maximum number of segments entered: $n := \text{rows}(a)$ $\quad n = 5$

Number of beams entered: $b := \text{cols}(a)$ $\quad b = 4$

The following expressions adjusts the sizes of vectors **P** and **w** to the same size as vector **a**:

$$P_{n,b} := \textbf{if}\left(\text{rows}(P) < \text{rows}(a), 0\ \boldsymbol{kip}, P_{n,b}\right)$$

$$w_{n,b} := \textbf{if}\left(\text{rows}(w) < \text{rows}(a), 0\ \frac{\boldsymbol{lbf}}{\boldsymbol{ft}}, w_{n,b}\right)$$

Range variable i from 1 to n; range variable i1 from 2 to n; and range variable j from 1 to b:

$i := 1..n$ $\qquad i1 := 2..n$ $\qquad j := 1..b$

Sum of the loads to the left side of each segment:

$$VL_{1,j} := 0\ \boldsymbol{kip} \qquad VL_{i1,j} := VL_{i1-1,j} + w_{i1-1,j} \cdot a_{i1-1,j} + P_{i1-1,j}$$

$$VL^{\mathrm{T}} = \begin{bmatrix} 0 & 16.9 & 45.2 & 51.95 & 51.95 \\ 0 & 7.86 & 15.72 & 23.58 & 31.44 \\ 0 & 39.15 & 39.15 & 39.15 & 39.15 \\ 0 & 56.25 & 57.5 & 57.5 & 57.5 \end{bmatrix} kip$$

Sum of the loads to the right side of each segment:

$$VR_{1,j} := w_{1,j} \cdot a_{1,j} \qquad VR_{i1,j} := VR_{i1-1,j} + P_{i1-1,j} + w_{i1,j} \cdot a_{i1,j}$$

$$VR^{\mathrm{T}} = \begin{bmatrix} 9.1 & 34.9 & 51.95 & 51.95 & 51.95 \\ 0.36 & 8.22 & 16.08 & 23.94 & 31.8 \\ 39.15 & 39.15 & 39.15 & 39.15 & 39.15 \\ 3.75 & 57.5 & 57.5 & 57.5 & 57.5 \end{bmatrix} kip$$

Sum of the moments due to loads, at the right end of each segment:

$$MR_{1,j} := \frac{w_{1,j} \cdot \left(a_{1,j}\right)^2}{2} \qquad MR_{i1,j} := MR_{i1-1,j} + \frac{VL_{i1,j} + VR_{i1,j}}{2} \cdot a_{i1,j}$$

$$MR^{\mathrm{T}} = \begin{bmatrix} 15.9 & \\ & \ddots \end{bmatrix} kip \cdot ft$$

Left end reactions:

$$R_{L_j} := \frac{MR_{n,j}}{\sum a^{\langle j \rangle}}$$

$$R_L^{\mathrm{T}} = \begin{bmatrix} 34.47 & 15.9 & 19.575 & 15.625 \end{bmatrix} kip$$

Right end reactions:

$$R_{R_j} := \sum_i \left(w_{i,j} \cdot a_{i,j} + P_{i,j}\right) - R_{L_j}$$

$$R_R^{\mathrm{T}} = \begin{bmatrix} 17.48 & 15.9 & 19.575 & 41.875 \end{bmatrix} kip$$

Shear at the left end of each segment:

$$V_{L_{i,j}} := R_{L_j} - VL_{i,j}$$

$$V_L^{\mathrm{T}} = \begin{bmatrix} 34.47 & 17.57 & -10.73 & -17.48 & -17.48 \\ 15.9 & 8.04 & 0.18 & -7.68 & -15.54 \\ 19.575 & -19.575 & -19.575 & -19.575 & -19.575 \\ 15.625 & -40.625 & -41.875 & -41.875 & -41.875 \end{bmatrix} \boldsymbol{kip}$$

Shear at the right end of each segment:

$$V_{R_{i,j}} := R_{L_j} - VR_{i,j}$$

$$V_R^{\mathrm{T}} = \begin{bmatrix} 25.37 & \\ & \ddots \end{bmatrix} \textbf{\textit{kip}}$$

Moments at the right end of each segment:

$$M_{1,j} := \frac{1}{2} \cdot \left(V_{L_{1,j}} + V_{R_{1,j}}\right) \cdot a_{1,j}$$

$$M_{i1,j} := M_{i1-1,j} + \frac{1}{2} \cdot \left(V_{L_{i1,j}} + V_{R_{i1,j}}\right) \cdot a_{i1,j}$$

$$M^{\mathrm{T}} = \begin{bmatrix} 104.72 & \\ & \ddots \end{bmatrix} \textbf{\textit{kip}} \cdot \textbf{\textit{ft}}$$

Matrix U with elements equal to 1 if the corresponding element in VL is greater than or equal to 0 kip or with elements equal to 0 if the corresponding element is less than 0 kip:

$$U_{i,j} := \mathbf{if}\left(V_{L_{i,j}} > 0 \cdot \textbf{\textit{kip}}, 1, 0\right)$$

$$U^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Index of the segment where shear passes through zero:

$$u_j := \left(\sum U^{\langle j \rangle} \right) \qquad u^T = [2\ \ 3\ \ 1\ \ 1]$$

Distance from the left end of the segment where shear passes through 0 to the point where shear passes through 0:

$$a'_j := \mathbf{if}\left(w_{u_j,j} = 0 \cdot \frac{kip}{ft}, a_{u_j,j}, \mathbf{if}\left(\frac{V_{L_{u_j,j}}}{w_{u_j,j}} > a_{u_j,j}, a_{u_j,j}, \frac{V_{L_{u_j,j}}}{w_{u_j,j}} \right) \right)$$

$$a'^T = [9.761\ \ 3\ \ 13.5\ \ 15]\ ft$$

Distance from the left reaction to the left end of each segment:

$$S_{L_{1,j}} := 0\ ft \qquad S_{L_{i1,j}} := S_{L_{i1-1,j}} + a_{i1-1,j}$$

$$S_L^T = \begin{bmatrix} 0 & 3.5 & 13.5 & 27 & 27 \\ 0 & 6 & 12 & 18 & 24 \\ 0 & 27 & 27 & 27 & 27 \\ 0 & 15 & 20 & 20 & 20 \end{bmatrix} ft$$

Distance from the left end reaction to the point of zero shear and maximum moment:

$$X_{L_j} := S_{L_{u_j,j}} + a'_j$$

$$X_L^T = [13.261\ \ 15\ \ 13.5\ \ 15]\ ft$$

Maximum bending moment:

$$M_{max_j} := \mathbf{if}\left(\left(u_j > 1 \right), \left(M_{u_j-1,j} \right), 0 \right) + \left(V_{L_{u_j,j}} \cdot a'_j - \frac{1}{2} \cdot w_{u_j,j} \cdot \left(a'_j \right)^2 \right)$$

$$M_{max}^T = [190.47\ \ 141.75\ \ 132.131\ \ 206.25]\ kip \cdot ft$$

Summary

Input Variables

The index number of each row in the transformed matrices a, w and P corresponds to the beam number. For example, row 0 represents beam number 0, row 1 represents beam number 1, and so forth.

Segment lengths: $$a^{\mathrm{T}} = \begin{bmatrix} 3.5 & 10 & 13.5 & 0 & 0 \\ 6 & 6 & 6 & 6 & 6 \\ 27 & 0 & 0 & 0 & 0 \\ 15 & 5 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{ft}$$

Uniformly distributed loads: $$w^{\mathrm{T}} = \begin{bmatrix} 2.6 & 1.8 & 0.5 & 0 & 0 \\ 0.06 & 0.06 & 0.06 & 0.06 & 0.06 \\ 1.45 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0 & 0 & 0 \end{bmatrix} \frac{\boldsymbol{kip}}{\boldsymbol{ft}}$$

Concentrated loads at right end of segments: $$P^{\mathrm{T}} = \begin{bmatrix} 7.8 & 10.3 & 0 & 0 & 0 \\ 7.5 & 7.5 & 7.5 & 7.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 52.5 & 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{kip}$$

Computed Variables

Beam numbers: $$j = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Span lengths: $$L_j = \begin{bmatrix} 27 \\ 30 \\ 27 \\ 20 \end{bmatrix} \boldsymbol{ft}$$

Left Reactions: $$R_{L_j} = \begin{bmatrix} 34.47 \\ 15.9 \\ 19.575 \\ 15.625 \end{bmatrix} \boldsymbol{kip}$$

Right Reactions: $$R_{R_j} = \begin{bmatrix} 17.48 \\ 15.9 \\ 19.575 \\ 41.875 \end{bmatrix} \boldsymbol{kip}$$

Distances from the left reaction to the point of zero shear: $$X_{L_j} = \begin{bmatrix} 13.261 \\ 15 \\ 13.5 \\ 15 \end{bmatrix} \boldsymbol{ft}$$

Maximum moment: $$M_{max_j} = \begin{bmatrix} 190.47 \\ 141.75 \\ 132.131 \\ 206.25 \end{bmatrix} \boldsymbol{kip} \cdot \boldsymbol{ft}$$