CHAPTER 3: Reinforced Concrete Slabs and Beams

3.4 Reinforced Concrete Beams - Size Selection

Description

This application calculates the spacing for shear reinforcement of a concrete beam supporting a uniformly distributed load. The application uses the strength design method of ACI 318-89, and calculates shear strength of the concrete using the simplified method of ACI Section 11-3, Eq. (11-3).

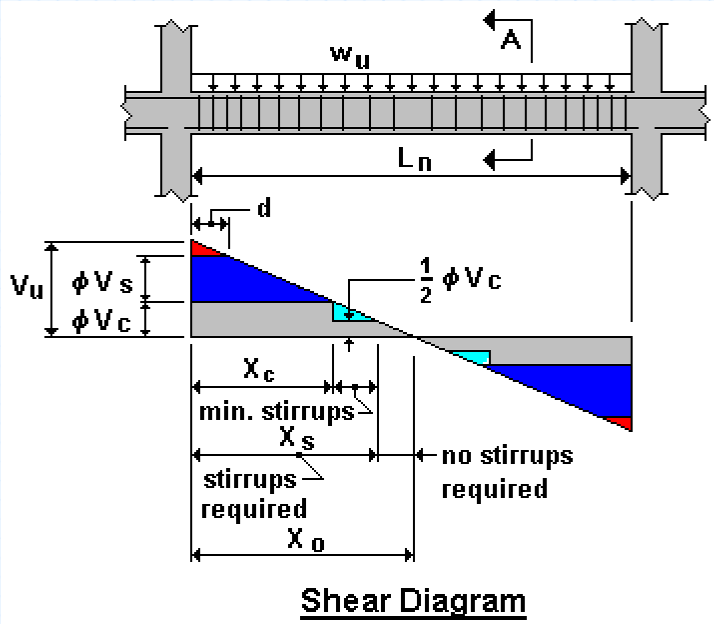
The required input includes strength of concrete, strength of reinforcement, unit weight of concrete, length of beam between the face of support and the point where shear passes through zero, uniformly distributed factored load, effective beam depth, beam web width, flange thickness and cross sectional area of the ties or stirrups.

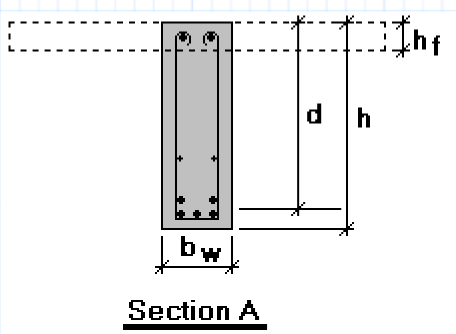
A summary of input and calculated values is shown on pages 13 and 14.

Reference: ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)

Input

Notation





Input Variables

Enter the following variables.

Clear span length: $L_n := 24\ ft$

Factor k, 1.15 for 1st interior support or 1 for all other supports: $k := 1.15$

Uniformly distributed factored load: $w_u := 6.5\ \frac{kip}{ft}$

Effective depth:	$d := 17.5\ in$
Beam web width:	$b_w := 12\ in$
Beam flange thickness:	$h_f := 4\ in$
Stirrup spacing factor:	$SpF := 1\ in$
Area of shear reinforcement within distance s:	$A_v := 0.22\ in^2$

Final stirrup spacing is calculated to a multiple of the specified stirrup spacing factor SpF. Since stirrup spacing may be governed by minimum spacing requirements, the smallest stirrup size should be tried first and increased only if the spacing is too small.

Computed Variables

The following variables are calculated in this document.

V_{uf}	factored shear force at face of support
V_{ud}	critical factored shear force, at distance d from face of support
V_c	nominal shear strength provided by concrete (ACI 318, 9.3.2.3, 11.3.1.1, Eq. (11-3), 11.5.4.3)
V_s	nominal shear provided by shear reinforcement (ACI 318, 9.3.2.3, 11.5.6, Eq. (11-17))
X_o	distance from face of support to the point where shear passes through zero
X_s	distance from face of support to the point where shear is equal to 1/2 the useable concrete shear strength
X_c	distance from face of support to the point where shear is equal to the useable concrete shear strength
s	spacing of shear reinforcement in direction parallel to longitudinal reinforcement
sp	spacing of shear reinforcement in direction parallel to longitudinal reinforcement from face of support to center of each stirrup

Material Properties

Enter values for f'_c, f_y, w_c, and k_v if different from that shown.

Specified compressive strength of concrete: $f'_c := 4\ ksi$

Specified yield strength of reinforcement (f_y may not exceed 60 ksi, ACI 318 11.5.2): $f_y := 60\ ksi$

Unit weight of concrete: $w_c := 145\ pcf$

Shear strength reduction factor for lightweight concrete k_v = 1 for normal weight, 0.75 for all-lightweight and 0.85 for sand-lightweight concrete (ACI 318, 11.2.1.2.): $k_v := 1$

Strength reduction factor for shear (ACI 318, 9.3.2.3): $\phi_v := 0.85$

Limit the value of f'_c for computing shear strength to 10 ksi (ACI 318, 11.1.2):

$$f'_c := \text{if}\left(f'_c > 10 \cdot ksi, 10 \cdot ksi, f'_c\right) \qquad f'_c = 4\ ksi$$

Limit the effective yield strength of shear reinforcement to 60 ksi as required by ACI 318, Section 11.5.2:

$$f_y := \text{if}\left(f_y > 60 \cdot ksi, 60 \cdot ksi, f_y\right) \qquad f_y = 60\ ksi$$

The following values are computed from the entered material properties.

Nominal concrete shear strength per unit area:

$$v_c := k_v \cdot 2 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi \qquad v_c = 126.491\ psi$$

If the member is subject to significant axial tension the concrete shear strength may be defined as 0. Shear reinforcement will then support the total shear (ACI 318 11.3.1.3).

Nominal shear strength per unit area for shear reinforcement spaced at d/4 or less:

$$v_{s\_max} := 8 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi \qquad v_{s\_max} = 505.964\ psi$$

Nominal shear strength per unit area for shear reinforcement spaced at greater than d/4 and less than d/2:

$$v_{s\_max2} := 4 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi \qquad v_{s\_max2} = 252.982\ psi$$

Calculations

Factored shear force at face of support:

$$V_{uf} := k \cdot \frac{w_u \cdot L_n}{2} = 89.7\ kip$$

Critical factored shear force, at distance d from face of support:

$$V_{ud} := V_{uf} - w_u \cdot d = 80.221\ kip$$

Maximum useable shear strength with shear reinforcement:

$$\phi V_{n\_max} := \phi_v \cdot \left(v_c + v_{s\_max}\right) \cdot b_w \cdot d = 112.893\ kip$$

Note: If ϕV_{n_max} is less than V_{u_d} the beam size must be increased.

If you do not increase the beam size the following expression will increase the entered value of the beam width to the minimum value necessary to support the shear force using the maximum permissible shear reinforcement.

$$b_w := \textbf{if}\left(\phi V_{n\_max} > V_{ud}, b_w, \frac{V_{ud}}{\phi V_{n\_max}} \cdot b_w\right) = 12\ in$$

$$\phi V_{n\_max} := \phi_v \cdot \left(v_c + v_{s\_max}\right) \cdot b_w \cdot d = 112.9\ kip$$

Useable shear strength provided by concrete (ACI 318, 11.3):

$$\phi V_c := \phi_v \cdot v_c \cdot b_w \cdot d = 22.579\ kip$$

Defined factor k_c for determining maximum permissible factored shear force before shear reinforcement is required (ACI 318, 11.5.5.1 (c)):

$$k_c := \textbf{if}\left(\left(d \le 10\ in\right) + \left(d \le b_w\right) + \left(d \le 2.5 \cdot h_f\right), 1, 0.5\right) = 0.5$$

Note: For wide shallow beams the full concrete shear strength of the beam is used in determining the maximum permissible factored shear force before shear reinforcement is required. For all other beams half the concrete shear strength is used. If the member is a slab or footing, or a joist as defined in ACI 318, Section 8.11, the user may define the value of k_c as equal to 1 at this point:

Maximum permissible factored shear force before shear reinforcement is required:

$$k_c \cdot \phi V_c = 11.289\ kip$$

Maximum spacing of shear reinforcement determined by maximum permissible spacing of d/2 (ACI 318, 11.5.4) or by minimum required shear reinforcement area (ACI 318, 11.5.5):

$$s'_{max} := \textbf{if}\left(V_{ud} \le k_c \cdot \phi V_c, 0 \cdot in, min\left(\begin{bmatrix} 0.5 \cdot d \\ \dfrac{A_v \cdot f_y}{50 \cdot psi \cdot b_w} \end{bmatrix}\right)\right) = 8.75\ in$$

Maximum spacing rounded off to a multiple of SpF:

$$s_{max} := \text{floor}\left(\frac{s'_{max}}{SpF}\right) \cdot SpF = 8\ in$$

Minimum shear capacity:

$$\phi V_{n\_min} := \phi V_c + \textbf{if}\left(s_{max} = 0 \cdot in, 0 \cdot kip, \frac{\phi_v \cdot A_v \cdot f_y \cdot d}{s_{max}}\right) = 47.1\ kip$$

Defined variable "Case":

Case = 0: no shear reinforcement required
Case = 1: minimum shear reinforcement required
Case = 2: greater than minimum shear reinforcement is required

$$Case := \textbf{if}\left(V_{ud} \le k_c \cdot \phi V_c, 0, \textbf{if}\left(V_{ud} \le \phi V_{n\_min}, 1, 2\right)\right)$$

$$Case = 2$$

Minimum stirrup spacing:

$$s'_{min} := \begin{array}{l} \text{if } Case = 0 \\ \quad \| 0\ in \\ \text{else if } Case = 1 \\ \quad \| s'_{max} \\ \text{else} \\ \quad \left\| min\left(\begin{bmatrix} \text{if}\left(V_{ud} \geq \phi V_{n\_max}, \frac{d}{4}, \frac{d}{2}\right) \\ \frac{\phi_v \cdot (A_v \cdot f_y \cdot d)}{V_{ud} - \phi V_c} \end{bmatrix}\right) \right. \end{array} = 3.406\ in$$

Minimum spacing rounded off to a multiple of SpF:

$$s_{min} := \text{floor}\left(\frac{s'_{min}}{SpF}\right) \cdot SpF = 3\ in$$

If the minimum spacing is too small or too large the shear reinforcement area should be revised.

First possible spacing larger than d/4 rounded off to multiple of SpF:

$$s1 := \text{floor}\left(\frac{d}{4 \cdot SpF}\right) \cdot SpF + SpF = 5\ in$$

Minimum useable spacing for stirrup spacings greater than d/4:

$$s'_{min2} := \text{if}\left(\max\left(\begin{bmatrix} s1 \\ \frac{A_v \cdot f_y \cdot d}{\phi_v \cdot v_{s\_max2} \cdot b_w \cdot d} \end{bmatrix}\right) > \frac{d}{2}, \frac{d}{2}, \max\left(\begin{bmatrix} s1 \\ \frac{A_v \cdot f_y \cdot d}{\phi_v \cdot v_{s\_max2} \cdot b_w \cdot d} \end{bmatrix}\right)\right) = 5.115\ in$$

Spacings larger than d/4 and less than the minimum spacing required to develop half the maximum shear reinforcement stress provide no useable additional shear strength.

Minimum useable spacing for stirrup spacings greater than d/4 rounded to a multiple of SpF:

$$s_{min2} := \text{floor}\left(\frac{s'_{min2}}{SpF}\right) \cdot SpF = 5\ in$$

Number of stirrups at a spacing less than or equal to d/4, greater than s_{min} and less than s_{max}:

$$n := \text{if}\left(\left(s_{min} > \frac{d}{4}\right) + \left(s_{min} = 0 \cdot in\right) + \left(s_{min} = s_{max}\right), 0, \frac{s1 - SpF - s_{min}}{SpF}\right) = 1$$

Minimum stirrup spacing s_{min} plus any additional stirrup spacings less than or equal to d/4:

$$j1 := 0 .. n \qquad s_{j1} := s_{min} + j1 \cdot SpF \qquad s = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \, in$$

Stirrup spacings from s_{min2} or s_{min} to s_{max}:

$$s2 := \text{if}\left(\left(s_{min2} \le s_{max}\right) \cdot \left(s_{min2} > s_{min}\right), s_{min2}, s_{min}\right) = 5 \, in$$

$$n1 := \text{if}\left(s_{min2} = s_{n}, n, n+1\right) = 2$$

$$j2 := n1 .. n1 + \text{if}\left(V_{ud} \ge k_c \cdot \phi V_c, \frac{s_{max} - s2}{SpF}, 0\right)$$

$$s_{j2} := \text{if}\left(V_{ud} \le k_c \cdot \phi V_c, 0 \cdot in, s2 + \text{if}\left(s2 = s_{max}, 0 \cdot in, (j2 - n - 1) \cdot SpF\right)\right)$$

Stirrup spacings from s_{min} to s_{max}:

$$j := 0 .. \text{last}(s)$$

$$s^{T} = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \, in$$

Useable strength of shear reinforcement at each stirrup spacing:

$$\phi V_{s_j} := \text{if}\left(V_{ud} \le k_c \cdot \phi V_c, 0 \cdot kip, \frac{\phi_v \cdot A_v \cdot f_y \cdot d}{s_j}\right)$$

$$\phi V_s^{T} = \begin{bmatrix} 65.5 & 49.1 & 39.3 & 32.7 & 28.1 & 24.5 \end{bmatrix} \, kip$$

Total useable shear strength at each stirrup spacing:

$$\phi V_n := \phi V_c + \phi V_s$$

$$\phi V_n^{T} = \begin{bmatrix} 88 & 71.7 & 61.8 & 55.3 & 50.6 & 47.1 \end{bmatrix} \, kip$$

Total useable shear capacity from face of support to the point of zero shear:

$u := \text{last}(s) + 2 \qquad u = 7$

$\phi V_{n_{u-1}} := k_c \cdot \phi V_c \qquad \phi V_{n_u} := k_c \cdot \phi V_c$

$\phi V_n^{T} = [88 \;\; 71.7 \;\; 61.8 \;\; 55.3 \;\; 50.6 \;\; 47.1 \;\; 11.3 \;\; 11.3] \; \textbf{\textit{kip}}$

$j3 := 0..u$

Distance from the left reaction to the point where shear force is less than the limiting concrete shear strength and shear reinforcement is not required (ACI 318, 11.5.1):

$$X_s := \textbf{if}\left(V_{ud} \le k_c \cdot \phi V_c, 0 \cdot \textit{ft}, \frac{V_{uf} - k_c \cdot \phi V_c}{w_u}\right) = 12.063 \; \textit{ft}$$

Distance from the left reaction to the point where shear is less than or equal to the useable concrete shear capacity:

$$X_c := \textbf{if}\left(V_{uf} \le \phi V_c, 0 \cdot \textit{ft}, \frac{V_{uf} - \phi V_c}{w_u}\right) = 10.326 \; \textit{ft}$$

Distance from face of support to the point where shear passes through 0:

$$X_o := \frac{V_{uf}}{w_u} = 13.8 \; \textit{ft}$$

Distances from the face of support to each point where stirrup spacing changes, and to the point of zero shear rounded up to the nearest foot:

$x_{s_0} := 0 \; \textit{in}$

$$x_{s_{j+1}} := x_{s_j} + s_j \cdot \text{ceil}\left(\frac{V_{uf} - \textbf{if}\left(s_j = s_{max}, k_c \cdot \phi V_c, \phi V_{n_{j+1}}\right)}{w_u \cdot s_j} - \frac{x_{s_j}}{s_j}\right)$$

$x_{s_u} := \text{ceil}\left(\frac{X_o}{\textit{ft}}\right) \cdot \textit{ft} \qquad x_{s_u} = 14 \; \textit{ft}$

$x_s^{T} = [0 \;\; 36 \;\; 52 \;\; 67 \;\; 73 \;\; 80 \;\; 152 \;\; 168] \; \textit{in}$

Length covered by each stirrup spacing:

$$L_{s_j} := x_{s_{j+1}} - x_{s_j}$$

$$L_s^{T} = [36\ \ 16\ \ 15\ \ 6\ \ 7\ \ 72]\ in$$

Number of stirrups at each spacing:

$$N1_j := \mathbf{if}\left(V_{ud} > k_c \cdot \phi V_c, \frac{L_{s_j}}{s_j}, 0\right)$$

$$N1^{T} = [12\ \ 4\ \ 3\ \ 1\ \ 1\ \ 9]$$

$$s^{T} = [3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8]\ in$$

Spacing of shear reinforcement from face of support:

$$sp_0 := \frac{s_0}{2} \qquad sp := \text{augment}\left(sp, s^{T}\right)^{T}$$

$$sp^{T} = [1.5\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8]\ in$$

Number of stirrups or ties at each spacing from face of support:

$$N_0 := \mathbf{if}\left(V_{ud} \le k_c \cdot \phi V_c, 0, 1\right)$$

$$N := \text{augment}\left(N, N1^{T}\right)^{T} \qquad N_1 := \mathbf{if}\left(N_1 \le 1, 0, N_1 - 1\right)$$

$$N^{T} = [1\ \ 11\ \ 4\ \ 3\ \ 1\ \ 1\ \ 9]$$

Factored shear force as a function of distance x from face of support:

$$V_u(x) := V_{uf} - w_u \cdot x$$

Range variable i and distances X_0 and X_1 for plotting shear diagram:

$$i := 0..1 \qquad x_0 := 0\ ft \qquad x_1 := X_o$$

Factored Shear Force V_u and Shear capacity ϕV_n in kips versus Distance from the Left Reaction to the Point of Zero Shear (in ft)

This chart is formatted for values of X_o less than 16 feet, and values of shear force up to 120 kips. The chart may be reformatted for any values outside these limits by changing the values of X_o and maximum shear force, which serve as "markers" defining the limits of the chart.

Summary

Input

Specified compressive strength of concrete:	$f'_c = 4\ ksi$
Unit weight of concrete:	$w_c = 145\ pcf$
Clear span length:	$L_n = 24\ ft$
Factor k, 1.15 for 1st interior support or 1 for all other supports:	$k = 1.15$
Effective depth:	$d = 17.5\ in$
Area of shear reinforcement within distance s:	$A_v = 0.22\ in^2$
Specified yield strength of reinforcement:	$f_y = 60\ ksi$
Shear strength reduction factor for lightweight concrete:	$k_v = 1$
Uniformly distributed factored load:	$w_u = 6.5\ \frac{kip}{ft}$
Strength reduction factor for shear:	$\phi_v = 0.85$
Beam web width:	$b_w = 12\ in$
Beam flange thickness:	$h_f = 4\ in$
Stirrup spacing factor:	$SpF = 1\ in$

Computed Variables

Critical shear force: $V_{ud} = 80.2$ ***kip***

Maximum useable shear strength with shear reinforcement: $\phi V_{n\_max} = 112.9$ ***kip***

Number of ties or stirrups at each spacing: $N^{T} = [1 \ \ 11 \ \ 4 \ \ 3 \ \ 1 \ \ 1 \ \ 9]$

Stirrup or tie spacing from face of support: $sp^{T} = [1.5 \ \ 3 \ \ 4 \ \ 5 \ \ 6 \ \ 7 \ \ 8]$ ***in***

Total number of ties: $\sum N = 30$

Maximum useable shear strength without shear reinforcement: $\phi V_c = 22.6$ ***kip***

Notes

1) For differing input variables some N values may be 0, which means that the particular spacing is not useable.
2) A minimum of 3 spaces will always be listed. The 1st spacing is half the minimum spacing, the 2nd is the minimum spacing, and the last spacing is the maximum. If the spacing is uniform throughout, the second and third spacing will be equal.