CHAPTER 4: Reinforced Concrete Columns

4.1 Rectangular Tied Columns

Description

Reinforced concrete columns are often designed with the help of computer programs or tables or graphs. The tables and graphs provide the usable axial load and moment capacities over the complete range (from maximum axial load capacity at the minimum moment to zero axial load and maximum moment capacity). A separate chart or table must be generated for each combination of concrete and reinforcement strengths. Even when the graphs or charts are prepared in a non-dimensionalized form there are still an infinite number of possible combinations of reinforcement, column sizes and material strengths.

This application computes the complete range of usable factored axial load and moment for a square or rectangular column of any size, reinforced with any number and size of reinforcing bars, and with any concrete and reinforcement strengths permissible under the requirements of ACI 318.

This application uses the actual dimensions of the column and the number and size of reinforcing bars to generate the column interaction chart and the tabular listings of usable factored load versus moment. All computations are made in accordance with the requirements of the Strength Design Method of ACI 318.

Required input includes the strength of the concrete and reinforcement, the column width and depth, the number of reinforcing bars, the bar size, and the clear cover.

A summary of input and computed values is shown on pages 9-11.

Reference:

ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)

Input

Input Variables

Width of member: $b := 12\ in$

Thickness of member: $h := 24\ in$

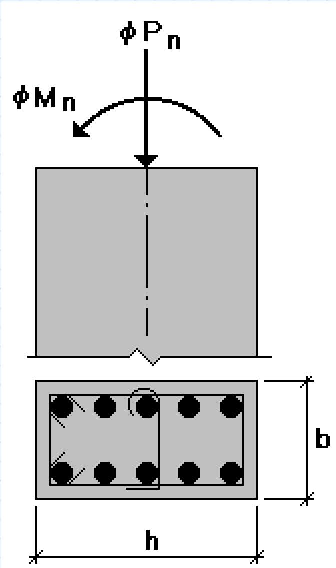
Bar size number (bar number, BarNo): $x := 10$

$BarNo := x$

Number of bars on "h" face, minimum of 2 bars: $N_h := 3$

Number of bars on "b" face, minimum 2 bars: $N_b := 2$

Clear reinforcement cover: $cl := 2\ in$



Computed Variables

A_g gross area of concrete

A_{st} total area of longitudinal reinforcement

ρ ratio of the total area of longitudinal reinforcement to the gross area of concrete

N_{total} total number of reinforcing bars

E_c modulus of elasticity of concrete for values of w_c between 90 pcf and 155 pcf (ACI 318, 8.5.1)

ε_y strain in reinforcement at yield stress f_y

β_1 factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3)

c_b distance to neutral axis at balanced tension/compression failure (ACI 318, Section 10.3.2)

ϕ	strength reduction factor (ACI 318, 9.3.2.2)
ϕP_n	usable axial load strength at given eccentricity
ϕM_n	usable moment strength with given axial load
P_o	nominal axial load strength at zero eccentricity
P_b	nominal axial load strength at balanced conditions (ACI 318, Section 10.3.2)
ϕP_{max}	maximum usable axial load (ACI 318, Eq. (10-2))

Material Properties & Constants

Enter f'_c, f_y and w_c if different from that shown.

Specified compressive strength of concrete:	$f'_c := 4\ ksi$
Specified yield strength of reinforcement:	$f_y := 60\ ksi$
Weight of concrete:	$w_c := 147\ pcf$
Modulus of elasticity of reinforcement (ACI 318, 8.5.2):	$E_s := 29000\ ksi$
Strain in concrete at compression failure (ACI 318, 10.3.2):	$\varepsilon_c := 0.003$

Reinforcing bar number designations, diameters and areas:

$$No := [0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18]^T$$

$$d_b := [0\ 0\ 0\ 0.375\ 0.5\ 0.625\ 0.75\ 0.875\ 1.00\ 1.128\ 1.27\ 1.41\ 0\ 0\ 1.693\ 0\ 0\ 0\ 2.257]^T\ in$$

$$A_b := [0\ 0\ 0\ 0.11\ 0.20\ 0.31\ 0.44\ 0.60\ 0.79\ 1.00\ 1.27\ 1.56\ 0\ 0\ 2.25\ 0\ 0\ 0\ 4.00]^T\ in^2$$

The following variables are computed from the entered material properties.

Modulus of elasticity of concrete for values of w_c between 90 pcf and 155 pcf (ACI 318, 8.5.1):

$$E_c := \left(\frac{w_c}{pcf}\right)^{1.5} \cdot 33 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi \qquad E_c = 3720\ ksi$$

Strain in reinforcement at yield stress f_y:

$$\varepsilon_y := \frac{f_y}{E_s} \qquad \varepsilon_y = 0.00207$$

β_1 factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3):

$$\beta_1 := \mathbf{if}\left(\left(f'_c \geq 4\ ksi\right) \cdot \left(f'_c \leq 8\ ksi\right), 0.85 - 0.05 \cdot \frac{f'_c - 4\ ksi}{ksi}, \mathbf{if}\left(\left(f'_c \leq 4\ ksi\right), 0.85, 0.65\right)\right)$$

$$\beta_1 = 0.85$$

Defined Units

$$pcf := lbf \cdot ft^{-3}$$

Calculations

Reinforcement yield strengths must be less than 80 ksi in accordance with ACI 318, Section 9.4.

$f_y := \text{if}\left(f_y > 80\ ksi, 80\ ksi, f_y\right)$ $\quad f_y = 60\ ksi$

Gross area of section Ag (in in²):

$A_g := b \cdot h$ $\quad A_g = 288\ in^2$

Total number of reinforcing bars:

$N_{total} := 2 \cdot \left(N_h + N_b - 2\right)$ $\quad N_{total} = 6$

Total area of longitudinal reinforcement A_{st} and reinforcement ratio ρ where ρ must range between a minimum of 1% and a maximum of 8%:

$A_{st} := N_{total} \cdot A_{b_x}$ $\quad A_{st} = 7.62\ in^2$

$\rho := \frac{A_{st}}{A_g}$ $\quad \rho = 2.646\%$

Distance from face of column to center of reinforcing bar:

$d' := cl + \frac{d_{b_x}}{2}$ $\quad d' = 2.635\ in$

Reinforcing bar locations from centroid of reinforcement on tension face:

$n := N_h - 1$ $\quad n = 2$

$i := 0 .. n$

$s_i := \left(\frac{h - 2 \cdot d'}{N_h - 1} \cdot i\right)$ $\quad s^T = [0\ \ 9.365\ \ 18.73]\ in$

Distance to neutral axis at balanced tension/compression failure:

$$c_b := \frac{\varepsilon_c \cdot (h - d')}{\varepsilon_y + \varepsilon_c} \qquad c_b = 12.645\ in$$

User specified lower value for neutral axis distance:

$$c_{min} := \frac{h}{5} \qquad c_{min} = 4.8\ in$$

Note ⇒ The value of c may be raised or lowered to show all required compression values of ϕP_n. A value of c = 0 inches will call for all tension values of ϕP_n.

User specified variables m, q and α:

$$m := 16 \qquad q := 2 \qquad \alpha := \frac{1}{2}$$

Note ⇒ Values of ϕP_n and ϕM_n are calculated for tension (or least compression) face reinforcement stresses at $-f_y$, and from $-\alpha f_y$ to $+f_y$ at intervals of f_y**.**$(1 + \alpha)/(m - 1)$, and for values of c from c_b to c_{min} at 1/q inch intervals. The values of m, q and α may be adjusted to show all required points on the interaction chart.

Stress range in tension face reinforcement for which values of ϕP_n and ϕM_n are calculated.

Lower stress: Stress interval: Limiting upper stress, $+f_y$:

$$-\alpha \cdot f_y = -30\ ksi \qquad \frac{f_y \cdot (1 + \alpha)}{m - 1} = 6\ ksi \qquad f_y = 60\ ksi$$

Range in values of c for which values of ϕP_n and ϕM_n are calculated:

Lower value of c = c_{min}: Intervals of c:

$$c_{min} = 4.8\ in \qquad \frac{1}{q} \cdot in = 0.5\ in$$

Limiting upper value of c = c_b:

Limiting upper value of c = c_b:

$$c_b = 12.645\ in$$

Range variable j defining all points for which values of ϕP_n and ϕM_n are calculated:

$$j := 0 .. m + q \cdot \left(\frac{c_b - c_{min}}{in} \right)$$

Total number of calculated values of ϕP_n and ϕM_n:

$$\text{ceil}\left(m + q \cdot \left(\frac{c_b - c_{min}}{in} \right) \right) = 32$$

Note ⇒ If this value is larger than 35, the last page of this document may need reformatting to provide ample space for the tabulated values of ϕP_n and ϕM_n.

Distance from extreme compression fiber to neutral axis:

$$c_j := \textbf{if}\left(j = 0, \frac{\varepsilon_c \cdot (h - d')}{\varepsilon_c - \varepsilon_y}, \textbf{if}\left((j \le m), \frac{\varepsilon_c \cdot (h - d')}{\varepsilon_c - \varepsilon_y \cdot \left(\alpha - \left(\frac{1 + \alpha}{m - 1} \cdot (j - 1) \right) \right)}, c_b - \frac{j - m}{q} \cdot in \right) \right)$$

Strains in tension face reinforcing bars :

$$\varepsilon_{s_{0,j}} := \frac{\varepsilon_c \cdot (h - d')}{c_j} - \varepsilon_c \qquad \varepsilon_{s_{i,j}} := \varepsilon_{s_{0,j}} - \frac{s_i}{h - d'} \cdot \left(\varepsilon_c + \varepsilon_{s_{0,j}} \right)$$

Stresses in all reinforcing bars:

$$f_{s_{i,j}} := \textbf{if}\left(\varepsilon_{s_{i,j}} \cdot E_s < -f_y, -f_y, \textbf{if}\left(\varepsilon_{s_{i,j}} \cdot E_s > f_y, f_y, \varepsilon_{s_{i,j}} \cdot E_s \right) \right)$$

Depth of rectangular stress block, a:

$$a_j := \textbf{if}\left(\beta_1 \cdot c_j \le h, \beta_1 \cdot c_j, h\right)$$

Reinforcement stresses reduced by 0.85.f'c for bars within rectangular stress block depth a:

$$f_{se_{i,j}} := \textbf{if}\left(\left(h - d' - s_i\right) < a_j, f_{s_{i,j}} + 0.85 \cdot f'_c, f_{s_{i,j}}\right)$$

Nominal axial load, Pn:

$$k := 1 .. n-1$$

$$P_{n_j} := 0.85 \cdot f'_c \cdot a_j \cdot b - A_{b_x} \cdot \left(N_b \cdot \left(f_{se_{0,j}} + f_{se_{n,j}}\right) + \left(N_h > 2\right) \cdot 2 \cdot \left(\sum_k f_{se_{k,j}}\right)\right)$$

Nominal moment, Mn:

$$M_{n_j} := 0.85 \cdot f'_c \cdot a_j \cdot b \cdot \left(h - d' - \frac{a_j}{2}\right) - A_{b_x} \cdot \left(N_b \cdot f_{se_{n,j}} \cdot \left(h - 2 \cdot d'\right) + \left(N_h > 2\right) \cdot 2 \cdot \left(\sum_k \left(f_{se_{k,j}} \cdot s_k\right)\right)\right) - P_{n_j} \cdot \left(\frac{h}{2} - d'\right)$$

Nominal axial load strength Po at zero eccentricity:

$$P_o := P_{n_0} \qquad P_o = \left(1.41 \cdot 10^3\right)\ kip$$

Nominal axial load Pb and moment Mb at balanced tension/compression failure:

$$P_b := P_{n_m} \qquad M_b := M_{n_m}$$

$$P_b = 441.143\ kip \qquad M_b = 473.266\ kip \cdot ft$$

Strength reduction factor ϕ (ACI 318, 9.3.2.2):

$$P_1 := \frac{0.10 \cdot f'_c \cdot A_g}{0.7} \qquad P_1 = 164.571\ kip$$

$$\gamma := \frac{h - 2 \cdot d'}{h} \qquad \gamma = 0.78$$

$$P_{2_j} := \frac{0.7 \cdot P_{n_j}}{0.10 \cdot f'_c \cdot A_g} \qquad P_{3_j} := 0.9 - \frac{0.7 \cdot P_{n_j}}{P_b} \cdot 0.2$$

$$\phi_j := \textbf{if}\left(P_{n_j} \ge P_1, 0.7, \textbf{if}\left(P_{n_j} \le 0 \cdot \textbf{\textit{kip}}, 0.9, \textbf{if}\left((\gamma \ge 0.7) \cdot \langle f_y \le 60 \cdot \textbf{\textit{ksi}} \rangle + \langle P_b \ge P_1 \rangle, 0.9 - P_{2_j} \cdot 0.2, P_{3_j}\right)\right)\right)$$

Design axial load and bending moment ϕPn and ϕMn:

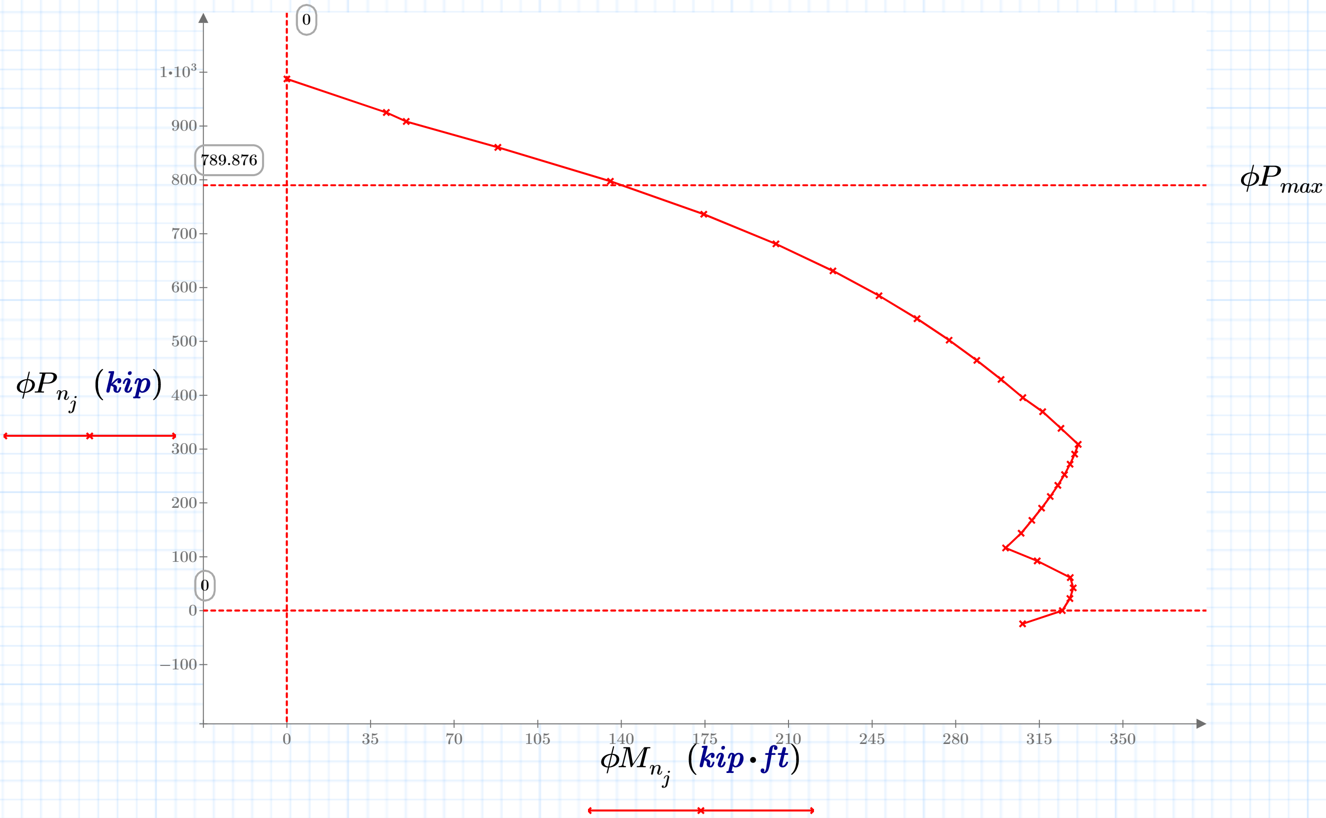
$$\phi P_{n_j} := \phi_j \cdot P_{n_j} \qquad \phi M_{n_j} := \phi_j \cdot M_{n_j}$$

Maximum usable axial load:

$$\phi P_{max} := \phi_0 \cdot 0.8 \cdot P_o \qquad \phi P_{max} = 789.876\ kip$$

Summary

Width of member:	$b = 12\ in$
Thickness of member:	$h = 24\ in$
Bar size number:	$BarNo = 10$
Number of bars on h face:	$N_h = 3$
Total number of bars:	$N_{total} = 6$
Reinforcement ratio:	$\rho = 2.646\ 1\%$
Clear reinforcement cover of longitudinal bars:	$cl = 2\ in$
Concrete strength:	$f'_c = 4\ ksi$
Reinforcement strength:	$f_y = 60\ ksi$
Unit weight of concrete:	$w_c = 147\ pcf$
Number of bars on b face:	$N_b = 2$
Maximum usable axial load:	$\phi P_{max} = 789.9\ kip$
Area of reinforcement:	$A_{st} = 7.62\ in^2$



Note

1) If curve does not cross ϕPn = 0, lower the value of cmin on page 4.
2) The chart may require reformatting for changes in input.

Reinforcing bar stress on "tension face":

$$f_{s_{0,j}} = \begin{bmatrix} -60 \\ -30 \\ -24 \\ -18 \\ -12 \\ -6 \\ \vdots \end{bmatrix} ksi$$

Neutral axis depth c:

$$c_j = \begin{bmatrix} 68.843 \\ 32.61 \\ 29.504 \\ 26.938 \\ 24.783 \\ 22.948 \\ \vdots \end{bmatrix} in$$

Usable axial load and moment:

$$\phi_j \cdot P_{n_j} = \begin{bmatrix} 987.344 \\ 925.088 \\ 908.428 \\ 860.287 \\ 797.355 \\ 736.129 \\ 681.05 \\ 630.929 \\ 584.863 \\ 542.158 \\ 502.269 \\ 464.762 \\ 429.289 \\ 395.566 \\ 369.405 \\ 338.522 \\ 308.8 \\ 290.619 \\ 271.918 \\ 252.628 \\ \vdots \end{bmatrix} \mathbf{kip}$$

$$\phi_j \cdot M_{n_j} = \begin{bmatrix} 0 \\ 41.627 \\ 49.953 \\ 88.314 \\ 135.44 \\ 174.537 \\ 204.738 \\ 228.608 \\ 247.896 \\ 263.825 \\ 277.262 \\ 288.834 \\ 299 \\ 308.103 \\ 316.396 \\ 324.075 \\ 331.286 \\ 329.805 \\ 327.894 \\ 325.552 \\ \vdots \end{bmatrix} \mathbf{kip \cdot ft}$$