3.3 Continuous Beams - Flexural Reinforcement

Description

This application calculates bending moments for continuous rectangular or T-shaped beams using ACI coefficients and determines the required number and size of reinforcing bars. Intermediate values calculated include the service and factored loads per unit length of beam, minimum beam width for bottom bars placed in a single layer, and maximum beam width for crack control. The application uses the strength design method of ACI 318-89.

The input required includes the strengths of the concrete and the reinforcement, the unit weight of concrete, design live load per unit area, superimposed dead load per unit area, beam dimensions and slab thicknesses, crack control factor, ratio of the shortest top bar cutoff length to the span length, percentage of bottom bars continuing from the point of inflection into the support, span length, span type, tributary slab width per beam, clear concrete cover of flexural reinforcement and estimated top and bottom bar sizes. Three continuous beams with their first three spans are shown for illustrative purposes, however any practical number of beams may be entered at one time. The application covers any combination of span types which meet the limitations for use of ACI coefficients. The beam size and loads are assumed to be the same for all spans of each continuous beam entered.

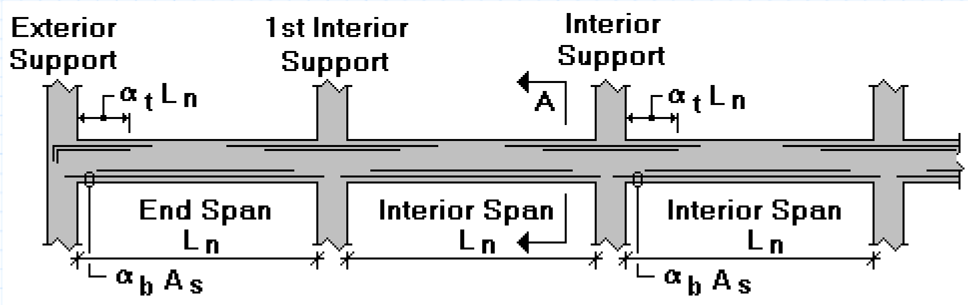
A summary of input and calculated values is shown on pages 25-29.

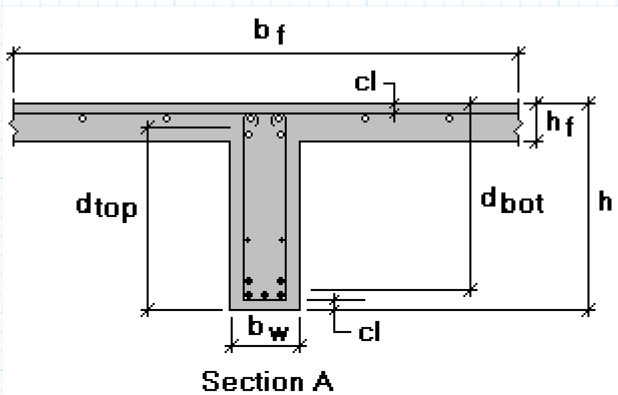
Reference:

ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)

Input

Notation





Input Variables

Enter uniformly distributed loads.

Service live load per unit area: $w_l := 200 \cdot psf$

Service dead load per unit area excluding slab weight: $w_{sd} := 15 \cdot psf$

Enter section dimensions with one beam size for all spans of each continuous beam.

Flange width: $b_f := [34\ \ 68\ \ 79]^T \cdot in$

Web width: $b_w := [12\ \ 12\ \ 14]^T \cdot in$

Beam thickness: $h := [22\ \ 22\ \ 22]^T \cdot in$

Slab thickness: $h_f := [4.5\ \ 4.5\ \ 4.5]^T \cdot in$

Numbers Designating "Span Type"	
Simple Span	0
End Span, Spandrel Beam Exterior Support	1
End Span, Column Exterior Support	11
End Span, Unrestrained Exterior Support	12
Interior Span	2
Cantilever Span	3

Enter span lengths and span types as three column matrices, with the number of rows equal to the number of continuous beam entered. The first column of the matrix must be end spans, the second column may be an adjacent interior span or the second end span of a two span continuous beam, and the third column may be an end span for a three span continuous beam or an interior span for four or more spans. Enter zeros for span length and type in the third columns for two span continuous beams.

Span lengths: $L_n := \begin{bmatrix} 10 & 10 & 0 \\ 20 & 24 & 24 \\ 24 & 20 & 22 \end{bmatrix} \cdot ft$

Type of span: $SpanType := \begin{bmatrix} 12 & 12 & 0 \\ 1 & 2 & 1 \\ 11 & 2 & 11 \end{bmatrix}$

Negative moments are calculated using the average length of adjacent spans. The larger adjacent span may not be more than 20% longer than the shorter span. (See ACI 318, Section 8.3.3 (b).)

Enter estimated or final bar size numbers. Exterior support top bars should be in the first column, first interior support top bars in the second column, and second interior support top bars in the third column. Exterior span bottom bars should be in the first column, first interior span bottom bars in the second column, and second interior span bottom bars in the third column.

Top bars:

$$xt := \begin{bmatrix} 0 & 3 & 0 \\ 4 & 7 & 8 \\ 7 & 7 & 7 \end{bmatrix}$$

Bottom Bars:

$$xb := \begin{bmatrix} 5 & 5 & 0 \\ 7 & 8 & 9 \\ 10 & 8 & 10 \end{bmatrix}$$

The bar sizes xt and xb are initially assumed. Since it is possible that a larger or smaller bar size may be required, and the effective depths may change, the final bar sizes may be substituted for a final check.

Enter the ratios α_t and α_b.

Specified ratio of the shortest top bar cutoff length to the clear span: $\alpha_t := 0.15$

Specified percentage of bottom bars continuing from the point of inflection into the support: $\alpha_b := 25\%$

Note ⇒ Ratios α_t and α_b should be determined from the user's standard detail for bar cutoff points.

These ratios are used to determine the available lengths for development of reinforcing bars and the maximum useable bar sizes.

Enter tributary slab widths, with one value for all three spans of each continuous beam entered.

Tributary slab width: $SW := \begin{bmatrix} 3 \\ 5.75 \\ 8 \end{bmatrix} \cdot ft$

Computed Variables

w_L	live load per unit length of beam
w_D	dead load per unit length of beam
w_u	total factored load per unit length of beam
M_u	factored load moments using ACI coefficients
d_{bot}	effective depth to centroid of bottom reinforcement
d_{top}	effective depth to centroid of top reinforcement
A_{neg}	required area of negative (top) reinforcement
A_{pos}	required area of positive (bottom) reinforcement
ϕM_{neg}	negative moment capacity
ϕMpos	positive moment capacity
MinSp	minimum permissible spacing of top reinforcement
MaxSp	maximum spacing of bottom reinforcement for crack control
Minbw	minimum beam width for bars in a single layer
Maxbw	maximum beam width for crack control
NumbTop	number of top bars
TopBarSize	size of top bars
NumbBot	number of bottom bars
BotBarSize	size of bottom bars

Material Properties and Constants

Enter values for f'_c, f_y, w_c, w_{rc}, k_v and k_w if different from that shown.

Specified compressive strength of concrete: $f'_c := 4 \cdot ksi$

Specified yield strength of reinforcement (f_y may not exceed 60 ksi, ACI 318 11.5.2):	$f_y := 60 \cdot ksi$
Unit weight of concrete:	$w_c := 145 \cdot pcf$
Weight of reinforced concrete:	$w_{rc} := 150 \cdot pcf$
Shear strength reduction factor for lightweight concrete k_v = 1 for normal weight, k_v = 0.75 for all-lightweight and k_v = 0.85 for sand-lightweight concrete (ACI 318, 11.2.1.2.):	$k_v := 1$
Weight factor for increasing development and splice lengths k_w = 1 for normal weight and k_w = 1.3 for lightweight aggregate concrete (ACI 318, 12.2.4.2):	$k_w := 1$
Modulus of elasticity of reinforcement (ACI 318, 8.5.2):	$E_s := 29000 \cdot ksi$
Strain in concrete at compression failure (ACI 318, 10.3.2):	$\varepsilon_c := 0.003$
Strength reduction factor for flexure (ACI 318, 9.3.2.1):	$\phi_f := 0.9$
Strength reduction factor for shear (ACI 318, 9.3.2.3):	$\phi_v := 0.85$
Clear concrete cover of longitudinal reinforcement:	$cl := 2 \cdot in$
Crack control factor (175 kip/in interior, 145 kip/in exterior (ACI 318,10.6.4):	$z := 175 \cdot \frac{kip}{in}$

Reinforcing bar number designations, diameters and areas:

$$No := [0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10\ \ 11\ \ 12\ \ 13\ \ 14\ \ 15\ \ 16\ \ 17\ \ 18]^{T}$$

$$d_b := [0\ \ 0\ \ 0\ \ 0.375\ \ 0.5\ \ 0.625\ \ 0.75\ \ 0.875\ \ 1.00\ \ 1.128\ \ 1.27\ \ 1.41\ \ 0\ \ 0\ \ 1.693\ \ 0\ \ 0\ \ 0\ \ 2.257]^{T} \cdot in$$

$$A_b := [0\ \ 0\ \ 0\ \ 0.11\ \ 0.20\ \ 0.31\ \ 0.44\ \ 0.60\ \ 0.79\ \ 1.00\ \ 1.27\ \ 1.56\ \ 0\ \ 0\ \ 2.25\ \ 0\ \ 0\ \ 0\ \ 4.00]^{T} \cdot in^{2}$$

Bar numbers, diameters and areas are stored in vector rows (or columns in the transposed vectors shown). The index number of each row (or column) corresponds to a particular bar number. Individual bar numbers, diameters, areas and development lengths and splices of a specific bar can be referred to and displayed by using the vector subscripts as show below.

Example: $No_5 = 5$ $\quad d_{b_5} = 0.625\ in$ $\quad A_{b_5} = 0.31\ in^2$

Limit the value of f'c for computing shear and development lengths to 10 ksi by substituting f'c_max for f'c (ACI 318, 11.1.2, 12.1.2):

$$f'_{c\_max} := \text{if}\left(f'_c > 10 \cdot ksi, 10 \cdot ksi, f'_c\right)$$

The following values are computed from the entered material properties.

Nominal "one way" shear strength per unit area in concrete (ACI 318, 11.3.1.1, Eq. (11-3), 11.5.4.3):

$$v_c := k_v \cdot 2 \cdot \sqrt{\frac{f'_{c\_max}}{psi}} \cdot psi \qquad v_c = 126\ psi$$

Modulus of elasticity of concrete for values of w_c between 90 pcf and 155 pcf (ACI 318, 8.5.1):

$$E_c := \left(\frac{w_c}{pcf}\right)^{1.5} \cdot 33 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi \qquad E_c = 3644\ ksi$$

Strain in reinforcement at yield stress:

$$\varepsilon_y := \frac{f_y}{E_s} \qquad \varepsilon_y = 2.06897 \cdot 10^{-3}$$

Factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3):

$$\beta_1 := \text{if}\left(\left(f'_c \ge 4 \cdot \boldsymbol{ksi}\right) \cdot \left(f'_c \le 8 \cdot \boldsymbol{ksi}\right), 0.85 - 0.05 \cdot \frac{f'_c - 4 \cdot \boldsymbol{ksi}}{\boldsymbol{ksi}}, \text{if}\left(\left(f'_c \le 4 \cdot \boldsymbol{ksi}\right), 0.85, 0.65\right)\right)$$

$$\beta_1 = 0.85$$

Reinforcement ratio producing balanced strain conditions (ACI 318, 10.3.2):

$$\rho_b := \frac{\beta_1 \cdot 0.85 \cdot f'_c}{f_y} \cdot \frac{E_s \cdot \varepsilon_c}{E_s \cdot \varepsilon_c + f_y} \qquad \rho_b = 2.851 \ 1\%$$

Maximum reinforcement ratio (ACI 318, 10.3.3):

$$\rho_{max} := \frac{3}{4} \cdot \rho_b \qquad \rho_{max} = 2.138 \ 1\%$$

Minimum reinforcement ratio for beams (ACI 318, 10.5.1, Eq. (10-3)):

$$\rho_{min} := \frac{200}{f_y} \cdot \frac{\boldsymbol{lbf}}{\boldsymbol{in}^2} \qquad \rho_{min} = 0.333 \ 1\%$$

Shrinkage and temperature reinforcement ratio
(ACI 318, 7.12.2.1):

$$\rho_{temp} := \text{if}\left(f_y \le 50 \cdot \boldsymbol{ksi}, .002, \text{if}\left(f_y \le 60 \cdot \boldsymbol{ksi}, .002 - \frac{f_y}{60 \cdot \boldsymbol{ksi}} \cdot .0002, \text{if}\left(\frac{.0018 \cdot 60 \cdot \boldsymbol{ksi}}{f_y} \ge .0014, \frac{.0018 \cdot 60 \cdot \boldsymbol{ksi}}{f_y}, \right.\right.\right.$$

$$\rho_{temp} = 0.18 \ 1\%$$

Flexural coefficient K, for rectangular beams or slabs, as a function of ρ (ACI 318, 10.2):
(Moment capacity $\phi M_n = K(\rho)F$, where $F = bd^2$)

$$K(\rho) := \phi_f \cdot \rho \cdot \left(1 - \frac{\rho \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y$$

Factors for adjusting minimum beam and slab thickness h_{min} for use of lightweight concrete and yield strengths other than 60 ksi (ACI 318, 9.5.2.1, see footnotes to Table 9.5 (a)):

Adjustment factor for minimum thickness for concrete weights between 90 and 120 pcf:

$$q_1 := \mathbf{if}\left(w_c \le 112 \cdot \boldsymbol{pcf}, 1.65 - 0.005 \cdot \frac{w_c}{\boldsymbol{pcf}}, \mathbf{if}\left(w_c \le 120 \cdot \boldsymbol{pcf}, 1.09, 1\right)\right)$$

$$q_1 = 1$$

Adjustment factor for minimum thickness for yield strengths other than 60 ksi:

$$q_2 := 0.4 + \frac{f_y}{100 \cdot \boldsymbol{ksi}} \qquad q_2 = 1$$

Adjustment factor for minimum thickness combining factors for concrete weight and for yield strengths other than 60 ksi:

$$Q := q_1 \cdot q_2 \qquad Q = 1$$

Basic tension development length l_{dbt} (ACI 318, 12.2.2, 12.2.3.6):

No. 3 through No. 11 bars: $n := 3..11$

$$X1_n := 0.04 \cdot A_{b_n} \cdot \frac{f_y}{\sqrt{f'_{c\_max} \cdot \boldsymbol{lbf}}} \qquad X2_n := 0.03 \cdot d_{b_n} \cdot \frac{f_y}{\sqrt{\frac{f'_{c\_max}}{\boldsymbol{psi}} \cdot \boldsymbol{psi}}}$$

$$l_{dbt_n} := \mathbf{if}\left(X1_n > X2_n, X1_n, X2_n\right)$$

$$l_{dbt}^{T} = [0\ \ 0\ \ 0\ \ 10.7\ \ 14.2\ \ 17.8\ \ 21.3\ \ 24.9\ \ 30\ \ 37.9\ \ 48.2\ \ 59.2]\ in$$

No. 14 bars: $l_{dbt_{14}} := 0.085 \cdot \dfrac{f_y \cdot in^2}{\sqrt{f'_{c\_max} \cdot lbf}}$ $\quad l_{dbt_{14}} = 80.6\ in$

No. 18 bars $l_{dbt_{18}} := 0.125 \cdot \dfrac{f_y \cdot in^2}{\sqrt{f'_{c\_max} \cdot lbf}}$ $\quad l_{dbt_{18}} = 118.6\ in$

Tension development length (ACI 318, 12.2.1):

No. 3 through No. 11 bars:

$$l_{dt_n} := \text{if}\left(k_w \cdot l_{dbt_n} \geq 12 \cdot in, k_w \cdot l_{dbt_n}, \text{if}\left(k_w \cdot l_{dbt_n} > 0 \cdot in, 12 \cdot in, 0 \cdot in\right)\right)$$

$$l_{dt}^{T} = [0\ \ 0\ \ 0\ \ 12\ \ 14.2\ \ 17.8\ \ 21.3\ \ 24.9\ \ 30\ \ 37.9\ \ 48.2\ \ 59.2]\ in$$

No. 14 bars: $l_{dt_{14}} := k_w \cdot l_{dbt_{14}}$ $\quad l_{dt_{14}} = 80.6\ in$

No. 18 bars: $l_{dt_{18}} := k_w \cdot l_{dbt_{18}}$ $\quad l_{dt_{18}} = 118.6\ in$

Defined Units

$pcf := lbf \cdot ft^{-3}$ $\quad psf := lbf \cdot ft^{-2}$

Calculations

Definition of range variables i and j; "SpanType" shortened to "S":

$i := 0..\text{last}\left(L_n^{\langle 0\rangle}\right)$ $\quad j := 0..2$ $\quad S := SpanType$

Dead load and live load per unit length of beam:

$$w_L := SW \cdot w_l$$

$$w_L^T = [0.6\ \ 1.15\ \ 1.6]\ \frac{kip}{ft}$$

$$w_D := \overrightarrow{SW \cdot \left(w_{sd} + w_{rc} \cdot h_f\right) + w_{rc} \cdot \left(b_w \cdot \left(h - h_f\right)\right)}$$

$$w_D^T = [0.433\ \ 0.628\ \ 0.825]\ \frac{kip}{ft}$$

Factored design load wu and service load ws per unit length of beam:

$w_s := w_L + w_D$ $\quad w_u := 1.4 \cdot w_D + 1.7 \cdot w_L$

$$w_s^T = [1.033\ \ 1.778\ \ 2.425]\ \frac{kip}{ft}$$

$$w_u^T = [1.626\ \ 2.835\ \ 3.875]\ \frac{kip}{ft}$$

Moments using ACI coefficients (see ACI 318, Section 8.3):

Exterior support: $M_{u_{i,0}} := \textbf{if}\left(S_{i,0} = 1, \frac{1}{24}, \textbf{if}\left(S_{i,0} = 12, 0, \frac{1}{16}\right)\right) \cdot w_{u_i} \cdot \left(L_{n_{i,0}}\right)^2$

Exterior span: $M_{u_{i,1}} := \textbf{if}\left(S_{i,0} = 12, \frac{1}{11}, \frac{1}{14}\right) \cdot \left(w_{u_i} \cdot \left(L_{n_{i,0}}\right)^2\right)$

1st interior support:

$$M_{u_{i,2}} := \mathbf{if}\left(S_{i,1} = 2, \frac{1}{10}, \frac{1}{9}\right) \cdot w_{u_i} \cdot \left(\frac{L_{n_{i,0}} + L_{n_{i,1}}}{2}\right)^2$$

2nd span (interior for 3 or more spans, end span for 2 spans)

$$M_{u_{i,3}} := \mathbf{if}\left(S_{i,1} = 2, \frac{1}{16}, \mathbf{if}\left(S_{i,1} = 12, \frac{1}{11}, \frac{1}{14}\right)\right) \cdot w_{u_i} \cdot \left(L_{n_{i,1}}\right)^2$$

3rd support (interior for 4 or more spans, 1st interior for 3 spans, or exterior for 2 spans)

$$km_i := \mathbf{if}\left(\left(S_{i,1} = 2\right) \cdot \left(S_{i,2} = 2\right), \frac{1}{11}, \mathbf{if}\left(\left(S_{i,2} = 1\right) + \left(S_{i,2} = 11\right) + \left(S_{i,2} = 12\right), \frac{1}{10}, \mathbf{if}\left(S_{i,1} = 1, \frac{1}{24}, \mathbf{if}\left(S_{i,1} = 11, \ldots\right.\right.\right.\right.$$

$$M_{u_{i,4}} := km_i \cdot w_{u_i} \cdot \left(\frac{L_{n_{i,1}} + L_{n_{i,2}}}{2}\right)^2$$

3rd span (interior for 4 or more spans, end span for 3 spans)

$$M_{u_{i,5}} := \mathbf{if}\left(S_{i,2} = 2, \frac{1}{16}, \mathbf{if}\left(S_{i,2} = 12, \frac{1}{11}, \frac{1}{14}\right)\right) \cdot w_{u_i} \cdot \left(L_{n_{i,2}}\right)^2$$

Factored load moments:

$$M_u = \begin{bmatrix} 0 & 14.777 & 18.061 & 14.777 & 0 & 0 \\ 47.247 & 80.995 & 137.205 & 102.053 & 163.285 & 116.632 \\ 139.511 & 159.441 & 187.564 & 96.882 & 170.9 & 133.974 \end{bmatrix} \mathbf{kip} \cdot ft$$

Negative moments at factored load:

$$M_{neg} := \text{augment}\left(M_u^{\langle 0 \rangle}, \text{augment}\left(M_u^{\langle 2 \rangle}, M_u^{\langle 4 \rangle}\right)\right)$$

$$M_{neg} = \begin{bmatrix} 0 & 18.061 & 0 \\ 47.247 & 137.205 & 163.285 \\ 139.511 & 187.564 & 170.9 \end{bmatrix} \mathbf{kip} \cdot ft$$

Positive moments at factored load:

$$M_{pos} := \text{augment}\left(M_u^{\langle 1 \rangle}, \text{augment}\left(M_u^{\langle 3 \rangle}, M_u^{\langle 5 \rangle}\right)\right)$$

$$M_{pos} = \begin{bmatrix} 14.777 & 14.777 & 0 \\ 80.995 & 102.053 & 116.632 \\ 159.441 & 96.882 & 133.974 \end{bmatrix} kip \cdot ft$$

Effective depths from extreme compression fiber to centroid of tension reinforcement:

$$d_{top_{i,j}} := \left(h_i - cl - \frac{1}{2} \cdot d_{b_{xt_{i,j}}}\right)$$

$$d_{top} = \begin{bmatrix} 20 & 19.813 & 20 \\ 19.75 & 19.563 & 19.5 \\ 19.563 & 19.563 & 19.563 \end{bmatrix} in$$

$$d_{bot_{i,j}} := h_i - cl - \frac{1}{2} \cdot d_{b_{xb_{i,j}}}$$

$$d_{bot} = \begin{bmatrix} 19.688 & 19.688 & 20 \\ 19.563 & 19.5 & 19.436 \\ 19.365 & 19.5 & 19.365 \end{bmatrix} in$$

Formulas shown below are based on ACI 318 Sections 10.2 and 10.3, "Design assumptions" and "General principles and requirements," respectively.

Required negative moment reinforcement ratios

Negative moment reinforcement ratio required for flexure:

$$\rho 1_{neg_{i,j}} := \left(1 - \sqrt{1 - \frac{2 \cdot M_{neg_{i,j}}}{\phi_f \cdot b_{w_i} \cdot \left(d_{top_{i,j}}\right)^2 \cdot 0.85 \cdot f'_c}}\right) \cdot \left(\frac{0.85 \cdot f'_c}{f_y}\right)$$

$$\max\left(\rho 1_{neg}\right) = 0.861 \ 1\%$$

The larger of the negative moment reinforcement ratio required for flexure or ρ_{min}:

$$\rho_{neg_{i,j}} := \mathbf{if}\left(\rho 1_{neg_{i,j}} \geq \rho_{min}, \rho 1_{neg_{i,j}}, \mathbf{if}\left(\left(\rho 1_{neg_{i,j}} < \rho_{min}\right) \cdot \left(M_{neg_{i,j}} \neq 0 \cdot kip \cdot ft\right), \rho_{min}, 0\right)\right)$$

$$\rho_{neg} = \begin{bmatrix} 0 & 0.333 & 0 \\ 0.333 & 0.708 & 0.861 \\ 0.612 & 0.84 & 0.76 \end{bmatrix} 1\%$$

Required positive moment reinforcement ratios

Maximum stress block depths:

$$a_{max} := 0.75 \cdot \beta_1 \cdot \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_y} \cdot d_{bot}$$

$$a_{max} = \begin{bmatrix} 7.428 & 7.428 & 7.546 \\ 7.381 & 7.357 & 7.333 \\ 7.306 & 7.357 & 7.306 \end{bmatrix} in$$

Maximum reinforcement ratio ρT developed by the full width of the flange alone, and the maximum useable flange moment ϕMT:

$$\rho_{T_{i,j}} := \mathbf{if}\left(a_{max_{i,j}} \ge h_{f_i}, \frac{h_{f_i}}{d_{bot_{i,j}}} \cdot 0.85 \cdot \frac{f'_c}{f_y}, \rho_{max}\right)$$

$$\rho_T = ?\ 1\%$$

$$\phi M_{T_{i,j}} := K\left(\rho_{T_{i,j}}\right) \cdot \left(b_{f_i}\right) \cdot \left(d_{bot_{i,j}}\right)^2$$

$$\phi M_T = \begin{bmatrix} 680.324 & 680.324 & 692.516 \\ 1350.894 & 1346.018 & 1341.024 \\ 1551.518 & 1563.756 & 1551.518 \end{bmatrix} \mathbf{kip \cdot ft}$$

Beam flange reinforcement ratio:

$$\rho_{flange_{i,j}} := \mathbf{if}\left(M_{pos_{i,j}} < \phi M_{T_{i,j}}, \left(1 - \sqrt{1 - \frac{2 \cdot M_{pos_{i,j}}}{\phi_f \cdot b_{f_i} \cdot \left(d_{bot_{i,j}}\right)^2 \cdot 0.85 \cdot f'_c}}\right) \cdot \frac{0.85 \cdot f'_c}{f_y}, \rho_{T_{i,j}}\right)$$

$$\rho_{flange} = \begin{bmatrix} 0.025 & 0.025 & 0 \\ 0.07 & 0.088 & 0.102 \\ 0.121 & 0.072 & 0.101 \end{bmatrix} 1\%$$

Beam web reinforcement ratio:

$$\rho_{web_{i,j}} := \text{if}\left(M_{pos_{i,j}} > \phi M_{T_{i,j}}, \left(1 - \sqrt{1 - \frac{2 \cdot \left(M_{pos_{i,j}} - \left(\frac{b_{f_i} - b_{w_i}}{b_{f_i}}\right) \cdot \phi M_{T_i}\right)}{\phi_f \cdot b_{w_i} \cdot \left(d_{bot_{i,j}}\right)^2 \cdot 0.85 \cdot f'_c}}\right) \cdot \frac{0.85 \cdot f'_c}{f_y}, \rho_{flange_{i,j}}\right)$$

$$\rho_{web} = \begin{bmatrix} 0.025 & 0.025 & 0 \\ 0.07 & 0.088 & 0.102 \\ 0.121 & 0.072 & 0.101 \end{bmatrix} 1\%$$

Required positive moment reinforcement ratio for flexure expressed as a ratio of the beam web area, b_w x d_{bot}:

$$\rho 1_{pos_{i,j}} := \frac{b_{f_i} - b_{w_i}}{b_{w_i}} \cdot \rho_{flange_{i,j}} + \rho_{web_{i,j}}$$

$$\rho 1_{pos} = \begin{bmatrix} 0.071 & 0.071 & 0 \\ 0.394 & 0.501 & 0.577 \\ 0.682 & 0.407 & 0.572 \end{bmatrix} 1\%$$

Maximum permissible T-beam reinforcement ratio expressed as a ratio of the beam web area:

$$\rho_{T\_max_{i,j}} := \frac{b_{f_i} - b_{w_i}}{b_{w_i}} \cdot \rho_{T_{i,j}} + \rho_{max}$$

$$\rho_{T\_max}{}^{\mathrm{T}} = \begin{bmatrix} 4.513 & 8.221 & 8.252 \\ 4.513 & 8.241 & 8.209 \\ 4.476 & 8.261 & 8.252 \end{bmatrix} 1\%$$

The larger of the positive moment reinforcement ratio required for flexure or ρ_{min}:

$$\rho_{pos_{i,j}} := \textbf{if}\left(\rho1_{pos_{i,j}} \ge \rho_{min}, \rho1_{pos_{i,j}}, \textbf{if}\left(\left(\rho1_{pos_{i,j}} < \rho_{min}\right) \cdot \left(S_{i,j} \ne 0\right), \rho_{min}, 0\right)\right)$$

$$\rho_{pos} = \begin{bmatrix} 0.333 & 0.333 & 0 \\ 0.394 & 0.501 & 0.577 \\ 0.682 & 0.407 & 0.572 \end{bmatrix} 1\%$$

Required reinforcement areas for negative and positive moments:

$$A_{neg_{i,j}} := \rho_{neg_{i,j}} \cdot b_{w_i} \cdot d_{top_{i,j}}$$

$$\frac{A_{neg}}{4} = \begin{bmatrix} 0 & 0.198 & 0 \\ 0.198 & 0.416 & 0.503 \\ 0.419 & 0.575 & 0.52 \end{bmatrix} in^2$$

$$A_{neg} = \begin{bmatrix} 0 & 0.793 & 0 \\ 0.79 & 1.662 & 2.014 \\ 1.675 & 2.301 & 2.081 \end{bmatrix} in^2$$

$$A_{pos_{i,j}} := \rho_{pos_{i,j}} \cdot b_{w_i} \cdot d_{bot_{i,j}}$$

$$A_{pos} = \begin{bmatrix} 0.788 & 0.788 & 0 \\ 0.926 & 1.172 & 1.346 \\ 1.849 & 1.111 & 1.551 \end{bmatrix} in^2$$

Index numbers x1 of top bar sizes governed by either development or the required minimum of four top bars:

$$\begin{aligned} x1(i,j) := \; & n \leftarrow 3 \\ & \text{while } \left(\left(1 + 0.4 \cdot \left(d_{top_{i,j}} \ge 12 \cdot in\right)\right) \cdot l_{dt_n} \le \alpha_t \cdot L_{n_{i,j}}\right) \wedge \left(A_{b_n} \le \frac{A_{neg_{i,j}}}{4}\right) \\ & \quad n \leftarrow n + 1 \\ & \text{if } n = 3 \\ & \quad n \leftarrow 1 \\ & \text{return } n - 1 \end{aligned}$$

Maximum size of top bars (from No. 3 to No. 11) as limited by the requirement of a minimum of four bars or by the requirement of development lengths less than the distance to the first specified bar cutoff point α L_n:

Top bar sizes and individual bar areas:

$$TopBarSize_{i,j} := x1(i,j) \qquad TopBarSize = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 5 & 6 \\ 5 & 6 & 6 \end{bmatrix}$$

$$TopA1_{i,j} := A_{b_{TopBarSize_{i,j}}}$$

Notes

1) The distance from the face of support to the first bar cutoff point must be less than or equal to the value of $\alpha_t L_n$.
2) "TopBarsSize" will equal 0 when there is no moment, or when the specified α_t L_n is less than the development length of a No. 3 bar.

Required number of bars for development, and to meet minimum requirement of 4 top bars:

$$NumbTop_{i,j} := \text{ceil}\left(0.98 \cdot \frac{A_{neg_{i,j}}}{\left(TopA1_{i,j} = 0 \cdot in^2\right) \cdot in^2 + TopA1_{i,j}}\right)$$

$$NumbTop = \begin{bmatrix} 0 & 8 & 0 \\ 8 & 6 & 5 \\ 6 & 6 & 5 \end{bmatrix}$$

Top reinforcement area provided A_{s_top} compared to area required A_{neg}:

$$A_{s\_top} := \overrightarrow{NumbTop \cdot TopA1}$$

$$A_{s\_top} = \begin{bmatrix} 0 & 0.88 & 0 \\ 0.88 & 1.86 & 2.2 \\ 1.86 & 2.64 & 2.2 \end{bmatrix} in^2 \qquad A_{neg} = \begin{bmatrix} 0 & 0.793 & 0 \\ 0.79 & 1.662 & 2.014 \\ 1.675 & 2.301 & 2.081 \end{bmatrix} in^2$$

Top reinforcement ratio provided ρ_{top}:

$$\rho_{top_{i,j}} := \frac{A_{s\_top_{i,j}}}{b_{w_i} \cdot d_{top_{i,j}}}$$

$$\rho_{top} = \begin{bmatrix} 0 & 0.37 & 0 \\ 0.371 & 0.792 & 0.94 \\ 0.679 & 0.964 & 0.803 \end{bmatrix} 1\%$$

Top bar spacing, distributed over the lesser of b_f or 1/10 L_n (ACI 318 10.6.6):

$$Sp_{i,j} := \frac{\left(b_{f_i} < 0.10 \cdot L_{n_{i,j}}\right) \cdot b_{f_i} + \left(b_{f_i} > 0.10 \cdot L_{n_{i,j}}\right) \cdot 0.10 \cdot L_{n_{i,j}}}{\left(NumbTop_{i,j} = 0\right) \cdot 1 + NumbTop_{i,j}}$$

$$Sp = \begin{bmatrix} 12 & 1.5 & 0 \\ 3 & 4.8 & 5.76 \\ 4.8 & 4 & 5.28 \end{bmatrix} in$$

Maximum permissible spacing for top bars in a single layer as a function of standard bar size No (ACI 318 10.6.4):

$$f(No) := \overrightarrow{\left(\frac{z}{0.6 \cdot f_y}\right)^3 \cdot \left(2 \cdot \left(cl + 0.5 \cdot d_{b_{No}}\right)^2\right)}^{-1}$$

$$MaxSp_{i,j} := \text{if}\left(TopBarSize_{i,j} = 0, 0 \cdot in, f\left(TopBarSize_{i,j}\right)\right)$$

$$MaxSp = \begin{bmatrix} 0 & 12.003 & 0 \\ 12.003 & 10.74 & 10.182 \\ 10.74 & 10.182 & 10.182 \end{bmatrix} in$$

Minimum required spacing for top bars in a single layer (ACI 318 7.6.1):

$$MinSp_{i,j} := \text{if}\left(d_{b_{TopBarSize_{i,j}}} \le 1 \cdot in, d_{b_{TopBarSize_{i,j}}} + 1 \cdot in, 2 \cdot d_{b_{TopBarSize_{i,j}}}\right)$$

$$MinSp = \begin{bmatrix} 1 & 1.375 & 1 \\ 1.375 & 1.625 & 1.75 \\ 1.625 & 1.75 & 1.75 \end{bmatrix} in$$

Positive moment coefficients:

$$km_{i,j} := \text{if}\left(S_{i,j} = 12, \frac{1}{11}, \text{if}\left(\left(S_{i,j} = 1\right) + \left(S_{i,j} = 11\right), \frac{1}{14}, \text{if}\left(S_{i,j} = 2, \frac{1}{16}, 0\right)\right)\right)$$

$$km = \begin{bmatrix} 0.091 & 0.091 & 0 \\ 0.071 & 0.063 & 0.071 \\ 0.071 & 0.063 & 0.071 \end{bmatrix}$$

$$\frac{1}{11} = 0.091 \qquad \frac{1}{14} = 0.071 \qquad \frac{1}{16} = 0.063$$

Positive moment development length, with M_n conservatively assumed to be equal to factored load moment M_u divided by ϕ_f, and $l_a = d_{bot}$ (ACI 318, 12.11.3):

$$l_{d\_pos_{i,j}} := \text{if}\left(S_{i,j} = 0, 0 \cdot ft, \frac{\alpha_b \cdot \sqrt{8 \cdot km_{i,j}} \cdot L_{n_{i,j}}}{\phi_f} + d_{bot_{i,j}}\right)$$

$$l_{d\_pos} = \begin{bmatrix} 4.01 & 4.01 & 0 \\ 5.83 & 6.339 & 6.659 \\ 6.653 & 5.553 & 6.233 \end{bmatrix} ft$$

Index numbers x2 of bottom bar sizes governed by either development or the required minimum number of 2 bars:

$$x2(i,j) := \left\| \begin{array}{l} n \leftarrow 3 \\ \text{while } \left(l_{dt_n} \le l_{d\_pos_{i,j}}\right) \wedge \left(A_{b_n} \le \dfrac{A_{pos_{i,j}}}{2}\right) \\ \quad \left\| \, n \leftarrow n+1 \right. \\ \text{if } n = 3 \\ \quad \left\| \, n \leftarrow 1 \right. \\ \text{return } n-1 \end{array} \right.$$

Bottom bar sizes:

$$BotBarSize_{i,j} := x2(i,j)$$

$$BotBarSize = \begin{bmatrix} 5 & 5 & 0 \\ 6 & 6 & 7 \\ 8 & 6 & 7 \end{bmatrix}$$

Areas of individual bottom bars:

$$BotA1_{i,j} := A_{b_{BotBarSize_{i,j}}}$$

$$BotA1 = \begin{bmatrix} 0.31 & 0.31 & 0 \\ 0.44 & 0.44 & 0.6 \\ 0.79 & 0.44 & 0.6 \end{bmatrix} in^2$$

Required number of bars for development and to meet minimum requirement of 2 bottom bars:

$$NumbBot_{i,j} := \text{ceil}\left(0.98 \cdot \frac{A_{pos_{i,j}}}{\left(BotA1_{i,j} = 0 \cdot in^2\right) \cdot in^2 + A_{b_{BotBarSize_{i,j}}}}\right)$$

$$NumbBot = \begin{bmatrix} 3 & 3 & 0 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

Bottom reinforcement area provided A_{s_bot} compared to bottom reinforcement required A_{pos}:

$$\overrightarrow{NumbBot \cdot BotA1} = \begin{bmatrix} 0.93 & 0.93 & 0 \\ 1.32 & 1.32 & 1.8 \\ 2.37 & 1.32 & 1.8 \end{bmatrix} in^2$$

$$A_{s\_bot} := \overrightarrow{NumbBot \cdot BotA1}$$

$$A_{s\_bot} = \begin{bmatrix} 0.93 & 0.93 & 0 \\ 1.32 & 1.32 & 1.8 \\ 2.37 & 1.32 & 1.8 \end{bmatrix} in^2 \qquad A_{pos} = \begin{bmatrix} 0.788 & 0.788 & 0 \\ 0.926 & 1.172 & 1.346 \\ 1.849 & 1.111 & 1.551 \end{bmatrix} in^2$$

$$NB := NumbBot$$

Flange and web reinforcement ratios with actual reinforcement used:

$$\rho_{bot_{i,j}} := \frac{A_{s\_bot_{i,j}}}{b_{w_i} \cdot d_{bot_{i,j}}}$$

$$\rho_{bot} = \begin{bmatrix} 0.004 & 0.004 & 0 \\ 0.006 & 0.006 & 0.008 \\ 0.009 & 0.005 & 0.007 \end{bmatrix}$$

$$\rho'_{flange_{i,j}} := \text{if}\left(\frac{b_{w_i}}{b_{f_i}} \cdot \rho_{bot_{i,j}} < \rho_{T_{i,j}}, \frac{b_{w_i}}{b_{f_i}} \cdot \rho_{bot_{i,j}}, \rho_{T_i}\right)$$

$$\rho'_{flange} = \begin{bmatrix} 0.001 & 0.001 & 0 \\ 9.923 \cdot 10^{-4} & 9.955 \cdot 10^{-4} & 0.001 \\ 0.002 & 8.569 \cdot 10^{-4} & 0.001 \end{bmatrix}$$

$$\rho'_{web_{i,j}} := \text{if}\left(\rho'_{flange_{i,j}} < \rho_{T_{i,j}}, \rho'_{flange_{i,j}}, \frac{A_{s\_bot_{i,j}} - \rho_{T_i} \cdot \left(b_{f_i} - b_{w_i}\right) \cdot d_{bot_{i,j}}}{b_{w_i} \cdot d_{bot_{i,j}}}\right)$$

$$\rho'_{web} = \begin{bmatrix} 0.139 & 0.139 & 0 \\ 0.099 & 0.1 & 0.136 \\ 0.155 & 0.086 & 0.118 \end{bmatrix} 1\%$$

Minimum required b_w for bottom bars in single layer, assuming No. 4 stirrups and a minimum clear distance d_b or 1 inch between bars (ACI 318 7.6.1):

$$Minb_{w_{i,j}} := \left(3 \cdot in + NB_{i,j} \cdot d_{b_{BotBarSize_{i,j}}}\right) + \left(NB_{i,j} - \left(NB_{i,j} > 0\right) \cdot 1\right) \cdot \left(\left(d_{b_{BotBarSize_{i,j}}} > 1 \cdot in\right) \cdot \left(d_{b_{BotBarSize_{i,j}}} - 1 \cdot in\right.\right.$$

$$Minb_w = \begin{bmatrix} 6.875 & 6.875 & 3 \\ 7.25 & 7.25 & 7.625 \\ 8 & 7.25 & 7.625 \end{bmatrix} in$$

Maximum permissible b_w for bottom bars in single layer:

$$Maxb_{w_{i,j}} := \left(\frac{z}{0.6 \cdot f_y}\right)^3 \cdot \frac{\left(d_{b_{BotBarSize_{i,j}}} + 2 \cdot cl\right)^{-1}}{in}$$

$$Maxb_w = \begin{bmatrix} 24.837 & 24.837 & 28.718 \\ 24.183 & 24.183 & 23.563 \\ 22.974 & 24.183 & 23.563 \end{bmatrix} in$$

Effective depths using selected bar sizes:

$$d'_{top_{i,j}} := h_i - cl - \frac{1}{2} \cdot d_{b_{TopBarSize_{i,j}}}$$

$$d'_{top} = \begin{bmatrix} 20 & 19.813 & 20 \\ 19.813 & 19.688 & 19.625 \\ 19.688 & 19.625 & 19.625 \end{bmatrix} in$$

$$d'_{bot_{i,j}} := h_i - cl - \frac{1}{2} \cdot d_{b_{BotBarSize_{i,j}}}$$

$$d'_{bot} = \begin{bmatrix} 19.688 & 19.688 & 20 \\ 19.625 & 19.625 & 19.563 \\ 19.5 & 19.625 & 19.563 \end{bmatrix} \, in$$

Ratios of initial (assumed) effective depths to final (actual) effective depths:

$$RatioTop := \overrightarrow{\frac{d_{top}}{d'_{top}}} \qquad RatioBot := \overrightarrow{\frac{d_{bot}}{d'_{bot}}}$$

$$RatioTop = \begin{bmatrix} 1 & 1 & 1 \\ 0.997 & 0.994 & 0.994 \\ 0.994 & 0.997 & 0.997 \end{bmatrix} \qquad RatioBot = \begin{bmatrix} 1 & 1 & 1 \\ 0.997 & 0.994 & 0.994 \\ 0.993 & 0.994 & 0.99 \end{bmatrix}$$

Moment capacities:

$$\phi M_{neg_{i,j}} := K\left(\rho_{top_{i,j}}\right) \cdot b_{w_i} \cdot \left(d_{top_{i,j}}\right)^2$$

$$\phi M_{neg} = \begin{bmatrix} 0 & 75.895 & 0 \\ 75.648 & 152.291 & 177.035 \\ 153.926 & 212.636 & 179.942 \end{bmatrix} \, kip \cdot ft$$

$$\phi M_{pos_{i,j}} := \left(K\left(\rho'_{flange_{i,j}}\right) \cdot \left(b_{f_i} - b_{w_i}\right) + K\left(\rho'_{web_{i,j}}\right) \cdot b_{w_i}\right) \cdot \left(d_{bot_{i,j}}\right)^2$$

$$\phi M_{pos} = \begin{bmatrix} 81.382 & 81.382 & 0 \\ 115.184 & 114.813 & 155.54 \\ 203.705 & 114.954 & 155.228 \end{bmatrix} \, kip \cdot ft$$

Calculated factored load moments:

$$M_{pos} = \begin{bmatrix} 14.777 & 14.777 & 0 \\ 80.995 & 102.053 & 116.632 \\ 159.441 & 96.882 & 133.974 \end{bmatrix} \boldsymbol{kip \cdot ft}$$

$$M_{neg} = \begin{bmatrix} 0 & 18.061 & 0 \\ 47.247 & 137.205 & 163.285 \\ 139.511 & 187.564 & 170.9 \end{bmatrix} \boldsymbol{kip \cdot ft}$$

Summary

Specified compressive strength of concrete:	$f'_c = 4\ \boldsymbol{ksi}$
Specified yield strength of reinforcement:	$f_y = 60\ \boldsymbol{ksi}$
Unit weight of concrete:	$w_c = 145\ \boldsymbol{pcf}$
Unit weight of reinforced concrete:	$w_{rc} = 150\ \boldsymbol{pcf}$
Service live load per unit area:	$w_l = 200\ \boldsymbol{psf}$
Service dead load per unit area excluding slab weight:	$w_{sd} = 15\ \boldsymbol{psf}$
Specified ratio of the shortest top bar cutoff length to the clear span:	$\alpha_t := 0.15$
Shear strength reduction factor for lightweight concrete:	$k_v = 1$
Weight factor for increasing development and splice lengths for lightweight aggregate concrete:	$k_w = 1$

Clear concrete cover of reinforcement: $cl = 2\ in$

Crack control factor
(175 kip/in interior, 145 kip/in exterior): $z = 175\ \frac{kip}{in}$

Specified percentage of bottom bars continuing from the point of inflection into the support: $\alpha_b := 25\%$

Flange width b_f, beam web width b_w, overall thickness h, and slab thickness h_f:

$$b_f = \begin{bmatrix} 34 \\ 68 \\ 79 \end{bmatrix} in \qquad b_w = \begin{bmatrix} 12 \\ 12 \\ 14 \end{bmatrix} in \qquad h = \begin{bmatrix} 22 \\ 22 \\ 22 \end{bmatrix} in \qquad h_f = \begin{bmatrix} 4.5 \\ 4.5 \\ 4.5 \end{bmatrix} in$$

Clear span lengths and span types are displayed as three column matrices, with the number of rows equal to the number of continuous beam entered. Numbers defining the variable SpanType for each beam are 0 for a simple span, 1 for the end span of a continuous beam with spandrel beam exterior support, 11 for and end span with column support, 12 for an end span with an unrestrained exterior support, 2 for an interior span of a continuous beam, and 3 for a cantilevered beam (ACI 318, 9.5.2.1). The tributary slab width is displayed as a vector since it is the same for all spans of the continuous beam being designed.

$$L_n = \begin{bmatrix} 3.048 & 3.048 & 0 \\ 6.096 & 7.315 & 7.315 \\ 7.315 & 6.096 & 6.706 \end{bmatrix} m \qquad SpanType = \begin{bmatrix} 12 & 12 & 0 \\ 1 & 2 & 1 \\ 11 & 2 & 11 \end{bmatrix}$$

$$SW = \begin{bmatrix} 3 \\ 5.75 \\ 8 \end{bmatrix} ft$$

Ratios of initial (assumed) effective depths for positive and negative moments to final (actual) effective depths:

$$RatioTop = \begin{bmatrix} 1 & 1 & 1 \\ 0.997 & 0.994 & 0.994 \\ 0.994 & 0.997 & 0.997 \end{bmatrix}$$

$$RatioBot = \begin{bmatrix} 1 & 1 & 1 \\ 0.997 & 0.994 & 0.994 \\ 0.993 & 0.994 & 0.99 \end{bmatrix}$$

If the ratios of initial to final effective depths differ significantly, the initial bar size should be changed to the calculated bar size for as final check.

Top bar spacing distributed over the lesser of 1/10 L_n or b_f:

$$Sp = \begin{bmatrix} 12 & 1.5 & 0 \\ 3 & 4.8 & 5.8 \\ 4.8 & 4 & 5.3 \end{bmatrix} in$$

Minimum permissible top bar spacing for placement, and maximum permissible top bar spacing for crack control for top bars in a single layer:

$$MinSp = \begin{bmatrix} 1 & 1.4 & 1 \\ 1.4 & 1.6 & 1.8 \\ 1.6 & 1.8 & 1.8 \end{bmatrix} in \qquad MaxSp = \begin{bmatrix} 0 & 12 & 0 \\ 12 & 10.7 & 10.2 \\ 10.7 & 10.2 & 10.2 \end{bmatrix} in$$

If actual spacing is less than the minimum permissible, the beam thickness must be increased or a second layer of bars must be used. If actual spacing is greater than the maximum spacing, a smaller bar size must be used.

Minimum permissible beam widths b_w for bottom bar placement, and maximum permissible beam widths b_w for crack control:

$$Minb_w = \begin{bmatrix} 6.875 & 6.875 & 3 \\ 7.25 & 7.25 & 7.625 \\ 8 & 7.25 & 7.625 \end{bmatrix} in$$

$$Maxb_w = \begin{bmatrix} 24.837 & 24.837 & 28.718 \\ 24.183 & 24.183 & 23.563 \\ 22.974 & 24.183 & 23.563 \end{bmatrix} in$$

If actual width is less than the minimum permissible, the beam width must be increased or a second layer of bars must be used. If actual width is greater than the maximum width, a smaller bar size or a second layer of bars must be used.

Number and size of top bars (exterior supports in 1st column, 1st interior supports in 2nd column, 3rd supports in 3rd column):

$$NumbTop = \begin{bmatrix} 0 & 8 & 0 \\ 8 & 6 & 5 \\ 6 & 6 & 5 \end{bmatrix} \qquad TopBarSize = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 5 & 6 \\ 5 & 6 & 6 \end{bmatrix}$$

Number and size of bottom bars (exterior spans in 1st column, 2nd spans in 2nd column, 3rd spans in 3rd column):

$$NumbBot = \begin{bmatrix} 3 & 3 & 0 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} \qquad BotBarSize = \begin{bmatrix} 5 & 5 & 0 \\ 6 & 6 & 7 \\ 8 & 6 & 7 \end{bmatrix}$$

$$xt = \begin{bmatrix} 0 & 3 & 0 \\ 4 & 7 & 8 \\ 7 & 7 & 7 \end{bmatrix}$$

Areas of top (negative) and bottom (positive) reinforcement provided:

$$A_{s\_top} = \begin{bmatrix} 0 & 0.88 & 0 \\ 0.88 & 1.86 & 2.2 \\ 1.86 & 2.64 & 2.2 \end{bmatrix} in^2$$

$$A_{s\_bot} = \begin{bmatrix} 0.93 & 0.93 & 0 \\ 1.32 & 1.32 & 1.8 \\ 2.37 & 1.32 & 1.8 \end{bmatrix} in^2$$

Theoretical calculated areas of reinforcement required:

$$A_{neg} = \begin{bmatrix} 0 & 0.793 & 0 \\ 0.79 & 1.662 & 2.014 \\ 1.675 & 2.301 & 2.081 \end{bmatrix} \, in^2$$

$$A_{pos} = \begin{bmatrix} 0.788 & 0.788 & 0 \\ 0.926 & 1.172 & 1.346 \\ 1.849 & 1.111 & 1.551 \end{bmatrix} \, in^2$$