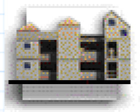


CHAPTER 1: Analysis of Beams

1.3 Single Span Beams - Shear and Moment

Description

This application computes the reactions and the maximum bending moment, and plots the shear and bending moment for a single span beam, with or without end moments, loaded with any practical number of uniformly distributed and concentrated loads. The user must divide the beam into segments with each segment supporting a single uniformly distributed load over its length and/or a concentrated load at the right end of each segment.

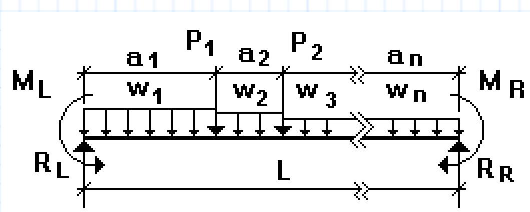
The user must enter the length of each segment, the uniformly distributed loads on each segment, the concentrated loads at the right end of each segment, and the left and right end moments.

A summary of input and calculated values is shown on pages 8.

The user should be familiar with subscript notation, entering numbers as vectors, and using the **transpose** and **vectorize** operators on the palette.

Notation

n is the last segment



Input Variables

a segment lengths

w uniformly distributed loads on segments

P concentrated loads at right end of segments

M_L left end moment

M_R right end moment

Computed Variables

The following variables are calculated in this document:

L span length

R_L beam reaction at left end

R_R beam reaction at right end

X_L distance from the left reaction to the point of maximum moment

M_{max} maximum bending moment

V(x) shear at distance x from left reaction

M(x) bending moment at distance x from right reaction

Defined Units

$$plf := \frac{lbf}{ft}$$

ORIGIN set equal to 1 to agree with customary usage. ORIGIN is a PTC Mathcad variable, the index number of the first element of a vector or matrix.

$$ORIGIN := 1$$

Input

Enter segment lengths, uniformly distributed loads, and concentrated loads starting from the left reaction, and the left and right end moments:

Segment lengths: $a := \begin{bmatrix} 3.5 & 10 & 13.5 \end{bmatrix}^{T} \cdot ft$

Span length: $L := \sum a$ $\qquad L = 27\ ft$

Uniformly distributed loads: $w := \begin{bmatrix} 2.6 & 1.8 & 0.5 \end{bmatrix}^{T} \cdot \frac{kip}{ft}$

Concentrated loads at the right end of segments: $p := \begin{bmatrix} 7.8 & 10.3 \end{bmatrix}^{T} \cdot kip$

Left end moment: $M_L := 97.5 \cdot kip \cdot ft$

Right end moment: $M_R := 121 \cdot kip \cdot ft$

Calculations

Beam reactions, location of the point of zero shear, and maximum bending moment are computed within this section.

Maximum number of segments entered: $n := \text{rows}(a)$ $\qquad n = 3$

The following expressions adjusts the sizes of vectors P and w to the same size as vector a:

$P := p$ $\qquad P_n := \textbf{if}\left(\text{rows}(p) < \text{rows}(a), 0 \cdot kip, p_n\right)$

$W := w$ $\qquad W_n := \textbf{if}\left(\text{rows}(w) < \text{rows}(a), 0 \cdot \frac{lbf}{ft}, w_n\right)$

Range variable i from 1 to n; range variable i1 from 2 to n:

$$i := 1 .. n \qquad i1 := 2 .. n$$

Sum of the loads to the left side of each segment:

$$VL_1 := 0 \cdot kip \qquad VL_{i1} := VL_{i1-1} + W_{i1-1} \cdot a_{i1-1} + P_{i1-1}$$

$$VL^{T} = [0 \;\; 16.9 \;\; 45.2]\ kip$$

Sum of the loads to the right side of each segment:

$$VR_1 := W_1 \cdot a_1 \qquad VR_{i1} := VR_{i1-1} + P_{i1-1} + W_{i1} \cdot a_{i1}$$

$$VR^{T} = [9.1 \;\; \ldots]\ kip$$

Sum of the moments due to loads, at the right end of each segment:

$$MR_1 := \frac{W_1 \cdot (a_1)^2}{2} \qquad MR_{i1} := MR_{i1-1} + \frac{VL_{i1} + VR_{i1}}{2} \cdot a_{i1}$$

$$MR^{T} = [15.9 \;\; \ldots]\ kip \cdot ft$$

Left end reaction:

$$R_L := \frac{MR_n}{\sum a} + \left(\frac{M_L - M_R}{L}\right) \qquad R_L = 33.6\ kip$$

Right end reaction:

$$R_R := \sum_i \left(W_i \cdot a_i + P_i\right) - R_L \qquad R_R = 18.35\ kip$$

Shear at the left end of each segment:

$$V_{L_i} := R_L - VL_i$$

$$V_L^{T} = [33.6 \;\; 16.7 \;\; -11.6] \; kip$$

Shear at the right end of each segment:

$$V_{R_i} := R_L - VR_i$$

$$V_R^{T} = [24.5 \;\; \ldots] \;\; kip$$

Moments at the left end of each segment:

$$M_{s_1} := -M_L$$

$$M_{s_{i1}} := M_{s_{i1-1}} + \frac{1}{2} \cdot \left(V_{L_{i1-1}} + V_{R_{i1-1}}\right) \cdot a_{i1-1}$$

$$M_s^{T} = [-97.5 \;\; \ldots] \;\; kip \cdot ft$$

Matrix U with elements equal to 1 if the corresponding element in V_L is greater than or equal to 0 kip or with elements equal to 0 if the corresponding element is less than 0 kip:

$$U_i := \mathbf{if}\left(V_{L_i} > 0 \cdot \boldsymbol{kip}, 1, 0\right)$$

$$U^{T} = [1 \;\; 1 \;\; 0]$$

Index of the segment where shear passes through zero:

$$u := \left(\sum U\right) \qquad u = 2$$

Distance from the left end of the segment where shear passes through 0 to the point where shear passes through 0:

$$a' := \text{if}\left(W_u = 0 \cdot \frac{kip}{ft}, a_u, \text{if}\left(\frac{V_{L_u}}{W_u} > a_u, a_u, \frac{V_{L_u}}{W_u}\right)\right)$$

$$a' = 9.278\ ft$$

Distance from the left reaction to the left end of each segment:

$$S_{L_1} := 0 \cdot ft \qquad S_{L_{i1}} := S_{L_{i1-1}} + a_{i1-1}$$

$$S_L^{T} = [0\ \ 3.5\ \ 13.5]\ ft$$

Distance from the left end reaction to the point of zero shear and maximum moment:

$$X_L := S_{L_u} + a' \qquad X_L = 12.778\ ft$$

Maximum bending moment:

$$M_{max} := M_{s_u} + \left(V_{L_u} \cdot a' - \frac{1}{2} \cdot W_u \cdot a'^2\right)$$

$$M_{max} = 81.639\ kip \cdot ft$$

Vectors V and xv for plotting shear diagram:

$$V_{2 \cdot i - 1} := V_{L_i} \qquad V_{2 \cdot i} := V_{R_i}$$

$$x_{v_{2 \cdot i - 1}} := S_{L_i} \qquad x_{v_{2 \cdot i}} := S_{L_i} + a_i$$

Range variable i2 for plotting shear diagram:

$$i2 := 1 .. 2 \cdot n$$

Number of equally spaced points for plotting moment diagram:

$$N := 100$$

Note ⇒ N may be any reasonable number greater than or equal to 1.

Range variable i3 and equally spaced points x_m for plotting moment diagram:

$$i3 := 1 \,..\, N \qquad\qquad x_{m_{i3}} := i3 \cdot \frac{L}{N}$$

Distances to the left end of each segment added to vector x_m:

$$x_m := \left(\text{augment}\left(x_m^{\mathrm{T}}, S_L^{\mathrm{T}}\right)^{\mathrm{T}}\right)$$

Distance to the point of zero shear added as last element in vector x_m:

$$x_{m_{N+n+1}} := X_L$$

Range variable i4 for all points to be plotted:

$$i4 := 1 \,..\, N+n+1$$

Sort the elements of vector x_m in ascending order:

$$x_m := \text{sort}\left(x_m\right)$$

Moment at distance x from the left reaction, as a function of x:

$$M(x) := \sum_i \left(\left(\left(x > S_{L_i}\right) \cdot \left(x \le \left(S_{L_i} + a_i\right)\right) + (i = 1) \cdot (x = 0 \cdot ft) \right) \cdot \left(M_{s_i} + V_{L_i} \cdot \left(x - S_{L_i}\right) - \frac{1}{2} \cdot W_i \cdot \left(x - S_{L_i}\right)^2 \right) \right)$$

Summary

Input Variables

Segment lengths: $a^{T} = [3.5 \;\; 10 \;\; 13.5] \; ft$

Uniformly distributed loads: $W^{T} = [2.6 \;\; 1.8 \;\; 0.5] \; \dfrac{kip}{ft}$

Concentrated loads at right end of segments: $P^{T} = [7.8 \;\; 10.3 \;\; 0] \; kip$

Left end moment: $M_L = 97.5 \; kip \cdot ft$

Right end moment: $M_R = 121 \; kip \cdot ft$

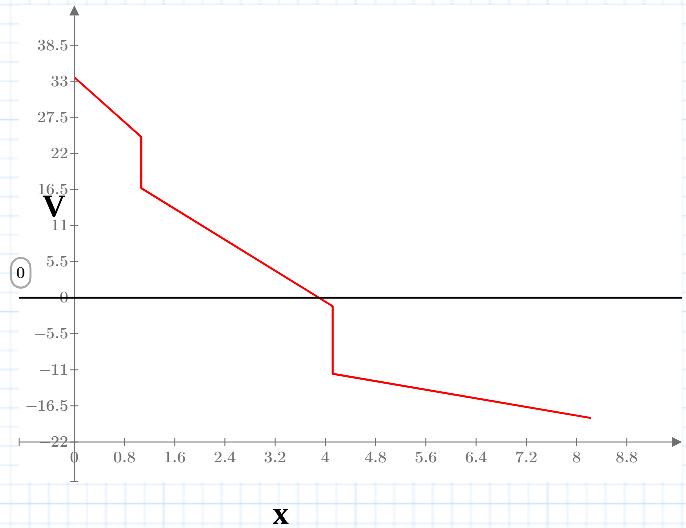
Computed Variables

Span lengths:	Left Reactions:	Right Reactions:
$L = 27 \; ft$	$R_L = 33.6 \; kip$	$R_R = 18.35 \; kip$

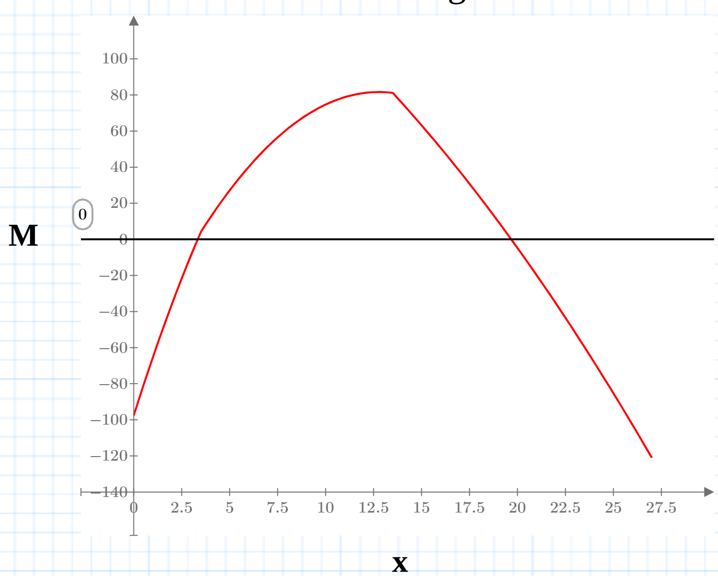
Distances from the left reaction to the point of zero shear: $X_L = 3.895 \; m$

Maximum moment: $M_{max} = 81.639 \; kip \cdot ft$

Shear Diagram



Moment Diagram



Note ⇒ PTC Mathcad plots may be enlarged to show small scale detail.