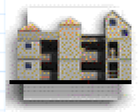

CHAPTER 3: Reinforced Concrete Slabs and Beams

# 3.2 Reinforced Concrete Beams - Size Selection

## Description

This application determines the sizes of rectangular beams to satisfy the flexural requirements, shear requirements and minimum thickness limits of ACI 318-89 using the strength design method of ACI 318-89.

The required input includes the strength of the concrete and the reinforcement, the maximum values for factored load bending moments and shears, and the span lengths and types of span for the spans which will determine minimum depths. Three beam sizes are shown for illustrative purposes, however any practical number of spans may be entered at one time.

A summary of input and calculated values are shown on pages 11 and 12.

**Reference:**

ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)

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## Input

### *Input Variables*

Enter Mu, Vu, L and SpanType as vectors with the number of rows equal to the number of beam sizes to be determined.

Critical factored moments:

$$M_u := \begin{bmatrix} 190 & 85 & 75 \end{bmatrix}^T \cdot kip \cdot ft$$

Critical factored shears:

$$V_u := \begin{bmatrix} 13 & 6.5 & 10.5 \end{bmatrix}^T \cdot kip$$

Span length, clear span for monolithic construction:

$$L := \begin{bmatrix} 20 & 20 & 22 \end{bmatrix}^T \cdot ft$$

| Numbers Designating "Span Type" | |
|---|---|
| Simple Span | 0 |
| End Span, | 1 |
| Interior Span | 2 |
| Cantilever Span | 3 |

Span type:

$$SpanType := \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}^T$$

The moment, shear, span length and span type are the critical values that determine the beam size. The critical moment and shear do not necessarily occur at the same location.

Enter values for $b_{min}$, $h_{max}$, R, and d'.

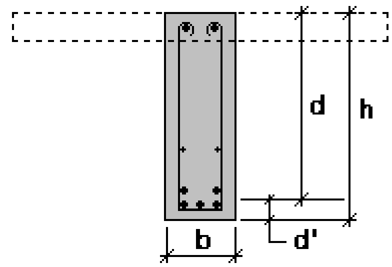
Specified minimum member width: $b_{min} := 8 \cdot in$

Specified maximum permissible member thickness: $h_{max} := 30 \cdot in$

Specified maximum ratio of h/b: $R := 2$

Estimated distance from the centroid of the tension reinforcement to the extreme fiber in tension: $d' := 2.5 \cdot in$

## Computed Variables




| | |
|---|---|
| b | width of compression face of member |
| h | overall thickness of member |
| d | distance from extreme compression fiber to centroid of tension reinforcement |
| $\phi M_n$ | useable moment capacity at factored load |
| $\phi V_n$ | useable shear capacity at factored load |

## Material Properties and Constants

Enter values for $f'_c$, $f_y$, $w_c$, $k_v$ and $k_s$ if different from that shown.

Specified compressive strength of concrete: $f'_c := 4 \cdot ksi$

Specified yield strength of reinforcement (fy may not exceed 60 ksi, ACI 318 11.5.2): $f_y := 60 \cdot ksi$

Unit weight of concrete: $w_c := 145 \cdot pcf$

Shear strength reduction factor (For lightweight concrete $k_v$ = 1, for normal weight, $k_v$ = 0.75, for all-lightweight and sand-lightweight concrete, $k_v$ = 0.85 (ACI 318, 11.2.1.2.)): $k_v := 1$

Enter factor for computing shear strength of stirrups. (For 50% of maximum shear reinforcement stress and minimum spacing at d/2, ks = 1. For 100% of maximum shear reinforcement stress and minimum stirrup spacing at d/4, ks = 2. (ACI 318, 11.5.4, 11.5.4.3)): $k_s := 1$

Modulus of elasticity of reinforcement (ACI 318, 8.5.2):

$E_s := 29000 \cdot ksi$

Strain in concrete at compression failure (ACI 318, 10.3.2):

$\varepsilon_c := 0.003$

Strength reduction factor for flexure (ACI 318, 9.3.2.1):

$\phi_f := 0.9$

Strength reduction factor for shear (ACI 318, 9.3.2.3):

$\phi_v := 0.85$

Sizing factor for rounding dimensions (to a multiple of SzF):

$SzF := 2 \cdot in$

Limit the value of f'c for computing shear and development lengths to 10 ksi by substituting f'c_max for f'c in formulas for computing shear (ACI 318, 11.1.2, 12.1.2):

$$f'_{c\_max} := \mathbf{if}\left(f'_c > 10 \cdot ksi, 10 \cdot ksi, f'_c\right)$$

The following values are computed from the entered material properties.

Nominal "one way" shear strength per unit area in concrete (ACI 318, 11.3.1.1, Eq (11-3), 11.5.4.3):

$$v_c := k_v \cdot 2 \cdot \sqrt{\frac{f'_{c\_max}}{psi}} \cdot psi \qquad v_c = 126\ psi$$

Nominal "one way" shear strength per unit area in concrete and shear reinforcement (ACI 318, 11.3.1.1, Eq. (11-3), 11.5.4.3):

$$v_c := k_v \cdot 2 \cdot \sqrt{\frac{f'_{c\_max}}{psi}} \cdot psi \qquad v_c = 126\ psi$$

$$v_s := k_s \cdot 4 \cdot \sqrt{\frac{f'_{c\_max}}{psi}} \cdot psi \qquad v_s = 253\ psi$$

Modulus of elasticity of concrete for values of $w_c$ between 90 pcf and 155 pcf (ACI 318, 8.5.1):

$$E_c := \left(\frac{w_c}{pcf}\right)^{1.5} \cdot 33 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi \qquad E_c = 3644\ ksi$$

Strain in reinforcement at yield stress:

$$\varepsilon_y := \frac{f_y}{E_s} \qquad \varepsilon_y = 0.00207$$

Factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3):

$$\beta_1 := \mathbf{if}\left(\left(f'_c \geq 4 \cdot ksi\right) \cdot \left(f'_c \leq 8 \cdot ksi\right), 0.85 - 0.05 \cdot \frac{f'_c - 4 \cdot ksi}{ksi}, \mathbf{if}\left(\left(f'_c \leq 4 \cdot ksi\right), 0.85, 0.65\right)\right)$$

$$\beta_1 = 0.85$$

Reinforcement ratio producing balanced strain conditions (ACI 318, 10.3.2):

$$\rho_b := \frac{\beta_1 \cdot 0.85 \cdot f'_c}{f_y} \cdot \frac{E_s \cdot \varepsilon_c}{E_s \cdot \varepsilon_c + f_y} \qquad \rho_b = 2.851\ 1\%$$

Maximum reinforcement ratio (ACI 318, 10.3.3):

$$\rho_{max} := \frac{3}{4} \cdot \rho_b \qquad \rho_{max} = 2.138\ 1\%$$

Preferred reinforcement ratio:

$$\rho_{pref} := 0.5 \cdot \rho_{max} \qquad \rho_{pref} = 1.069\ 1\%$$

Minimum reinforcement ratio for beams (ACI 318, 10.5.1, Eq. (10-3)):

$$\rho_{min} := \frac{200}{f_y} \cdot \frac{lbf}{in^2} \qquad \rho_{min} = 0.333\ 1\%$$

Shrinkage and temperature reinforcement ratio (ACI 318, 7.12.2.1):

$$\rho_{temp} := \text{if}\left(f_y \le 50 \cdot ksi, .002, \text{if}\left(f_y \le 60 \cdot ksi, .002 - \frac{f_y}{60 \cdot ksi} \cdot .0002, \text{if}\left(\frac{.0018 \cdot 60 \cdot ksi}{f_y} \ge .0014, \frac{.0018 \cdot 60 \cdot ksi}{f_y}\right.\right.\right.$$

$$\rho_{temp} = 0.18\ 1\%$$

Flexural coefficient K, for rectangular beams or slabs, as a function of ρ (ACI 318, 10.2):
(Moment capacity $\phi M_n = K(\rho)F$, where $F = bd^2$)

$$K(\rho) := \phi_f \cdot \rho \cdot \left(1 - \frac{\rho \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y$$

Factors for adjusting minimum beam and slab thickness $h_{min}$ for use of lightweight concrete and yield strengths other than 60 ksi (ACI 318, 9.5.2.1, see footnotes to Table 9.5 (a)):

Adjustment factor for minimum thickness for concrete weights between 90 and 120 pcf:

$$q_1 := \text{if}\left(w_c \le 112 \cdot pcf, 1.65 - 0.005 \cdot \frac{w_c}{pcf}, \text{if}\left(w_c \le 120 \cdot pcf, 1.09, 1\right)\right)$$

$$q_1 = 1$$

Adjustment factor for minimum thickness for yield strengths other than 60 ksi:

$$q_2 := 0.4 + \frac{f_y}{100 \cdot ksi} \qquad q_2 = 1$$

Adjustment factor for minimum thickness combining factors for concrete weight and for yield strengths other than 60 ksi:

$$Q := q_1 \cdot q_2 \qquad Q = 1$$

**Defined Units**

$$pcf := lbf \cdot ft^{-3} \qquad psf := lbf \cdot ft^{-2}$$

## Calculations

Minimum required shear area ShA (ACI 318, 9.3.2.3, 11.3.1.1, Eq. (11-3), 11.5.4.3):

$$i := 0 .. \text{last}\left(M_u\right) \qquad ShA_i := \frac{V_{u_i}}{\phi_v \cdot \left(v_c + v_s\right)}$$

$$ShA^{\mathrm{T}} = [40.304 \;\; 20.152 \;\; 32.553] \; \boldsymbol{in}^2$$

Minimum member thickness $h_{min}$ (unless deflections are checked) (ACI 318, 9.5.2.1):

$$S := SpanType$$

$$k_i := \textbf{if}\left(S_i = 0, 16, \textbf{if}\left(S_i = 1, 18.5, \textbf{if}\left(S_i = 2, 21, 8\right)\right)\right)$$

$$h_{min_i} := \frac{Q \cdot L_i}{k_i}$$

$$h_{min}^{\mathrm{T}} = [12.973 \;\; 11.429 \;\; 14.27] \; \boldsymbol{in}$$

Round $h_{min}$ up to nearest upper multiple of SzF unless lower multiple is within 1/2%:

$$h_{Rmin_i} := SzF \cdot \text{ceil}\left(\frac{0.995 \cdot h_{min_i}}{SzF}\right) \qquad h_{Rmin}^{\mathrm{T}} = [14 \;\; 12 \;\; 16] \; in$$

Required section coefficient F ($F = bd^2$):

$$F_i := \frac{M_{u_i}}{K\left(\rho_{pref}\right)} \qquad F^{\mathrm{T}} = [4361.024 \;\; 1950.984 \;\; 1721.457] \; \boldsymbol{in}^3$$

Calculate required member size:

Guess value of h:

$$h_i := \frac{h_{min_i} + h_{max}}{2} \qquad h^T = [21.486 \;\; 20.714 \;\; 22.135]\ in$$

Thickness h required to satisfy flexural requirement with preferred ratio of h/b:

$$f1(h, F) := \mathbf{root}\left(\frac{h}{R} \cdot (h - 2.5 \cdot \mathbf{in})^2 - F, h\right)$$

$$h_{f_i} := f1\left(h_i, F_i\right) \qquad h_f^T = [22.284 \;\; 17.452 \;\; 16.811]\ in$$

Round h up or down to the nearest multiple of SzF:

$$h_{f\_rd_i} := SzF \cdot \text{floor}\left(\frac{h_{f_i}}{SzF} + 0.5\right) \qquad h_{f\_rd}^T = [22 \;\; 18 \;\; 16]\ in$$

Member thickness h determined in step 3 or as limited by $h_{Rmin}$ or $h_{max}$:

$$h_i := \mathbf{if}\left(h_{f\_rd_i} \geq h_{max}, h_{max}, \mathbf{if}\left(h_{f\_rd_i} \leq h_{Rmin_i}, h_{Rmin_i}, h_{f\_rd_i}\right)\right)$$

$$h^T = [22 \;\; 18 \;\; 16]\ \mathbf{in}$$

Member widths determined by flexure and shear:

$$b_{f_i} := \frac{F_i}{\left(h_i - 2.5 \cdot \mathbf{in}\right)^2} \qquad b_f^T = [11.469 \;\; 8.121 \;\; 9.446]\ \mathbf{in}$$

$$b_{v_i} := \frac{ShA_i}{h_i - 2.5 \cdot \mathbf{in}} \qquad b_v^T = [2.067 \;\; 1.3 \;\; 2.411]\ \mathbf{in}$$

The larger member width determined by ratio R or $b_{min}$:

$$b_{1_i} := \mathbf{if}\left(b_{min} \ge \frac{h_i}{R}, b_{min}, \frac{h_i}{R}\right) \qquad b_1^{\mathrm{T}} = [11 \;\; 9 \;\; 8] \; in$$

The largest member width determined by shear, flexure, ratio R or $b_{min}$:

$$b_{2_i} := \mathbf{if}\left(b_{v_i} \ge b_{f_i}, \mathbf{if}\left(b_{v_i} \ge b_{1_i}, b_{v_i}, b_{1_i}\right), \mathbf{if}\left(b_{f_i} \ge b_{1_i}, b_{f_i}, b_{1_i}\right)\right)$$

$$b_2^{\mathrm{T}} = [11.469 \;\; 9 \;\; 9.446] \; in$$

Required member width b rounded up to the nearest multiple of SzF, unless lower multiple is within 1/2%:

$$b_i := SzF \cdot \mathrm{ceil}\left(\frac{0.995 \cdot b_{2_i}}{SzF}\right) \qquad b^{\mathrm{T}} = [12 \;\; 10 \;\; 10] \; in$$

Effective depth to the centroid of the tension reinforcement:

$$d := h - d' \qquad d^{\mathrm{T}} = [19.5 \;\; 15.5 \;\; 13.5] \; in$$

Theoretical reinforcement ratio required for flexure:

$$\rho_{1_i} := \left(1 - \left(\sqrt{1 - \frac{2 \cdot M_{u_i}}{\phi_f \cdot b_i \cdot \left(d_i\right)^2 \cdot 0.85 \cdot f'_c}}\right)\right) \cdot \frac{0.85 \cdot f'_c}{f_y}$$

$$\rho_1^{\mathrm{T}} = [1.016 \;\; 0.85 \;\; 1.003] \; 1\%$$

The larger of the theoretical reinforcement ratio or the minimum reinforcement ratio:

$$\rho_i := \mathbf{if}\left(\rho_{1_i} \le \frac{3}{4} \cdot \rho_{min}, \frac{4}{3} \cdot \rho_{1_i}, \mathbf{if}\left(\rho_{1_i} \ge \rho_{min}, \rho_{1_i}, \rho_{min}\right)\right)$$

$$\rho^{\mathrm{T}} = [1.016 \;\; 0.85 \;\; 1.003] \; 1\%$$

Reinforcement areas:

$$A_{s_i} := \rho_i \cdot b_i \cdot d_i$$

$$A_s^{\mathrm{T}} = [2.379 \;\; 1.317 \;\; 1.354] \; \boldsymbol{in}^2$$

Useable moment capacity at factored load:

$$\phi M_{n_i} := K\left(\rho_i\right) \cdot b_i \cdot \left(d_i\right)^2$$

$$\phi M_n^{\mathrm{T}} = [190 \;\; 85 \;\; 75] \; kip \cdot \boldsymbol{ft}$$

Useable shear capacity at factored load:

$$\phi V_{n_i} := \phi_v \cdot \left(v_c + v_s\right) \cdot b_i \cdot d_i$$

$$\phi V_n^{\mathrm{T}} = [75.477 \;\; 49.996 \;\; 43.545] \; \boldsymbol{kip}$$

Useable shear capacity of concrete:

$$\phi V_{c_i} := \phi_v \cdot v_c \cdot b_i \cdot d_i$$

$$\phi V_c^{\mathrm{T}} = [25.159 \;\; 16.665 \;\; 14.515] \; \boldsymbol{kip}$$

## Summary

### *Input*

| | |
|---|---|
| Specified compressive strength of concrete: | $f'_c = 4\ ksi$ |
| Specified yield strength of reinforcement: | $f_y = 60\ ksi$ |
| Unit weight of concrete: | $w_c = 145\ pcf$ |
| Shear strength reduction factor for lightweight concrete: | $k_v = 1$ |
| Factor for computing shear strength of stirrups: | $k_s = 1$ |
| Sizing factor for rounding dimensions to a multiple of SzF: | $SzF = 2\ in$ |
| Specified minimum member width: | $b_{min} = 8\ in$ |
| Specified maximum permissible member thickness: | $h_{max} = 30\ in$ |
| Estimated distance from the centroid of the tension reinforcement to the extreme fiber in tension: | $d' = 2.5\ in$ |
| Specified maximum ratio of h/b: | $R = 2$ |
| Preferred reinforcement ratio: | $\rho_{pref} = 0.011$ |
| Span length: | $L^{T} = [20\ \ 20\ \ 22]\ ft$ |

Span type: $SpanType^{T} = [1 \;\; 2 \;\; 1]$

Critical factored moments: $M_u^{T} = [190 \;\; 85 \;\; 75] \; kip \cdot ft$

Critical factored shears: $V_u^{T} = [13 \;\; 6.5 \;\; 10.5] \; kip$

**Notes**

1) Span type is 0 for simple spans, 1 for end spans, 2 for interior span and 3 for cantilever.
2) Shear strength reduction factor lightweight concrete, $k_v$ = 1 for normal weight, 0.85 for sand-lightweight and 0.75 for all-lightweight.
3) Factor for shear reinforcement spacing, $k_s$ = 1 for minimum spacing at d/2 and 50% of maximum $v_s$ and 2 for minimum spacing at d/4 and 100% of maximum $v_s$.

## ***Computed Variables***

Useable moment capacity at factored load: $\phi M_n^{T} = [190 \;\; 85 \;\; 75] \; kip \cdot ft$

Useable shear capacity at factored load: $\phi V_n^{T} = [75.477 \;\; 49.996 \;\; 43.545] \; kip$

Useable shear capacity of concrete: $\phi V_c^{T} = [25.159 \;\; 16.665 \;\; 14.515] \; kip$

## ***Beam dimensions selected, reinforcement ratios, reinforcement areas, and minimum and maximum permissible $\rho$ values***

Member width: $b^{T} = [12 \;\; 10 \;\; 10] \; in$

Member thickness: $h^{T} = [22 \;\; 18 \;\; 16] \; in$

Reinforcement areas: $A_s^{T} = [2.379 \;\; 1.317 \;\; 1.354] \; in^2$

Reinforcement ratio: $\rho^{T} = [1.016 \;\; 0.85 \;\; 1.003] \; 1\%$

Minimum required reinforcement ratio: $\rho_{min} = 0.333\ 1\%$

Maximum permissible reinforcement ratio: $\rho_{max} = 2.138\ 1\%$